1. Let $\beta = (12346)(1345)(2643)$

   (a) Write β in array notation.

   $$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 1 & 3 & 6 & 2 & 5 \end{bmatrix}$$

   (b) What is the order of β?

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   (c) Is β an even or odd permutation?

   even

   (d) What is β^{-1}?

   $\beta^{-1} = (12564)$

   (e) What is β^{101}?

   $\beta^{101} = (\beta^5)^{20} \cdot \beta = \beta$

   (f) What is the order of $\alpha = (134)(5678)$?

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2. Let H be the subgroup generated by $\langle 5 \rangle$. Write down all of the left cosets of H in Z_{20}.

   $\langle 5 \rangle = \{0, 5, 10, 15, \}$
   $1 + \langle 5 \rangle = \{1, 6, 11, 16, \}$
   $2 + \langle 5 \rangle = \{2, 7, 12, 17, \}$
   $3 + \langle 5 \rangle = \{3, 8, 13, 18, \}$
   $4 + \langle 5 \rangle = \{4, 9, 14, 19, \}$

3. Let G be a cyclic group of order 20, $G = \langle a \rangle$.

   (a) How many right cosets does the subgroup $\langle a^4 \rangle$ have in G?

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   (b) List them.

   $$\langle a^4 \rangle = \{e, a^4, a^8, a^{12}, a^{16}\}$$
   $$a\langle a^4 \rangle = \{a, a^5, a^9, a^{13}, a^{17}\}$$
   $$a^2\langle a^4 \rangle = \{a^2, a^6, a^{10}, a^{14}, a^{18}\}$$
   $$a^3\langle a^4 \rangle = \{a^3, a^7, a^{11}, a^{15}, a^{19}\}$$

4. How many elements of order 4 are there in S_7?

   $210 + 630 = 840$

5. How many elements of order 2 are there in S_7?

   $21 + 105 + 105 = 231$

6. Let G be a group of order 30.

   (a) What are the possible number of elements of order 5 in G?

       $0, 4, 8, 12, 16, 20, 24, 28$ (using Cor. to Thm 4.4)

   (b) Suppose further that G is cyclic. Does your answer change? If so, how?

       Yes, if G is cyclic then by Thm. 4.4 there are exactly $\phi(5) = 4$ elements of order 5.

7. Find 8^{242} (mod 13) without using your calculator program. Show work.

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8. Suppose $\phi : Z_{50} \to Z_{50}$ with $\phi(11) = 13$. Find a formula for $\phi(x)$. It is OK to use your calculator program if necessary, but write "calculator" next to the computation.

   $\phi(x) = 33x$ (mod 50)

9. Go back to each problem and write down any Theorems, Corollaries or Lemmas you used when solving them. Would they always apply to questions like this or only for particular types of questions like this?

10. When studying for the test, redo these problems until you can do them quickly without looking. Go back and write new problems similar to these (change numbers, groups, etc.) and do those. Be sure if you see a problem similar to this on the exam, you know exactly what to do.