28.12. **Model:** The electric field is uniform over the rectangle in the $xy$ plane.

**Solve:** (a) The area vector is perpendicular to the $xy$ plane. Thus

$$\vec{A} = (2.0 \text{ cm} \times 3.0 \text{ cm}) \hat{k} = (6.0 \times 10^{-4} \text{ m}^2) \hat{k}$$

The electric flux through the rectangle is

$$\Phi_e = \vec{E} \cdot \vec{A} = (50\hat{i} + 100\hat{k}) \cdot (6.0 \times 10^{-4} \hat{k}) \text{ N m}^2/\text{C} = 6.0 \times 10^{-2} \text{ N m}^2/\text{C}$$

(b) The electric flux is

$$\Phi_e = \vec{E} \cdot \vec{A} = (50\hat{i} + 100\hat{j}) \cdot (6.0 \times 10^{-4} \hat{k}) \text{ N m}^2/\text{C} = 0 \text{ N m}^2/\text{C}$$

**Assess:** In (b), $\vec{E}$ is in the plane of the rectangle. That is why the flux is zero.

28.14. **Model:** The electric field over the circle in the $xy$ plane is uniform.

**Solve:** The area vector of the circle is

$$\vec{A} = \pi r^2 \hat{k} = \pi (0.015 \text{ m})^2 \hat{k} = (7.07 \times 10^{-4} \text{ m}^2) \hat{k}$$

Thus, the flux through the area of the circle is

$$\Phi_e = \vec{E} \cdot \vec{A} = (1500\hat{i} + 1500\hat{j} + 1500 \hat{k}) \text{ N/C} \cdot (7.07 \times 10^{-4} \text{ m}^2) \hat{k}$$

Using $\hat{i} \cdot \hat{k} = \hat{j} \cdot \hat{k} = 0$ and $\hat{k} \cdot \hat{k} = 1$,

$$\Phi_e = (1500 \text{ N/C})(7.07 \times 10^{-4} \text{ m}^2) = 1.06 \text{ N m}^2/\text{C}$$

the rectangle. That is why the flux is zero.

28.20. **Visualize:** Please refer to Figure EX28.20. For any closed surface that encloses a total charge $Q_{in}$, the net electric flux through the closed surface is $\Phi_e = Q_{in}/\varepsilon_0$. For the closed surface of the torus, $Q_{in}$ includes only the $-1 \text{nC}$ charge. So, the net flux through the torus is due to this charge:

$$\Phi_e = \frac{-1.0 \times 10^{-9} \text{ C}}{8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2} = -113 \text{ N m}^2/\text{C}$$

This is inward flux.

28.22. **Solve:** For any closed surface enclosing a total charge $Q_{in}$, the net electric flux through the surface is

$$\Phi_e = \frac{Q}{\varepsilon_0} \Rightarrow Q_{in} = \varepsilon_0 \Phi_e = \left(8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2\right)(-1000 \text{ N m}^2/\text{C}) = -8.85 \text{nC}$$

28.28. **Visualize:** Please refer to Figure EX28.28.

**Solve:** For any closed surface that encloses a total charge $Q_{in}$, the net electric flux through the closed surface is $\Phi_e = Q_{in}/\varepsilon_0$. In the present case, the conductor is neutral and there is a point charge $Q$ inside the cavity. Thus $Q_{in} = Q$ and the flux is
\[ \Phi_e = \frac{Q}{\varepsilon_0} \]

28.30. **Model:** The electric field over the five surfaces is uniform.

**Visualize:** Please refer to Figure P28.30.

**Solve:** The electric flux through a surface area \( A \) is \( \Phi_e = \vec{E} \cdot \vec{A} = EA \cos \theta \) where \( \theta \) is the angle between the electric field and a line perpendicular to the plane of the surface. The electric field is perpendicular to side 1 and is parallel to sides 2, 3, and 5. Also the angle between \( \vec{E} \) and \( \vec{A}_4 \) is \( 60^\circ \). The electric fluxes through these five surfaces are

\[ \begin{align*}
\Phi_1 &= E_1A_1 \cos \theta_1 = (400 \text{ N/C})(2 \text{ m} \times 4 \text{ m}) \cos 180^\circ = -3200 \text{ N m}^2/\text{C} \\
\Phi_2 &= E_2A_2 \cos 90^\circ = \Phi_3 = \Phi_5 = 0 \text{ N m}^2/\text{C} \\
\Phi_4 &= E_4A_4 \cos \theta_4 = (400 \text{ N/C})\left[\left(\frac{2 \text{ m}}{\sin 30^\circ}\right) \times 4 \text{ m}\right] \cos 60^\circ = +6400 \text{ N m}^2/\text{C}
\end{align*} \]

**Assess:** Because the flux into these five faces is equal to the flux out of the five faces, the net flux is zero, as we found.

28.33. **Solve:** For any closed surface that encloses a total charge \( Q_\text{in} \), the net electric flux through the closed surface is \( \Phi_e = \frac{Q_\text{in}}{\varepsilon_0} \). The flux through the top surface of the cube is one-sixth of the total:

\[ \Phi_{e\text{surface}} = \frac{Q_\text{in}}{6\varepsilon_0} = \frac{10 \times 10^{-9} \text{ C}}{6\left(8.85 \times 10^{-12} \text{ C}^2/\text{N m}^2\right)} = 188 \text{ N m}^2/\text{C} \]

28.37. **Model:** The excess charge on a conductor resides on the outer surface.

**Visualize:**

**Solve:** (a) Consider a Gaussian surface surrounding the cavity just inside the conductor. The electric field inside a conductor in electrostatic equilibrium is zero, so \( \vec{E} \) is zero at all points on the Gaussian surface. Thus \( \Phi_e = 0 \). Gauss’s law tells us that \( \Phi_e = \frac{Q_\text{enc}}{\varepsilon_0} \), so the net charge enclosed by this Gaussian surface is \( Q_\text{in} = Q_\text{point} + Q_\text{wall} = 0 \).

We know that \( Q_\text{point} = +100 \text{ nC} \), so \( Q_\text{wall} = -100 \text{ nC} \). The positive charge in the cavity attracts an equal negative charge to the inside surface.

(b) The conductor started out neutral. If there is \(-100 \text{ nC} \) on the wall of the cavity, then the exterior surface of the conductor was initially \(+100 \text{ nC} \). Transferring \(-50 \text{ nC} \) to the conductor reduces the exterior surface charge by \(50 \text{ nC} \), leaving it at \(+50 \text{ nC} \).

**Assess:** The electric field inside the conductor stays zero.
28.40. **Model:** The excess charge on a conductor resides on the outer surface. The charge distribution on the two spheres is assumed to have spherical symmetry.

**Visualize:** Please refer to Figure P28.40. The Gaussian surfaces with radii $r = 8\,\text{cm}$, $10\,\text{cm}$, and $17\,\text{cm}$ match the symmetry of the charge distribution. So, $\vec{E}$ is perpendicular to these Gaussian surfaces and the field strength has the same value at all points on the Gaussian surface.

**Solve:** (a) Gauss’s law is $\Phi_e = A \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$. Applying it to a Gaussian surface of radius $8\,\text{cm}$,

$$Q_{in} = -\varepsilon_0 A \vec{E} \cdot d\vec{A} = -\left(8.85 \times 10^{-12} \, \text{C}^2/\text{m}^2\right)\left(15,000 \, \text{N/C}\right)\left(4\pi \left(0.08 \, \text{m}\right)^2\right) = -1.07 \times 10^{-8} \, \text{C}$$

Because the excess charge on a conductor resides on its outer surface and because we have a solid metal sphere inside our Gaussian surface, $Q_{in}$ is the charge that is located on the exterior surface of the inner sphere.

(b) In electrostatics, the electric field within a conductor is zero. Applying Gauss’s law to a Gaussian surface just inside the inside surface of the hollow sphere at $r = 10\,\text{cm}$,

$$\Phi_e = A \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0} \Rightarrow Q_{in} = 0 \, \text{C}$$

That is, there is no net charge. Because the inner sphere has a charge of $-1.07 \times 10^{-8} \, \text{C}$, the inside surface of the hollow sphere must have a charge of $+1.07 \times 10^{-8} \, \text{C}$.

(c) Applying Gauss’s law to a Gaussian surface at $r = 17\,\text{cm}$,

$$Q_{in} = \varepsilon_0 A \vec{E} \cdot d\vec{A} = \varepsilon_0 A \vec{E} \cdot d\vec{A} = \left(8.85 \times 10^{-12} \, \text{C}^2/\text{m}^2\right)\left(15,000 \, \text{N/C}\right)\left(4\pi \left(0.17 \, \text{m}\right)^2\right) = 4.82 \times 10^{-8} \, \text{C}$$

This value includes the charge on the inner sphere, the charge on the inside surface of the hollow sphere, and the charge on the exterior surface of the hollow sphere due to polarization. Thus,

$$Q_{exterior\ hollow} + \left(1.07 \times 10^{-8} \, \text{C}\right) + \left(-1.07 \times 10^{-8} \, \text{C}\right) = 4.82 \times 10^{-8} \, \text{C}$$

$$\Rightarrow Q_{exterior\ hollow} = 4.82 \times 10^{-8} \, \text{C}$$

28.41. **Model:** The charge distribution at the surface of the earth is assumed to be uniform and to have spherical symmetry.

**Visualize:**

Due to the symmetry of the charge distribution, $\vec{E}$ is perpendicular to the Gaussian surface and the field strength has the same value at all points on the surface.

**Solve:** Gauss’s law is $\Phi_e = A \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$. The electric field points inward (negative flux), hence

$$Q_{in} = -\varepsilon_0 A \vec{E} \cdot d\vec{A} = -\left(8.85 \times 10^{-12} \, \text{C}^2/\text{m}^2\right)\left(100 \, \text{N/C}\right)\left(4\pi \left(6.57 \times 10^6 \, \text{m}\right)^2\right) = -4.51 \times 10^5 \, \text{C}$$
28.55. Model: The long thin wire is assumed to be an infinite line of charge.

Visualize: Please refer to Figure CP28.55. The cube of edge length \( L \) is centered on the line charge with a linear charge density \( \lambda \). Although the line charge has cylindrical symmetry, we will take the cube as our Gaussian surface.

Solve: (a) The electric flux through an area \( d\mathbf{A} \) in the \( yz \) plane is \( d\Phi = \mathbf{E} \cdot d\mathbf{A} \). The electric field \( \mathbf{E} \) due to an infinite line of charge at a distance \( s \) from the line charge is

\[
\mathbf{E} = \frac{\lambda}{4\pi\varepsilon_0 r} \hat{r} = \frac{\lambda}{2\pi\varepsilon_0 \sqrt{y^2 + \left(\frac{L}{2}\right)^2}} \hat{r}
\]

Also \( d\mathbf{A} = L\dy \hat{i} \). Thus,

\[
d\Phi = \frac{\lambda}{2\pi\varepsilon_0} \left( \frac{1}{\sqrt{y^2 + \left(\frac{L}{2}\right)^2}} \right) L\dy \left( \hat{r} \cdot \hat{i} \right) = \frac{\lambda L \dy}{2\pi\varepsilon_0} \left( \frac{1}{\sqrt{y^2 + \left(\frac{L}{2}\right)^2}} \right) \cos \theta
\]

\[
= \frac{\lambda L \dy}{2\pi\varepsilon_0 \sqrt{y^2 + \left(\frac{L}{2}\right)^2}} \left( \frac{L/2}{\sqrt{y^2 + \left(\frac{L}{2}\right)^2}} \right) = \frac{\lambda L^2 \dy}{4\pi\varepsilon_0 \left[ y^2 + \left(\frac{L}{2}\right)^2 \right]}
\]

(b) The expression for the flux \( d\Phi \) can now be integrated to obtain the total flux through this face as follows:

\[
\Phi = \int d\Phi = \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{\lambda L^2 \dy}{4\pi\varepsilon_0 \left[ y^2 + \left(\frac{L}{2}\right)^2 \right]} = \frac{\lambda L^2}{4\pi\varepsilon_0} \left[ \frac{2}{L} \tan^{-1} \left( \frac{2y}{L} \right) \right]_{-\frac{L}{2}}^{\frac{L}{2}}
\]

\[
= \frac{\lambda L}{2\pi\varepsilon_0} \left[ \tan^{-1}(1) - \tan^{-1}(-1) \right] = \frac{\lambda L}{2\pi\varepsilon_0} \left[ \frac{\pi}{4} - \left( -\frac{\pi}{4} \right) \right] = \frac{\lambda L}{4\varepsilon_0}
\]

(c) Because there are four faces through which the flux flows, the net flux through the cube is

\[
\Phi_e = 4\Phi = \frac{\lambda L}{4\varepsilon_0} \left( \frac{Q_m}{L} \right) L = \frac{Q_m}{\varepsilon_0}
\]