\[ E = \frac{kQ}{r^2} \]

\[ +Q \]

\[ \frac{r}{F_{net}} \]

\[ |F_-| > |F_+| \]

\[ \vec{T} = \vec{r} \times \vec{F} \]

Uniform field

\[ |F_-| = |F_+| \rightarrow \text{no force} \]

\[ \vec{F} = q \vec{E} \]
Calc Review

$251x \to y = f(x) \text{ scalar function - 1 dimension}$

Complicating:

1. $3D$ universe $T(x, y, z) \rightarrow$ scalar Field

2. $\nabla (x, y, z) \rightarrow$ vector fields.
   So at every point $(x, y, z)$--there is an arrow of some length $|\nabla (x, y, z)|$, in a particular direction.

Denote it with arrows

Length of arrows give the magnitude, orientation gives the direction.
Characteristics of vector fields

Consider a closed surface, we want to know how much “stuff” is coming out from the surface.

The net outward flow \( \Phi \) equals the flux

\[ \Phi = \iiint_S \mathbf{F} \cdot d\mathbf{A} \]

where \( \mathbf{F} \) is the vector field, \( d\mathbf{A} \) is the differential area element, and \( S \) is the surface integral.

Flux is the average normal component of the surface area.
2. Define another quantity "circulation" 
\[ \mathbf{V} \times \text{path} \]

circulation = "average tangential component of \( \mathbf{V}(x,y,z) \)"
+ distance around a closed loop

Curve C

\[ \oint_C \mathbf{V} \cdot d\mathbf{r} = \int \mathbf{V}(x,y,z) \cdot d\mathbf{r} = \text{path integral} \]

\[ \oint_C \mathbf{V} \cdot d\mathbf{r} = \text{path element} \]

closed loop

cartesian x, y, z

\[ dV = dxdydz \]
\[ dA = dxdy, dxdz, dydz \]
\[ dl = dx, dy, dz \]
Cylindrical coordinates (polar coords + z)

\[ \mathrm{d}V = r \, \mathrm{d}r \, \mathrm{d}\theta \, \mathrm{d}z \]

\[ \mathrm{d}A = \text{top} = r \, \mathrm{d}r \, \mathrm{d}\theta \]

\[ \text{side} = R \, \mathrm{d}\theta \, \mathrm{d}z \]

\[ \mathrm{d}l = \mathrm{d}r \, R \, \mathrm{d}\theta \, \mathrm{d}z \]

\[
\text{dot: } \hat{A} \cdot \hat{B} = \frac{|A| \times |B| \times \cos \theta}{2} = \begin{cases} 2 \cdot \hat{r} = \hat{r} & 2 \cdot \hat{\theta} = \hat{\theta} \rightarrow 2 \cdot \hat{z} = \hat{z} \end{cases} \]

\[ f \cdot \hat{r} = 0, f \cdot \hat{\theta} = 0, f \cdot \hat{z} = 1 \]

\[
\text{cross: } \hat{A} \times \hat{B} = |A| \times |B| \times \sin \theta \times \begin{cases} \hat{\theta} = \hat{r}, \hat{\Phi} \times \hat{z} = 1, \hat{\Phi} \times \hat{e} = 0 \end{cases} \]

\[ \hat{\Phi} \times \hat{e} = 0 \]
Gauss's Law — to come

\[ \vec{F} = q \vec{E} \]

Electric flux is proportional to the number of field lines penetrating the surface:

\[ \Phi = EA \left( \frac{N}{\epsilon_0 \cdot m^2} \right) \] — scalar quantity

If \( \vec{E} \) is tangent to the surface, no fields penetrate:

\[ \Phi = EA \cos \theta = \vec{E} \cdot d\vec{s} \]

\[ d\vec{s} = \hat{n} dA \]

If \( \vec{E} \) is normal to the surface, all the field penetrate the surface:

\[ \Phi = \vec{E} \cdot \hat{n} dA \] — outward unit normal

If the field is not uniform, sum over small bits of the surface:

\[ \Phi = \sum \Delta \Phi_i = \sum \vec{E} \cdot \Delta \vec{s}_i = \sum \vec{E}_i \cdot \hat{n}_i \Delta A_i = \]

\[ = \lim_{\Delta A_i \to 0} \sum \vec{E} \cdot \Delta A_i \cos \theta = \int \vec{E} \cdot \hat{n} dA \]
Calculate the flux through a cube, with one corner at the origin, in a uniform \( \vec{E} \), \( \vec{E} = E_0 \vec{e}_z \).

Each face has area \( A = l^2 \).

\[ \vec{E} = E_0 \vec{e}_z \] everywhere.

\[ \oint \vec{E} \cdot d\vec{s} \] on all 6 faces:

\[ \begin{align*}
\vec{S}_{top} &= l^2 \vec{e}_x \\
\vec{S}_1 &= l^2 \vec{e}_y \\
\vec{S}_2 &= l^2 \vec{e}_z \\
\vec{S}_3 &= -l^2 \vec{e}_z \\
\vec{S}_4 &= -l^2 \vec{e}_x \\
\vec{S}_5 &= -l^2 \vec{e}_y \\
\end{align*} \]

\[ \oint \vec{E} \cdot d\vec{s} = 0 \]

\[ \oint = \oint_1 = \oint_2 = \oint_3 = \oint_4 = \oint_5 = \oint_6 = 0 \]
Now onto Gauss’s Law

Gaussian sphere of radius \( r \),

at the surface \( E = \frac{kq}{r^2} \) outward

\[ E = \frac{kq}{r^2} \]

Now let’s calculate the flux through our surface.

\[ \Phi = \oint E \cdot d\mathbf{S} = \oint E \mathbf{F} \cdot d\mathbf{A} = \oint E dA \]

\[ \Phi = E \oint dA \]

\[ \oint dA \text{ for a sphere} = 4\pi r^2 \]

\[ \Phi = \frac{q}{3\epsilon_0} \]

\[ \Phi = \frac{q}{3\epsilon_0} \]
Gauss's Law: The net flux through a closed surface is proportional to the net charge contained inside the surface.

\[ \Phi = \oint \vec{E} \cdot d\vec{S} = \frac{Q_{\text{closed}}}{\varepsilon_0} \]

independent of \( r \)