Sample Test Questions: Test 2

1. Graph the following equation by hand. Also, give the coordinates of the vertex and the domain and range.

\[ f(x) = 2(x + 3)^2 - 1 \]

\[ \text{vertex: } (-3, -1) \quad \text{domain: } \{ x = \text{All real numbers} \} \quad \text{range: } \{ y \geq -1 \} \]

2. Graph the following equation by hand. You must show your work to receive credit. Also, give the domain and range of the function.

\[ f(x) = -3x^2 + 6x + 5 \]

\[ \text{Domain: } \{ x = \text{All real numbers} \} \quad \text{Range: } \{ y \leq 8 \} \]

3. Give the coordinates of the vertex for the following quadratic equation. You may use a calculator to verify your work and to help with arithmetic, but the work needs to be shown.

\[ y + 5 = 3x^2 - 12x \]

\[ \text{Vertex } = (2, -17) \]
4. For the following exercises, simplify the radical expression as much as possible.
   a) \( \sqrt{45} = 3\sqrt{5} \)
   b) \( -\sqrt{72} = -6\sqrt{2} \)
   c) \( \sqrt{\frac{169}{16}} = \frac{13}{4} \)
   d) \( \sqrt{\frac{25}{6}} = \frac{5\sqrt{6}}{6} \)

5. Solve each of the given equations for \( x \). List all solutions, even those that are not real.
   a) \( (x - 5)^2 = 81 \)
   \[ x = -4 \text{ or } x = 14 \]
   b) \( 0 = 2(x - 4)^2 - 6 \)
   \[ x = 4 \pm \sqrt{3} \]
   c) \( 12x^2 + 17 = 2 \)
   \[ x = \pm \frac{i\sqrt{5}}{2} \]
6. Solve the following equation by completing the square.

\[ x^2 - 10x + 4 = 0 \]

\[ \begin{align*} 
-4 & \quad -4 \\
\frac{x^2 - 10x}{4} & = -4 \\
+25 & \quad +25 \\
\frac{x^2 - 10x + 25}{25} & = -4 + 25 \\
\frac{(x - 5)^2}{25} & = 21 \\
\sqrt{(x - 5)^2} & = \pm \sqrt{21} \\
x - 5 & = \pm \sqrt{21} \\
+5 & \quad +5 \\
x & = 5 \pm \sqrt{21} 
\end{align*} \]

7. Find the value of \( c \) which will make the expression become a perfect-square trinomial.

\[ x^2 + 3x + c \]

\[ c = \left( \frac{3}{2} \right)^2 = \frac{9}{4} \]

8. Use the quadratic formula to solve the following expressions.

a) \[ -4x^2 - 3x + 2 = 0 \]

\[ \frac{-(-3) \pm \sqrt{(-3)^2 - 4(-4)(2)}}{2(-4)} = \frac{3 \pm \sqrt{9 + 32}}{-8} = \frac{3 \pm \sqrt{41}}{-8} \]

b) \[ (2x + 5)(x - 4) = -15 \]

\[ 2x^2 - 3x - 5 = 0 \]

\[ \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-5)}}{2(2)} = \frac{3 \pm \sqrt{9 + 40}}{4} = \frac{3 \pm \sqrt{49}}{4} = \frac{3 \pm 7}{4} \]

so \( X = -1 \) or \( \frac{5}{2} \)
10. Simplify the following expressions as much as possible

a) \( \frac{b^{-5}}{b^8} \quad \frac{1}{b^{13}} \)

b) \( \frac{2^5 m^4}{2^3 b^{-5}} \quad 4m^4 b^5 \)

c) \((2x^{-1}y^{-3})(5x^6 y^{-2}) \quad \frac{10x^5}{y^5} \)

11. Simplify the following expression completely.
\[ \frac{(-5b^7 c^{-5})(4b^{-3} c^7)}{35b^2 c^9} \quad -\frac{4b^6}{7c^7} \]

12. Simplify the given expression as much as possible.
\[ \left( \frac{9b^{10} c^5}{5b^{10} c^{-10}} \right)^{-1/2} \quad \frac{\sqrt{5bc}}{3b^5 c^8} \]
13. Using the given function and coordinate plane, draw a sketch of the exponential equation by hand. State whether or not it is exponentially increasing/decaying and what the y-intercept is.

\[ f(x) = 5(2)^x \]

y-intercept: (0, 5)  
exponentially increasing/decaying: Increasing

14. Simplify the right hand side of the equation as much as possible. Tell the domain of the function.

\[ f(x) = \frac{x^2 + 9x + 20}{x^2 + 2x - 15} = \frac{x + 4}{x - 3} \text{ Domain } = \text{ All Real numbers except 3 and -5.} \]

15. Simplify the expression completely.

\[ \frac{x^2 - 49}{x^2 - 4x - 21} = \frac{x + 7}{x + 3} \]

16. Find the following product, and simplify the result.

\[ \frac{m^2 - 10m + 25 \cdot m + 8}{m^2 + 3m - 40 \cdot m^2 - 25} = \frac{1}{m + 5} \]
17. Find the following quotient, and simplify the result.
\[
\frac{3y^2 - 27}{y^2 - 12y + 35} + \frac{y + 3}{y^2 - 10y + 21} = \frac{3(y - 3)(y - 3)}{(y - 5)}
\]

18. Perform the indicated addition and simplify.
\[
\frac{5}{x^2 - 4} + \frac{6}{x^2 + 8x + 12} = \frac{11x + 18}{(x + 2)(x - 2)(x + 6)}
\]

19. Perform the indicated subtraction and simplify.
\[
\frac{3}{x^2 - x - 20} - \frac{2}{x^2 - 2x - 24} = \frac{x - 8}{(x - 5)(x + 4)(x - 6)}
\]

20. Simplify the following expression.
\[
\frac{3x - 3}{2x + 10} - \frac{6x^2 - 6}{4x + 20} = \frac{1}{(x + 1)}
\]

21. Solve the following equation
\[
\frac{4}{x + 2} + \frac{3}{x + 1} = \frac{3}{x^2 + 3x + 2} \quad x = -1
\]

BUT, x cannot = -1 (since the domain is restricted where x cannot be -1 or -2.

So, there are no solutions