Exam 1 is an in class exam to be given on Friday, October 30th.

- Exam 1 covers material through Chapter 4 (not including Thm 4.4).
- I will provide a copy of the Assumed Background Knowledge and Chapter 4 Theorem list.
- You may have one $3x5$ card (both sides) of notes. You may have no more than 2 worked out problems or theorem proofs on your note card. You will turn in your note card with your exam.
- If you need it, I will provide a copy of the back cover of your book (the Cayley tables for D_4 and D_3).
- Suggestions for study:
  - Review the theorems and proofs from the class and book. Work out the proofs on your own, then check with the book or notes.
  - Redo (not just look at) assigned homework problems.
  - Do additional problems from the text.
  - Work out the practice problems below.
  - Make a notecard.
- **Disclaimer:** The set of problems below is not meant to be an exhaustive list of the type of problems that may be on the exam, it is simply for your practice.

1. Circle TRUE or FALSE. Note to be TRUE, it must ALWAYS be true, no exceptions. If something is false, you should be able to come up with a counterexample.
   (a) TRUE FALSE Any subgroup of a cyclic group is cyclic.
   (b) TRUE FALSE $\mid D_n \mid = n$
   (c) TRUE FALSE The intersection of two subgroups is a subgroup.
   (d) TRUE FALSE The union of two subgroups is a subgroup.
   (e) TRUE FALSE A nonempty subset of a group that is closed is a subgroup.
   (f) TRUE FALSE If g is a group element and $g^n = e$, then $\mid g \mid = n$.
   (g) TRUE FALSE Z_n is a subgroup of Z
   (h) TRUE FALSE The set Q^* forms a group under the operation of addition.
   (i) TRUE FALSE The set Q^* forms a group under the operation of multiplication.

2. Let $S = \{(a, b)\text{where } a \in Q^* \text{and } b \in Q\}$, and define a new operation $\#$ as follows: $(a, b)\#(c, d) = (ac + d, b)$.
   (a) What is $(4, 5)\#(-2, 3)$?
   (b) Is $\#$ an operation on S (defined, well-defined, closed)? Carefully check each property and show which hold and which fail.

(c) Is # commutative on S? (Prove or give a counterexample.)

(d) Does S have an identity element under #? Explain. (If yes, what is the identity?)

3. Prove that a group of order 4 must be Abelian.

4. Let $G = \{a + b\sqrt{2} \mid$ a and b are rational numbers not both $0\}$. Prove that G is a group under ordinary multiplication.

5. Carefully prove the socks and shoes theorem (justify each step): Let a, b be elements of a group G, then $(ab)^{-1} = b^{-1}a^{-1}$.

6. Find a cyclic subgroup of order 4 in U(40).

7. Prove that if $f : A \to B$ and $g : B \to C$ are $1-1$ functions, then so is $(f \circ g)(x)$.

8. Find $C(D)$ in D_4.

9. What are all the subgroups of D_3?

10. For any elements a, and b from a group and any integer n, prove that $(a^{-1}ba)^n = a^{-1}b^n a$.

11. If a and b are distinct group elements, prove that either $a^2 \neq b^2$ or $a^3 \neq b^3$.

12. Prove that a group of order 5 must be cyclic.

13. Let G be a group and let H be a subgroup of G. For any *fixed* x in G, define $xHx^{-1} = \{xhx^{-1} \mid h \in H\}$. Prove that xHX^{-1} is a subgroup of G.

14. How many different subgroups does Z_{20} have? Write them all down.

15. Let G be the cyclic group $U(25)$ (under the operation of multiplication modulo 25).

    (a) Given that 2 is a generator of G, i.e., $\langle 2 \rangle = U(25)$, find all generators of $U(25)$.

    (b) let H be the subgroup of $U(25)$ generated by 2^2, $H = \langle 2^2 \rangle$. Find all other elements of $U(25)$ that generate H.

16. For more problems see the *Supplementary Exercises for Chapter 1-4* after Chapter 4 in your book (p.95). Note, we did not cover all of Chapter 1. Odd answers are in the back of the book.