MTH 344 **Assumptions** [1]

Retain this sheet for reference.

Notation $\mathbb{Z}$ is the set of all integers. $\mathbb{Q}$ is the set of all rational numbers. $\mathbb{R}$ is the set of all real numbers. $\mathbb{C}$ is the set of all complex numbers.

Putting a $*$ after any set of numbers means the set of all *nonzero* elements of that set. For example, $\mathbb{R}^*$ is the set of all nonzero real numbers.

Putting a $+$ after any of real numbers means the set of all *positive* elements of that set. For example, $\mathbb{R}^+$ is the set of all positive real numbers. Recall that the number 0 is neither positive nor negative.

Assumptions Here are the assumptions we take as our starting place in the course:

1. $+$, $-$ and $\times$ are defined and well-defined on any subset of $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$ and $\mathbb{C}$.
2. $+$, $-$ and $\times$ are closed on $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$ and $\mathbb{C}$ (Of course, $+$, $-$ and $\times$ are not necessarily closed on a subset of those sets. It depends on the subset.)
3. $\div$ is defined and well-defined on any subset of $\mathbb{Z}^*$, $\mathbb{Q}^*$, $\mathbb{R}^*$ and $\mathbb{C}^*$.
4. $\times$ and $\div$ are closed on $\mathbb{Q}^*$, $\mathbb{R}^*$ and $\mathbb{C}^*$ ($\times$ and $\div$ are not necessarily closed on a subset of those sets. It depends on the subset.)
5. $+$, $\times$ and $\div$ are closed on $\mathbb{Q}^+$ and $\mathbb{R}^+$ ($+$, $\times$ and $\div$ are not necessarily closed on a subsets of those sets. It depends on the subset.)
6. $+$ and $\times$ are associative and commutative on any subset of $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$ and $\mathbb{C}$.
7. The distributive property holds on any subset of $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$ and $\mathbb{C}$.

   Additional Assumptions
8. $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$ and $\mathbb{C}$ are Abelian groups under $+$.
9. $\mathbb{Q}^*$, $\mathbb{R}^*$ and $\mathbb{C}^*$ are Abelian groups under $\times$.
10. $+_n$ and $\times_n$ are associative binary operations on $\mathbb{Z}_n$.
11. $\mathbb{Z}_n$ is an Abelian group under $+_n$.
12. If $a, b \in \mathbb{C}$ and $ab = 0$, then either $a = 0$ or $b = 0$. (Note that this assumption applies automatically to any subset of $\mathbb{C}$, including $\mathbb{Q}$ and $\mathbb{R}$ and $\mathbb{Z}$.)
13. Arithmetic facts (like $2 + 3 = 5$ and–for now–rules of signs, for example)
14. Basic facts about the familiar functions of college algebra and trig, ln, sin, etc. including the assumption that they *are* functions.
15. The quadratic formula.
16. $\sqrt{2}$ is not an element of $\mathbb{Q}$. In general, $\sqrt{p}$ is not an element of $\mathbb{Q}$ when p is a prime number.
17. For every $a \in \mathbb{C}$, $\sqrt{a} \in \mathbb{C}$. For every $a \in \mathbb{R}^+$, $\sqrt{a} \in \mathbb{R}^+$.

[1] Shamelessly stolen from Mike Ward