\[ E \rightarrow \textbf{K} \rightarrow \textbf{M} \textbf{Q} \]

\[ Q = 50 \mu C \]
\[ K = 100 \text{ Nm} \]
\[ \varepsilon = 5 \times 10^{-5} \text{ N/C} \]

\[ \text{Turn on the } E \text{ field} \]

\[ \Gamma = \text{ frictionless} \]

1) What is the max displacement of the block?

2) What is the new equlibrium position?

At equilibrium, net force = 0

initial electric force \[ \vec{F} = q \vec{E} \]

\[ \sum F = 0 \]

\[ kx = qE \]

\[ x = \frac{qE}{k} \]

\[ x = 2.5 \text{ meters} \]

\[ 2) \text{ What scalar equation do you have up your sleeve?} \]

\[ E_C = E_F \]

\[ k = 0 \]

\[ u_i, gp = 0 \]

\[ u_i, el = 0 \]

\[ u_f - u_i = -\int_{u_i}^{u_f} \vec{F} \cdot d\vec{x} = -\int q \vec{E} \cdot d\vec{x} = -q \varepsilon X_{\text{max}} \]

\[ u_i, el = -q \varepsilon X_{\text{max}} \]

\[ u_f, el = -q \varepsilon X_{\text{max}} \]
Calculation of potentials from point charges:

Consider an isolated point charge $q$.

Calculate change in potential in going from $A$ to $B$:

$$\Delta V = V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{r}$$

Expand this segment:

$$\vec{F} = \frac{k q}{r^2}$$

$$\vec{r} \cdot d\vec{r} = d\ell \cos \theta$$

$$\cos \theta = \frac{d\ell}{\ell}$$

$$d\ell \cos \theta = d\ell$$

$$\Delta V = V_B - V_A = -\int_A^B \frac{k q}{r^2} \, d\ell$$

$$= -kq \int_A^B \frac{d\ell}{r^2}$$
\[ \Delta V = V_B - V_A = -kq \left( \frac{1}{r_B} - \frac{1}{r_A} \right) \]

Note that this is independent of the path, as it must since the electric force is conservative.

By convention, we'll choose \( V = 0 \) when \( r = \infty \)

\[ V_B - V_A = -kq \int_{r_B}^{\infty} \frac{dr}{r} = kq - 0 = V_B \]

For a point charge \( V(r) = \frac{kq}{r} \)
for a point charge \( V = \frac{kq}{r} \)

\[
V(r) = \sum \frac{kq_i}{r_i^2} \\
E(r) = \sum \frac{kq}{r^2}
\]

assemble a group of charges

1) bring in a charge \( q_1 \) from infinity
   - how much work is involved?
   \[
   W = \frac{kq_1}{r}
   \]
   - no force \( \Rightarrow \) no work \( \Rightarrow \)
   \( \Delta U = 0 \)

2) now bring in a second charge

\[
q_2 \quad U = q_2 V_1 = \frac{kq_1 q_2}{r}
\]
If \( q_1 \) and \( q_2 \) have the same sign,

\( U \) is positive since

\[ \Delta U = - \int F \cdot dl = -W_c \]

\( W_c < 0 \) \( \Rightarrow \) work must be done on

The system to arrange the charges

If \( q_1 \) and \( q_2 \) are oppositely charged

\( U \) is negative \( \Rightarrow \) \( W_c \) is positive \( \Rightarrow \)

The system does the work

(electric fields have energy)
How much energy is needed to assemble this configuration

\[ U = \sum_{\text{all pairs}} \frac{k q_i q_j}{r_{ij}} \]

1) first bring in charge \( q' \) – \( U = 0 \) – since energy requires 2 charges

2) now bring in \( q' \) \( U = -\frac{2kq^2}{a} \) – we do the work

3) now bring in \( q \) \( U = \frac{6kq^2}{b} + \frac{3kq^2}{\sqrt{a^2 + b^2}} \) – we do the work
1) Now bring in $-2g$

$$U = \frac{-2 + q^2}{b} - \frac{6kq^2}{a} - \frac{4kq^2}{\sqrt{a^2 + b^2}}$$

Add together the 6 terms

$$U = \frac{-4kq}{a} + \frac{4kq}{b} - \frac{kq}{\sqrt{a^2 + b^2}}$$

$$= -3.96 \text{J}$$

$a = -2 \text{ m}$

$b = 4 \text{ m}$

$q = 6 \text{ mC}$
\[ \Delta U = - \int \mathbf{E} \cdot d\mathbf{r} \]
\[ \mathbf{E} = -\nabla U \]
\[ \mathbf{E}_x = - \frac{\partial \phi}{\partial x} (U) \]
\[ \mathbf{E}_y = - \frac{\partial \phi}{\partial y} (U) \]
\[ \mathbf{E}_z = - \frac{\partial \phi}{\partial z} (U) \]

For spherical symmetry:
\[ \mathbf{E}_r = - \hat{r} \frac{\partial \phi}{\partial r} \]

For a point particle, \( V = \frac{kq}{r} \):
\[ \mathbf{E} = - \hat{r} \frac{\partial \phi}{\partial r} = - \hat{r} \frac{kq}{r} \frac{d}{dr} \left( \frac{1}{r} \right) \]
\[ \mathbf{E} = - \hat{r} \frac{kq}{r^2} \left( - \frac{1}{r^2} \right) = \frac{kq}{r^2} \hat{r} \]
Find \( \vec{E}(x) \)

For a point charge \( V = \frac{kq}{r} \)

\[
V = \frac{kq}{x-a} - \frac{kq}{x+a}
\]

\[
v(x) = \frac{kq(x+a) - kq(x-a)}{(x-a)(x+a)} = \frac{2kqa}{(x-a)(x+a)} = \frac{2kqa}{(x^2-a^2)}
\]

\[
\frac{1}{\varepsilon_0} \frac{dV}{dx} = -\frac{1}{\varepsilon_0} \frac{d}{dx} \left( \frac{2kqa}{(x^2-a^2)} \right) = \frac{2kqa - 2x}{(x^2-a^2)^2}
\]

\[
\vec{E}(x) = \frac{4kqa \cdot \vec{z}}{(x^2-a^2)^2} = \frac{4kqa \cdot \vec{z}}{(x+a)(x-a)^2}
\]

what if \( x \approx a \)

\[
\vec{E} = \frac{4kqa \cdot \vec{z}}{x^3}
\]