Instructions: Complete 5 of the 6 problems. Show all work, state any assumptions you make.

1) A rifle bullet with mass 8.00 g strikes and embeds itself in a block with mass 0.992 kg that rests on a frictionless, horizontal surface and is attached to a coil spring. The impact compresses the spring 15.0 cm. Calibration of the spring shows that a force of 0.750 N is required to compress the spring 0.250 cm. (a) Find the magnitude of the block's velocity just after impact. (b) What was the Initial speed of the bullet?

\[ m = 8.0 \text{ g} = 0.008 \text{ kg} \]
\[ M = 0.992 \text{ kg} \]
\[ \Delta x = 15 \text{ cm} = 0.15 \text{ m} \text{ (after collision with bullet)} \]

Spring measurement \[ F = 0.75N, \text{ when } \Delta x = 0.25 \text{ cm} = 0.0025 \text{ m} \Rightarrow F = k \Delta x \]
\[ k = \frac{F}{\Delta x} = \frac{0.75N}{0.0025 \text{ m}} = 300 \text{ N/m} \]

2) Use conservation of energy
\[ E_c = \frac{1}{2} k \Delta x_{\text{max}}^2 \]
\[ E_c = \frac{1}{2} (m+n) v^2 \]

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\[ \frac{1}{2} k \Delta x_{\text{max}}^2 = \frac{1}{2} (m+n) v^2 \]
\[ v = \sqrt{\frac{k}{(m+n)}} \Delta x_{\text{max}} = \sqrt{\frac{300}{1} (0.15)} = 2.60 \text{ m/s} \]

b) Use conservation of momentum
\[ m \vec{v}_o = (m+M) \vec{v} \]
\[ m \vec{v}_o = (0.008 + 0.992) \times (2.60 \text{ m/s}) \]
\[ \vec{v}_o = 3.214 \text{ m/s} \]

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2) The pulley in the figure has radius \( R \) and a moment of inertia \( I \). The rope does not slip over the pulley, and the pulley spins on a frictionless axle. The coefficient of kinetic friction between block \( A \) and the tabletop is \( \mu_k \). The system is released from rest, and block \( B \) descends. Block \( A \) has mass \( m_A \) and block \( B \) has mass \( m_B \). Use energy methods to calculate the speed of block \( B \) as a function of the distance \( d \) that it has descended. Express your answer in terms of the variables \( m_A, m_B, R, I, \mu_k, d \) and any appropriate constants.

\[
E_i = m_B gd
\]

\[
E_f = \frac{1}{2} (m_A + m_B) v_f^2 + \frac{1}{2} I \omega^2
\]

\[
\text{but } v = \omega R = \frac{v_f}{R}
\]

\[
E_f = \frac{1}{2} (m_A + m_B + I/R^2) v_f^2
\]

\[
W_{nc} = -\mu_k m_A g d
\]

\[
E_i + W_{nc} = E_f
\]

\[
m_B gd - \mu_k m_A gd = \frac{1}{2} (m_A + m_B + I/R^2) v_f^2
\]

\[
v_f = \sqrt{\frac{2gd (m_B - \mu_k m_A)}{m_A + m_B + I/R^2}}
\]
3) A 50.0-kg grindstone is a solid disk 0.530 m in diameter. You press an ax down on the rim with a normal force of 150 N. The coefficient of kinetic friction between the blade and the stone is 0.60, and there is a constant friction torque of 6.50 N m between the axle of the stone and its bearings. (a) How much force must be applied tangentially at the end of a crank handle 0.500 m long to bring the stone from rest to 120 rev/min in 9.00s? (b) After the grindstone attains an angular speed of 120 rev/min, what tangential force at the end of the handle is needed to maintain a constant angular speed of 120 rev/min? (c) How much time does it take the grindstone to come from 120 rev/min to rest if it is acted on by the axle friction alone?

\[ F_2 = \frac{I \alpha + M_K F_1 + T_{\text{axle}}}{R} = 65.8 \text{ N} \]

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\[ \alpha = \frac{120 \text{ rev/min}}{60 \text{ sec}} = 2 \text{ rad/sec} \]

\[ \alpha = \frac{1.386 \text{ rad/sec}^2}{9 \text{ sec}} \]

\[ t = \frac{\pi}{60 \text{ rev/sec}} = \frac{1.756}{60 \text{ rev/sec}} = 0.03 \text{ sec} \]

\[ t = 3.40 \text{ sec} \]
4) A 46.0-cm diameter wheel, consisting of a rim and six spokes, is constructed from a thin rigid plastic material having a linear mass density of 25.0 g/cm. This wheel is released from rest at the top of a hill 58.0 m high. How fast is it rolling when it reaches the bottom of the hill?

\[
\lambda = \frac{\text{mass}}{\text{length}} = \frac{\text{length}}{\text{length}}
\]

\[
\text{mass} = \lambda \times \text{length}
\]

First, find moment of inertia:

3 rods of length \( l = 46 \text{ cm} \), rotated at their center, \( I = \frac{1}{12} l^2 \).

So, \( I_{\text{spoke}} = (3)(\frac{1}{12})(\lambda L)^2 = (\frac{1}{4})(2.5)\text{ g/cm}(46 \text{ cm})^3 = 0.0608 \text{ kg m}^2 \) for rim.

\( I = MR^2 \), \( M = \lambda L = \lambda (2\pi R) \), so

\( I_{\text{rim}} = (2\pi R \lambda) R^2 = 2\pi R^3 = 2\pi (2.5 \text{ g/cm})(23 \text{ cm})^3 = 191 \text{ kg m}^2 \)

Then \( I_{\text{net}} = (0.0608 + 0.1911) \text{ kg m}^2 = 0.2519 \text{ kg m}^2 \)

Now use conservation of energy:

\[
E_i = mgh
\]

\[
E_f = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \quad \text{but} \quad v = WR \Rightarrow \omega = \frac{v}{R}
\]

\[
v^2 = \frac{2mgh}{(m + I/R^2)}
\]

we need the net moment of the wheel:

\[
m = \lambda (2\pi r) + 3\lambda d = \lambda (2\pi r + 3d)
\]

\[
= (2.5)(2\pi (0.23) + 3(0.46))
\]

\[
= 7.06 \text{ kg}
\]

\[
v = \sqrt{\frac{2mgh}{(m + I/R^2)}} = \sqrt{\frac{2(7.06)(9.8)(58)}{7.06 + (0.23)^2(62319)}} = 26.1 \text{ m/s}
\]
5) One yoga exercise, known as the "Downward-Facing Dog," requires stretching your hands straight out above your head and bending down to lean against the floor. This exercise is performed by a certain 755 N person, as shown in the simplified model in the figure below. When he bends his body at the hip to a 90.0 angle between his legs and trunk, his legs, trunk, head and arms have the dimensions indicated. Furthermore, his legs and feet weigh a total of 260N, and their center of mass is 41.0cm from his hip, measured along his legs. The person's trunk, head, and arms weigh 495N, and their center of gravity is 65.0cm from his hip, measured along the upper body. (a) Find the normal force that the floor exerts on each foot, assuming that the person does not favor either hand or either foot. (b) Find the normal force that the floor exerts on each hand, assuming that the person does not favor either hand or either foot. (c) Find the friction force on each hand, assuming that it is the same on both feet and on both hands (but not necessarily the same on the feet as on the hands). Hint: First treat his entire body as a system; then isolate his legs (or his upper body).

\[ \sin \theta_1 = \frac{135}{162.3} \]
\[ \theta_1 = 56.3^\circ \]

\[ W = 755 \text{ N} \]

\[ \sum F_y = 0 \Rightarrow \sum F_x = 0 \Rightarrow \sum \tau = 0 \]

\[ \sum F_y = 0 \Rightarrow n_1 - w_1 + w_2 = 755 \]
\[ \sum F_x = 0 \Rightarrow f_{s_1} = f_{s_2} \]
\[ \sum \tau = 0 \text{ will calculate torque about feet} \]

\[ \sum \tau = 0 \]
\[ x_1 w_1 + x_2 w_2 = l n_2 \]
\[ n_2 = \frac{x_1 w_1 + x_2 w_2}{l} = \frac{(272)(260) + (104)(465)}{1623} \]
\[ n_2 = 360.8 \text{ newtons} \]

Therefore, \( n_1 = 755 - 360.8 = 394.2 \text{ N} \)

force on hands = \( \frac{360.8}{2} = 180.4 \text{ newtons} \)

force on feet = \( \frac{394.2}{2} = 197.1 \text{ newtons} \)
6) A 1400 kg sedan goes through a wide intersection traveling from north to south when it is hit by a 2100 kg SUV traveling from east to west. The two cars become enmeshed due to the impact and slide as one thereafter. On-the-scene measurements show that the coefficient of kinetic friction between the tires of these cars and the pavement is 0.750, and the cars slide to a halt at a point 5.51m west and 6.55m south of the impact point. (a) How fast was sedan traveling just before the collision? (b) How fast was SUV traveling just before the collision?

\[ m_1 = 1400 \text{ kg} \]
\[ m_2 = 2100 \text{ kg} \]

\[
\tan \theta = \frac{5.51}{6.55} \\
\theta = 40.07^\circ
\]

\[
d = \sqrt{x^2 + y^2} = \sqrt{(5.51)^2 + (6.55)^2} = 8.56 \text{ meters}
\]

\[
\text{Now Work} = \text{Force} \times \text{distance} = M_k (m_1 + m_2) g d
\]
\[ = (0.75)(1400+2100)(9.8)(8.56 \text{ m})
\]
\[ = 220,200 \text{ Joules}
\]

This is the work done by friction, so immediately after the collision, this is how much kinetic energy the joined cars had, thus

\[ E = \frac{1}{2} m v^2 = 1 \times 11,217 \text{ m/s}^2
\]

Therefore, immediately after the collision, the velocity components are

\[ v_x = v \sin 40.07^\circ \]
\[ = 7.22 \text{ m/s} \]

\[ v_y = v \cos 40.07^\circ \]
\[ = 8.58 \text{ m/s} \]

Now kinetic energy is not conserved in this collision, but momentum is.

Therefore

\[ \text{y direction:} \quad (1400) v_{yi} = (3500) v_{fy} \]
\[ v_{yi} = \frac{(3500)}{(1400)}(8.58 \text{ m/s}) = 21.45 \text{ m/s} \]

\[ \text{x direction:} \quad (2100) v_{xi} = (3500) v_{fx} \]
\[ v_{xi} = \frac{(3500)}{(2100)}(7.22 \text{ m/s}) = 12.0 \text{ m/s} \]