A Modified Moore Method for Small Advanced Calculus Classes of Predominantly Future High School Teachers

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14th Annual Legacy of RL Moore Conference

Thursday, June 2, 2011

Mathematics Students at Western Oregon University

- WOU has about 6000 students
- 5-15 Mathematics graduates annually
- Typically 60-80% of those go on to teach high school
- Perhaps most importantly, very few of the graduates will choose research as a career

Typical Track at WOU for Future HS Teachers

1. Math Major
   - No mathematics pedagogy classes
2. Master of Arts in Teaching at WOU
   - All formal mathematics pedagogy is learned in this program

Mathematics Classes at WOU

- Calculus - largely traditional except that web-based homework is used in part
- Introduction to Proofs class freshman year
- Algebra (Group and Ring) junior year
- Advanced Calculus I & II and Senior Project I & II senior year
- (Also required: Linear Algebra, Mathematical Probability and Statistics, and four upper-division electives)

Nationally Recognized Teacher Preparation Program

- It is worth noting that the courses at WOU for future elementary and middle school teachers are exclusively guided-discovery/IBL courses; our Elementary and Middle School focus is nationally recognized for its success
- See for example, the Chronicle of Higher Education Article of July 11, 2010

Advanced Calculus I & II

- IBL courses
- Notes are at available at JIBLM, to be updated at the end of the month
- Development of calculus based on sequences, after Ken Ross, Jimmy Nanney
- Two-quarter class culminates in Fundamental Theorem of Calculus
- From year to year, students' career goals differ widely

Water

Bruce Lee: “Be Like Water."

- One technique does not fit all situations
- Adapt: Class to class, student to student

Each Student has One or More Optimal Ways to Think According to His/Her Career/Life Goals

- Future researchers might emphasize creativity
- Future teachers might emphasize multiple perspectives
- I feel that it is my job to assess these students somewhat differently, though fairly, based on their career goals - this is a delicate process
- Golden Rule V1.β: Treat individual students in a way that optimizes their education
- This is quite difficult at first, but becomes easier with experience

Minimize or Eliminate Talking Toward the Board

- We're not teaching the board, we're teaching the class; eye contact is crucial
- Leading questions are critical in the sense that the class be engaged
  - The presenter must detect incomplete understanding in any of his/her audience, as it is for any teacher
  - It is the responsibility of each audience member to always ask for clarification if s/he has incomplete understanding
  - Failure in either of these aspects can lead to a lower course average, by either reducing the class participation grade or diminishing available extra credit points

Assessment of Students' Work

- For the proofs presented to the class, students are assessed based on degree of difficulty/cleverness of the proof and understanding of the proof as evidenced by their answers to questions posed by classmates
- My scale is 5-12 points per proof
- Presenters are expected to ask leading questions of their audience
- I reserve the right to ask the presenter to ask questions of specific audience members, and to ask audience members questions myself

Components of Students' Grades

- Presented proofs and/or solutions 48% - this percentage is based on the total proofs points, scaled by the median total
- Quiz average 10%
- Midterm 1 score 12%
- Midterm 2 score 12%
- Final exam score 18%
- Informally assessed on pedagogy - $\pm$ - This will likely be formalized using a rubric similar to that on my webpage starting in Fall 2011

Primary Goals and Parameters

- Students must get 100% on all quizzes; these are strictly definitions and theorem statements; unlimited verbal or written retakes are permitted in my office within one week
- Six complete, precise proofs are expected of each student in the first term; eight in the second term
- Notes are allowed at the board for the first presentation, no notes in any subsequent proof or solution; I much prefer that they maintain a "scaffolding" and trust their mathematical abilities to fill in the details

"It's an Explanation, not a Calculation" (After Colleague Mike Ward)

- Memorizing full proofs leads to a false sense of security, but the outline may be memorized
- Strong emphasis is placed on the less-obvious aspects of the proofs
- Students are expected to focus on and provide particularly transparent exposition for these components
- These are the teachable moments for the class

The Denouement and Role of the Given Proof

- Presenters should summarize the proof or solution upon completion
- Equally important, the presenter should make an effort to place the theorem or problem properly in the scope of previously covered material, or as a precursor of upcoming topics
- For example, students should provide a foreshadowing of the role that the Bolzano-Weierstrass Theorem plays in the proof of Extreme Value Theorem; students are continually made aware of the fabric of the course
- This latter component often requires a bit of help, but it helps that students have the entire course packet at their disposal from the first day of class

How Much Help Should We Give?

- In a class of future teachers, I don't mind if students show me their proofs in my office, but it must be presented on the whiteboard, and I never give them more than leading questions
- Those leading questions press students to generate their own knowledge, but once their proof is written we turn to the pedagogy and presentational aspects

Students Use LaTeX

- Students learn LaTeXin my classes at various points beginning in the sophomore year
- In Advanced Calculus they are required to LaTeXtheir proofs and submit them through an internal course management system (Moodle), but solutions are not available on the web
- This not only provides further ownership of their work, but more importantly, it permits us to generate a course document which is effectively a textbook
- In this manner, students learn a typesetting language which they may use to produce professional-looking documents for use in their own teaching

Stagefright

- Even future teachers can go into shock (figuratively speaking) at the board in front of their classmates
- This despite having a fair amount experience presenting in WOU Mathematics classes prior to Advanced Calculus
- I believe this is often the result of the unfamiliarity with and abstractness of the material
- In these cases, I may let a student practice once or twice in my office, with another (friendly) Advanced Calculus student in attendance

Mitigating Ego and Diffidence Through Respect

- Students' skills are exposed in front of the classroom; there is no place to hide
- Dually, audience members enjoy some measure of anonymity as they analyze their classmates' work
- IBL Advanced Calculus must be conducted in an environment conducive to a respectful exchange of ideas, in the absence of disrespect or effrontery
- To this end the class as a whole must agree that personalities take a back seat to the content

Does it Have an Effect?

- In the last three years, since the first of the Moore Method Advanced Calculus for HS teachers, several in-service teachers have reported successful implementation of IBL methods in their HS classrooms (geometry, trigonometry)
- This year, a student majoring in Mathematics Education did her senior project on IBL methods used in a class for future teachers at WOU
- Most importantly, a deluge of anecdotes indicates that the students who have successfully completed pedagogically-oriented Advanced Calculus I & II are *better at separating conjecture from reasonable conclusions in all walks of life*

How much of an effect?

- Imagine four pictures of happy Advanced Calculus classes - but I didn't get their permissions to post all of their photos on the Web
- Many of these students have gone on to teach mathematics classes of their own
- And many more will do so

A compound effect!

- Nearly all students who successfully navigate our IBL Advanced Calculus and who go on to a teaching career can propagate the ideals of independence, leadership, and self-confidence to their own students
- This roughly exponentiates the number of critical thinkers in our populace
- Several past graduates who are not in school are in the process of proving *all* of the theorems in the 2008-09 course packet, just for fun
- And so on...

A Last Item: Linear Algebra II

- During Spring Term 2011, I offered what is best termed a pseudo-IBL course in Linear Algebra II
- The topics included theorems about matrix norms, the SVD, orthogonalization, Hermitian and normal matrices, and norms and inner products on function spaces
- Students' grades were largely determined by class presentations
- The course was essentially a bridge course for sophomores who'd already had an Introduction to Proofs class
- Eventually, this will result in a definition - theorem - exercise guided discovery list, incorporating MATLAB

Why abdicate my responsibility to impart mathematical wisdom through flawlessly thought-out lectures?

- The earlier that future teachers become comfortable in front of the classroom, the better
- Most importantly - in a classroom environment where questions are effectively mandatory, there is no hiding weakness
- This compels a level of preparedness which serves them well whether in teaching, research or any other career where solid reasoning is critical

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