MTH 105: Final Review Part 1

1. (Mindscape 11 from Section 1.4)**A fair fare**. Three strangers, Bob, Mary, and Ivan, meet at a taxi stand and decide to share a cab to cut down the cost. Each has a different destination, but all the destinations are on the highway leading from the airport, so no circuitous driving is required. Bob's destination is 10 miles away, Mary's is 20 miles, and Ivan's is 30 miles. The taxi costs $1.50 per mile including the tip, regardless of the number of passengers. How much should each person pay? (There is more than one way to think about this, but the answer should be fair in some mathematical way - not just "split the final fare 3 ways".)

   Bob pays $5, Mary pays $12.50 and Ivan pays $27.50

2. (Mindscape 7 from Section 8.4)**The dorm door**. A dormitory has an electronic lock. To unlock the door, students must enter their unique five-digit secret code into the keypad (made up of the digits from 0 to 9, each of which can be used more than once). How many different secret codes are there? Suppose there are 200 students living in the dorm. What is the probability of one of them randomly guessing a code and having the door unlock?

   There are $10^5 = 100,000$ different combinations to the door.
   If one student gueeses at random, the probability they are correct is $\frac{1}{100,000}$
   In 200 guess, the probability that one will get it correct is $\frac{200}{100,000}$

3. Suppose you have a stack of 10 lottery tickets where 3 are winners and the rest are losers. They are mixed together randomly.

   (a) What is the probability that the first ticket is a winner and the second is a loser?

   (1st winner AND 2nd loser) The AND tells us to multiply: $\frac{3}{10} \times \frac{7}{9} = \frac{21}{90} = 0.2\overline{3}$

   (b) What is the probability that in the top three tickets at least one is a winner?

   It is easier to think about the complementary event: the probability that none of the three are winters is $\frac{7}{10} \times \frac{6}{9} \times \frac{5}{8} = \frac{210}{720}$. To get our answer we subtract that from 1: $1 - \frac{210}{720} = \frac{510}{720} = 0.708\overline{3}$.

4. (Mindscape 12 from Section 8.3) **A bad block**. Suppose 1054 people died in Datasville last year. Why must there be a two-week period during which 40 of those people died?

   If no two week period had 40 deaths, then the most we could have had without violating that would be something like 19 one week then 20 the next, then 19 the following week, then 20 the next, and so on so we have 39 every two week period, but none with 40. However doing this would only give 26 weeks with 19 deaths and 26 weeks with 20

deaths for a total of 1014 deaths. Since 1054 people died, then we have to add more deaths into our weeks. Adding even one more death to a week would make a two week period with 40 deaths.

5. Suppose that the number of calories eaten per day by a college student follows a normal distribution with mean 2300 calories and standard deviation 750.

   (a) Between what two values do the middle 95% of students eat in calories in a day?

   800 to 3800 calories

   (b) What is the probability that a randomly selected student will eat more than 3050 calories in one day?

   0.016

   (c) Would it be unusual for a student to eat less than 500 calories in a day? Explain.

   Yes, it would be unusual becasue 500 is more than two standard deviations below the mean.