I. In this problem, we are working in the group D_4 with the operation of composition.

(a) Consider $\langle V \rangle$, the subgroup generated by V, in D_4. Calculate $\langle V \rangle$, which means list the elements, inside { }, of course.

(b) Calculate the left coset $D \circ \langle V \rangle = \{D \circ y : y \in \langle V \rangle\}$. Write your answer here and also in the appropriate space in the table below.

(c) Repeat the above with D replaced by each of the other elements of D_4. (That's 7 more sets to compute. Sorry about that, but it is a necessary evil. Divide up the work.) Write your answers in the table below.

(d) Calculate the right coset $\langle V \rangle \circ D = \{y \circ D : y \in \langle V \rangle\}$. (Note the D and y have changed places.) Then, as before, repeat that with D replaced by each of the other elements of D_4.Write your answers in the table below.

Left Cosets		*Right Cosets*	
$D \circ \langle V \rangle$	$= \{ \quad \}$	$\langle V \rangle \circ D$	$= \{ \quad \}$
$R_0 \circ \langle V \rangle$	$= \{ \quad \}$	$\langle V \rangle \circ R_0$	$= \{ \quad \}$
$R_{90} \circ \langle V \rangle$	$= \{ \quad \}$	$\langle V \rangle \circ R_{90}$	$= \{ \quad \}$
$R_{180} \circ \langle V \rangle$	$= \{ \quad \}$	$\langle V \rangle \circ R_{180}$	$= \{ \quad \}$
$R_{270} \circ \langle V \rangle$	$= \{ \quad \}$	$\langle V \rangle \circ R_{270}$	$= \{ \quad \}$
$H \circ \langle V \rangle$	$= \{ \quad \}$	$\langle V \rangle \circ H$	$= \{ \quad \}$
$V \circ \langle V \rangle$	$= \{ \quad \}$	$\langle V \rangle \circ V$	$= \{ \quad \}$
$D' \circ \langle V \rangle$	$= \{ \quad \}$	$\langle V \rangle \circ D'$	$= \{ \quad \}$

II. Again, we are working in the group D_4 with the operation of composition. Follow the directions of (I) except use the subgroup $K := \{R_0, R_{180}, V, H\}$ of D_4 in place of $\langle V \rangle$ *wherever* $\langle V \rangle$ *occurs* in the instructions. Part (a) has no analog here so you can skip that. Record answers in the table below.

Left Cosets		*Right Cosets*	
$R_0 \circ K$	$= \{ \quad \}$	$K \circ R_0$	$= \{ \quad \}$
$R_{90} \circ K$	$= \{ \quad \}$	$K \circ R_{90}$	$= \{ \quad \}$
$R_{180} \circ K$	$= \{ \quad \}$	$K \circ R_{180}$	$= \{ \quad \}$
$R_{270} \circ K$	$= \{ \quad \}$	$K \circ R_{270}$	$= \{ \quad \}$
$H \circ K$	$= \{ \quad \}$	$K \circ H$	$= \{ \quad \}$
$V \circ K$	$= \{ \quad \}$	$K \circ V$	$= \{ \quad \}$
$D \circ K$	$= \{ \quad \}$	$K \circ D$	$= \{ \quad \}$
$D' \circ K$	$= \{ \quad \}$	$K \circ D'$	$= \{ \quad \}$

III. In this problem we are working in the permutation group A_4 (a subgroup of S_4) with the operation of composition.

(a) List the elements of the group A_4. There are 12 of them. Ignore the table in the book.

(b) Consider $\langle(1\ 2\ 3)\rangle$, the subgroup generated by $(1\ 2\ 3)$. Calculate $\langle(1\ 2\ 3)\rangle$, which means list the elements, inside { }, of course.

(c) Calculate the left coset $(1\ 2)(3\ 4) \circ \langle(1\ 2\ 3)\rangle = \{(1\ 2)(3\ 4) \circ y : y \in \langle(1\ 2\ 3)\rangle\}$.

(d) Repeat the above with $(1\ 2)(3\ 4)$ replaced by each of the other elements of A_4. (That's 11 more sets to compute. Sorry about that, but it is a necessary evil. Divide up the work.) Record your answers in the table on the next page.

(e) Calculate the right coset $\langle(1\ 2\ 3)\rangle \circ (1\ 2)(3\ 4) = \{y \circ (1\ 2)(3\ 4) : y \in \langle(1\ 2\ 3)\rangle\}$. (Note the $(1\ 2)(3\ 4)$ and y have changed places.) Then, as before, repeat that with $(1\ 2)(3\ 4)$ replaced by each of the other elements of A_4. Record your answers in the table on the next page.

Left Cosets		*Right Cosets*	
$\epsilon\langle(123)\rangle$	$= \{ \quad \}$	$\langle(123)\rangle\epsilon$	$= \{ \quad \}$
$(123)\langle(123)\rangle$	$= \{ \quad \}$	$\langle(123)\rangle(123)$	$= \{ \quad \}$
$(132)\langle(123)\rangle$	$= \{ \quad \}$	$\langle(123)\rangle(132)$	$= \{ \quad \}$
$(124)\langle(123)\rangle$	$= \{ \quad \}$	$\langle(123)\rangle(124)$	$= \{ \quad \}$
$(142)\langle(123)\rangle$	$= \{ \quad \}$	$\langle(123)\rangle(142)$	$= \{ \quad \}$
$(134)\langle(123)\rangle$	$= \{ \quad \}$	$\langle(123)\rangle(134)$	$= \{ \quad \}$
$(143)\langle(123)\rangle$	$= \{ \quad \}$	$\langle(123)\rangle(143)$	$= \{ \quad \}$
$(234)\langle(123)\rangle$	$= \{ \quad \}$	$\langle(123)\rangle(234)$	$= \{ \quad \}$
$(243)\langle(123)\rangle$	$= \{ \quad \}$	$\langle(123)\rangle(243)$	$= \{ \quad \}$
$(12)(34)\langle(123)\rangle$	$= \{ \quad \}$	$\langle(123)\rangle(12)(34)$	$= \{ \quad \}$
$(13)(24)\langle(123)\rangle$	$= \{ \quad \}$	$\langle(123)\rangle(13)(24)$	$= \{ \quad \}$
$(14)(23)\langle(123)\rangle$	$= \{ \quad \}$	$\langle(123)\rangle(14)(23)$	$= \{ \quad \}$

Answer the following questions by using your results.

1. For each of the subgroups in (I), (II) and (III), look at your list of left cosets.

   (a) How many *different* left cosets are there?
   (I)
   (II)
   (III)

   (b) How many elements are in each coset?
   (I)
   (II)
   (III)

   (c) Is every element of the "big" group in at least one left coset? Is any element in two different cosets?
   (I)
   (II)
   (III)

2. Answer the above questions for the right cosets.

3. Are your left cosets the same as your right cosets?