# MTH 344 Exam 2 "Proof" Review

One aspect of successful test taking is to be able to quickly do proofs in a timed, test-like environment. Below are examples of proofs of reasonable length that may be asked on a 50 minute exam. My expectation would be that you could complete them start to finish in a rather short amount of time. This requires you have some idea of what a successful strategy would be and an idea of which Theorems, Proofs, or Definitions would be applicable. This only comes with practice. I recommend you do the problems below over and over until you fully understand each step and justification and could reproduce the proof without notes in a timely manner. Although you cannot practice proofs that will be exactly like the ones on your test, knowing a variety of proofs well helps with your recall and identification of key parts of many proofs.

1. Prove or disprove $D_4 \approx S_4$.

2. Prove or disprove $D_{12} \approx S_4$.

3. Prove Property 3 of the Chapter 7 Lemma: Let $H$ be a subgroup of $G$, and let $a$ and $b$ belong to $G$. Then $aH = bH$ if and only if $a \in bH$. You may use Properties 1 and/or 2 in your proof, but no other properties from the Lemma.

4. Let $G$ be a finite group and $a \in G$ a fixed element of $G$. Prove that the mapping $\phi : G \to G$ given by $\phi(x) = axa^{-1}$ is an isomorphism from $G$ onto itself. (Remember $a$ stays fixed for every $x$.)

5. Suppose $G$ is a group and $K$ and $H$ are subgroups of $G$. If $|K| = 15$ and $|H| = 6$, prove $K \cap H$ must be cyclic.

6. Given that $G$ is a finite abelian group of order 55 and $G$ has an element $a$ of order 5 and an element $b$ of order 11. Prove or disprove that $G$ must be cyclic.

7. Prove that $A_4$, the set of even permutations in $S_4$ is a subgroup of $S_4$.

8. Prove Property 4 of Theorem 6.2: Suppose $\phi : G \to \overline{G}$ is a group isomorphism. Prove that $G = \langle a \rangle$ if and only if $\overline{G} = \langle \phi(a) \rangle$. Do not cite Theorem 6.3, but you may use parts 1-3 of Theorem 6.2.