\[ \Phi = \oint E \cdot dS = \frac{Q_{	ext{enclosed}}}{\varepsilon_0} \]

\[ E = \frac{V}{2\varepsilon_0} \]

\[ E = \frac{V}{\varepsilon_0} \quad \text{Field in a parallel plate capacitor} \]
2. Any excess charge on a conductor must reside on the surface.

Gaussian surface,

From property #1, \( \mathbf{E}_{\text{inside}} = 0 \)

\[
\oint \mathbf{E} \cdot d\mathbf{s} = \frac{q_{\text{enclosed}}}{\varepsilon_0} \implies q_{\text{enclosed}} = 0
\]

Now expand the size of the Gaussian surface as close to the outside as possible; still \( \mathbf{E} = 0 \)

\[ \rightarrow \] Charge must reside on the surface
Electric field outside a conductor is \( \mathbf{E} \) to the surface and has magnitude \( \frac{Q}{\varepsilon_0} \).

For electrostatics, the tangential component of \( \mathbf{E} \) must be zero \( \Rightarrow \mathbf{E} \parallel \text{normal to the surface} \).

\[
\Phi = \oint \mathbf{E} \cdot d\mathbf{s} = \frac{Q_{\text{enclosed}}}{\varepsilon_0}
\]

Zero flux since \( \mathbf{E} = 0 \) inside a conductor.

\[
\Phi = \oint \mathbf{E} \cdot d\mathbf{s} = \mathbf{E} \cdot \mathbf{A} = \frac{Q_{\text{enclosed}}}{\varepsilon_0}
\]

\[
\mathbf{E} = \frac{Q_{\text{enclosed}}}{A} \cdot \frac{1}{\varepsilon_0} = \mathbf{E}_0
\]

Q.E.D.
New problem: Solid conducting sphere of radius $a$, with net charge $+2Q$, surrounded by conducting spherical shell of inner radius $b$, and outer radius $c$, and net charge $-Q$. Find $E$ everywhere.

Blue circles are really spheres.

$I$. $r < a$

$II$. $a < r < b$

$III$. $b < r < c$

$IV$. $r > c$

$I$. $E = 0$ inside a conductor

$\mathbf{E} = \oint \mathbf{E} \cdot d\mathbf{S} = \frac{Q_{\text{enclosed}}}{\varepsilon_0}$

$E_{\text{II}} = \frac{2kQ}{r^2}$

$E_{\text{III}} = 0$

inside a conductor $\rightarrow Q_{\text{enclosed}} = Q$

$E = \frac{kQ}{r^2}$

$E_{\text{IV}} = \frac{Q}{4\pi r^2}$
example sphere of radius $2a$, of a non-conducting material that has uniform energy density $\varepsilon_0$. Now create a spherical cavity of radius $a$, show that within the cavity, $E_x = 0$ and

$$E_y = \frac{\varepsilon_0}{2\varepsilon_0}$$

**Solution:**

We'll use Gauss's Law + principle of superposition.
Let's determine $E_+$ first.

\[ E_+ = \frac{q_{\text{en}}}{\varepsilon_0} \]

\[ E(4\pi R^2) = \frac{q_{\text{en}}}{\varepsilon_0} \]

\[ E_+ = \frac{q_{\text{en}} R}{3\varepsilon_0} \]

Now let's $E_-$.

\[ \Phi = \int E_+ dS = \frac{q_{\text{en}}}{\varepsilon_0} \]

\[ E(4\pi R^2) = -\frac{q_{\text{en}} R^3}{\varepsilon_0} \]

\[ E_- = -\frac{q_{\text{en}} R}{3\varepsilon_0} \]

\[ \Phi = \int E_- dS = \frac{q_{\text{en}}}{\varepsilon_0} \]

\[ \Phi = \int E_- dS = \frac{q_{\text{en}}}{\varepsilon_0} \]
Thus we see that

\[ \vec{r} + \alpha \hat{J} = \vec{R} \]

\[ \vec{r} = \vec{R} - \alpha \hat{J} \]

Now \[ \vec{E} = \vec{E}_- + \vec{E}_+ \]

\[ \vec{E}_{\text{total}} = \frac{-\vec{p}_r}{3\varepsilon_0} + \frac{\vec{p}_R}{3\varepsilon_0} = \frac{-\vec{p}(\vec{R}-\alpha \hat{J})}{3\varepsilon_0} + \frac{\vec{p}_R}{3\varepsilon_0} \]

\[ \vec{E}_{\text{total}} = \frac{-\vec{p}_R}{3\varepsilon_0} + \frac{\alpha \vec{J}}{3\varepsilon_0} + \frac{\vec{p}_R}{3\varepsilon_0} \]

\[ \vec{E}_{\text{total}} = \frac{\alpha \vec{J}}{3\varepsilon_0} \]

\[ E_x = 0 \]

\[ E_y = \frac{\alpha J}{3\varepsilon_0} \]

QED
Conductors in electrostatic equilibrium

\[ F = qE = ma \]

1. Electric field inside a conductor is zero everywhere. \[ F = -zeE \]

Consider an uncharged conducting rod in a uniform \( E \)

Inside the rod there is the external electric field, and the electric field from the induced polarization

\[ E_{\text{ext}} + E_{\text{induced}} = 0 \]