MTH 243 Introduction to Probability and Statistics

Final Exam Review

- The final exam is comprehensive, but focuses more on the material from chapters 5-8.
- The final will take place during the last class session Thursday 7/18. That is the only thing we will be doing.
- You may use your calculator, one sheet of paper with notes on both sides, the tables in the back of your book, and the last page in your book "Some Useful Formulas".
- Cell phones may not be used as calculators and must be put away.
- Study ideas – review vocabulary, homework problems, quizzes, in-class worksheets, do additional problems from the book – examples can be found in the text and more problems at the end of the chapter. Redo homework problems, don't just look them over.
- Office Hours Thursday 8-9am.

Topics

Topics since Exam 1 (weighted more heavily on the final)

- Probability
  - sample space
  - multiplication and addition rule
  - empirical vs. theoretical probability
  - computing probabilities
- Conditional probability
- Discrete and Continuous Random variables
- Probability distribution functions (Discrete random variables)
- Probability density functions (Continuous random variables)
- Expected Value
- The Binomial Distribution
  - The mean and standard deviation for a binomial distribution
  - How to use formulas and tables
- The Normal Distribution
  - The standard normal distribution, how to find probabilities
  - Converting a $N(\mu,\sigma^2)$ to a standard normal ($z = (x-\mu)/\sigma$)
  - Graphing a normal curve and labeling the axis correctly
  - The Central Limit Theorem
- Confidence Intervals
  - How to find confidence intervals for means with σ known or unknown
  - How to interpret your confidence interval and write a concluding statement
  - Concepts and related computations:
    - Confidence level
    - Margin of error
    - Determining sample size
- Hypothesis Testing

- How to conduct hypothesis tests for means and proportions
- How to translate a claim into null/alternate hypothesis
- How to interpret results and write a concluding statement about the claim
- Concepts and related computations:
    - Significance level
    - P-value
    - Critical region

<u>Topics from before Exam 1 (less emphasized, but don't forget the basics)</u>

- Population vs. Sample
- Parameter vs. Statistics
- Measures of Center
    - mean, median, mode
- Measures of Variation
    - range, standard deviation (sample vs. population), variance (sample vs. population)
- Measures of Relative Standing
    - z-scores, percentiles, quartiles, 5-number summary, interquartile range
- Graphs
    - box plots, histograms, stem and leaf plots
- Outliers
- Tables
    - frequency and relative frequency charts
- Shapes of graphs
    - symmetric, normal, uniform, skewed left, skewed right
- Normal graph – the empirical rule (page 19)
- "unusual" values

The following practice problems focus only on material in Chapters 5-8. For review of material in Chapters 1-4 go back and look at the midterm review sheet (posted on our website). Disclaimer: this is not meant to be a comprehensive list of all questions that might be asked. It is just to help you study.

1. A family wishes to have 3 children. Let X be the number of girls.
    a. Find the probability distribution function of X.
    b. What is the expected number of girls?

2. Suppose that X is the random variable representing the number of days a patient needs to be in the hospital (where 4 is the maximum possible). Suppose X has the probability distribution function:

$$p(x) = \frac{5-x}{10}\ for\ x = 1,2,3,4.$$

   a. What is the probability that a patient will be in the hospital for exactly three days?
   b. What is the expected number of days the patient will be in the hospital?
   c. If the patient is to receive from an insurance company \$200 for each of the first two days in the hospital and \$100 for each day after the first two days, what is the expected payment for a hospitalization?

3. It is believed that 20% of Americans do not have any health insurance. Suppose that this is true and let X equal the number with no health insurance in a random sample of n=15 Americans.
   a. Explain why X has a binomial distribution. What are n and p?
   b. Find the probability that the number with no health insurance is <u>at most 3</u>. (Note: AT MOST X means from 0 to X: 0,1,2,3…,X)
   c. Find the probability that the number with no health insurance is <u>at least 6</u>. (Note: AT LEAST X means X or more: X,X+1,…,N)
   d. What is the expected number of people in the sample who will have no health insurance? (e.g. the expected value of X)
   e. What is $P(2 \le X \le 5)$?
   f. What is the standard deviation of X?
   g. Would it be unusual for 6 of the people to have no health insurance? <u>Explain.</u> (Note: Explain means explain. Use some mathematics to support your explanations.)

4. If Z has a normal distribution with $\mu=0$ and $\sigma=1$, find:
   a. $P(Z \le 1.25)$
   b. $P(Z \ge -0.36)$
   c. $P(0.45 \le Z \le 2.79)$
   d. Find the 45^{th} percentile.
   e. Between which two values does the middle 80% of the data lie?

5. Is X has a normal distribution with $\mu=15$ and $\sigma=8$, find:
   a. $P(X \ge 19)$
   b. $P(10 \le X \le 20)$
   c. Find the 80^{th} percentile.
   d. Between which two values does the middle 90% lie?

6. A teacher gave the same test to two sections of an Algebra class. Both sets of scores were normally distributed. The scores for Class 1 had a mean of μ=80 and standard deviation of σ=5. The scores for Class 2 had a mean of μ=75 and a standard deviation of σ=10.
   a. Carefully sketch both normal curves on the <u>same axis (label your axis well)</u>. (Pay attention to the height of each curve – which should be taller?)
   b. A paper was misplaced. The teacher doesn't remember anything except that the score was more than 89. Which class is it more likely the test came from? Explain using probabilities.

7. Assume that the number of red M&M's in a regular size package follows a normal distribution with mean μ=5 and standard deviation σ=1.2.
   a. What is the probability that my package of M&M's has 7 or more red M&M's?
   b. Would it be unusual if my package only had 2 red M&M's?
   c. If I bought 20 bags of M&M's what is the probability that the average number of red M&M's per bag would be 4.7 or less?

8. Assume that the weights of full-term newborn babies in the U.S. follows a normal distribution with mean μ=7.5 lbs and standard deviation σ= 0.94 lbs.
   a. Mercy Hospital in St. Louis delivered 50 babies in the month of November. What is the probability that the average weight of the babies was 7.65 lbs or less?
   b. Mercy Hospital in St. Louis delivered 120 babies in the month of November. What is the probability that the average weight of the babies was 7.65 lbs or less?

9. Students took n=35 samples of water from a lake and measured the amount of sodium in parts per million. Assume that σ = 4.94. From their data they found $\bar{x} = 24.11$.
   a. Find a 90% confidence interval for μ, the mean amount of sodium in parts per million in the lake.
   b. What is the margin of error in your interval?
   c. The "Citizens for the Lake" group claimed that the mean amount of sodium in the lake is 20 parts per million. Is this claim supported by the confidence interval you found? Explain.

10. Researchers are trying to determine the average credit card debt, μ, of a student graduating from college. Suppose we know this follows a normal distribution. In a random sample of 30 graduating college students, the average credit card debt was found to be $\bar{x} = \$3{,}500$ and the sample standard deviation to be s= \$300. Find a 95% confidence interval for μ, the average credit card debt.

11. The Merchant's Association claims that the average outlet mall shopper will spend more than \$250 on the Friday after Thanksgiving (aka Black Friday). Let X be the amount spent by such a shopper and suppose it is known that the standard deviation of X is $\sigma=200$. In a random sample of 35 outlet mall shoppers on Black Friday, the average spending was found to be $\bar{x} = 270$.

    a. Do a hypothesis test at the significance level of $\alpha=0.05$ to determine if the Merchant's Association claim is true. Include a concluding statement in your answer.
    b. Find a 90% confidence interval for μ, the average spent by shoppers.
    c. Do your answers to a. and b. agree? Explain why or why not.

12. It has been said that a majority of people living in Roswell, New Mexico, believe that we have been visited by extraterrestrial life. In a random sample of n=120 residents of Roswell, 81 said they believe we have been visited by extraterrestrial life. Test this claim to a significance level of $\alpha=0.10$. Include a concluding statement in your answer.