**Faraday's Law**

\[ \text{EMF} = -\frac{d\Phi_B}{dt} \]

\[ \Phi_B = \int \mathbf{B} \cdot d\mathbf{A} \]

**Lenz's Law**

**Induced EMF**

\[ \mathcal{E} = -\frac{d\Phi_B}{dt} \]

\[ \mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d(Blw)}{dt} \]

\[ \mathcal{E} = -Bl \frac{dw}{dt} = -Blv \]
A new problem involves a conducting rail with uniform magnetic field $\mathbf{B}$, giving a rod initial velocity $v_0$. Assume $t=0$, $v(t=0) = v_0$.

Since $\mathbf{I}$ induced is CCW, $\mathbf{F}_m = \mathbf{I} \times \mathbf{B}$ which is to the left.

$$\mathbf{E} = -\frac{\partial \mathbf{\Phi}}{\partial t} = -\frac{1}{c} (\mathbf{Blv}) = -\mathbf{Blv}$$

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{R}}$$

$$\mathbf{F}_m = \mathbf{I} \times \mathbf{B} = -\left(\frac{\mathbf{Blv}}{\mathbf{R}}\right) \times \mathbf{B}$$

$$\mathbf{ma} = \mathbf{F} = \mathbf{F}_m = -\frac{\mathbf{B}^2 l^2 v}{\mathbf{R}}$$

$$\mathbf{ma} = -\frac{\mathbf{B}^2 l^2 v}{\mathbf{R}}$$
\[
\frac{dv}{dt} = a = -\frac{B^2 l^2}{mR} v
\]

\[
\int_{V_0}^V \frac{dv}{v} = -\int_{t=0}^t \frac{B^2 l^2}{mR} dt
\]

\[
\ln \frac{V}{V_0} = -\frac{B^2 l^2}{mR} t
\]

\[
\ln \frac{V_0}{v} = e^{-\frac{B^2 l^2}{mR} t}
\]

\[
\frac{V}{V_0} = e^{-\frac{B^2 l^2}{mR} t}
\]

\[
V(t) = V_0 e^{-\frac{B^2 l^2}{mR} t}
\]

\[
\frac{B^2 l^2}{mR} = \frac{1}{\text{sec}}
\]

\[
\frac{T M^2}{kg \text{ m}^2} = \frac{1}{\text{sec}}
\]

\[
F = q v B
\]

\[
T = \frac{N s}{cm}
\]

\[
U = \frac{1}{2} m v^2 = \frac{V^2}{2}
\]
what does inducing an EMF really mean?

current loop \perp \textbf{B},

\begin{align*}
\text{lets assume } \textbf{B} \text{ changes in time} \\
\text{if } \textbf{B} \text{ increases,} \\
\text{induced current flow in } \text{SSW} \text{ direction}
\end{align*}

\[ E = -\frac{\partial \Phi}{\partial t} = -\frac{\partial}{\partial t} (A \cdot \textbf{B}) = -A \frac{\partial \textbf{B}}{\partial t} \]

\[ \textbf{m} \cdot \textbf{I} = \text{VOLT} \]

what is the work done on \( q \) as it makes one full loop around circuit

\[ \text{Work} = \Delta \text{energy} = q \Delta V = \int E \, \text{induced} \]

what causes the charge to move in the first place?

\[ W = \int \textbf{F} \cdot d\mathbf{r} \]

\[ \textbf{F} = q \textbf{E} \]

\[ W = q E = q E (\text{mrrc}) = q \int E \cdot d\textbf{r} \text{ so } E \text{ is parallel to } d\textbf{r} \]
\[ W = \frac{q}{2} E (2\pi r) = \frac{q}{2} \oint \vec{E} \cdot d\vec{l} \]

\[ \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \]

Faraday's law

When you change the magnetic flux, you induce an electric field.

Inductance: What happens when the switch closes? Does \( I = \frac{V}{R} \) instantly?

"Self induced EMF"