Chapter 20 solutions

Q20.22. Reason:
(a) Yes, the field would be zero at a point on the line between the two charges, closer to the 10 nC charge.
(b) In this case the contributions from the two charges are in the same direction on the line between the charges, so there is no point between them at which the fields cancel each other.
Assess: In the first case the field contributions from the two charges are in opposite directions, so they can cancel out when the magnitudes are the same.

Problems

P20.1. Prepare: We will use the charge model. An electron has a negative charge of magnitude 1.6 × 10⁻¹⁹ C.
Solve: (a) In the process of charging by rubbing, electrons are removed from one material and transferred to the other because they are relatively free to move. Protons, on the other hand, are tightly bound in nuclei. So, electrons have been removed from the glass rod to make it positively charged.
(b) The number of electrons removed is
\[
\frac{5 \times 10^{-9}}{1.6 \times 10^{-19}} = 3.1 \times 10^{10}
\]
Assess: A large number of electrons are needed to create a modest charge.

P20.3. Prepare: Use the charge model. Each oxygen molecule has 16 protons (8 per atom), and there are 6.02 × 10²³ oxygen molecules in 1.0 mole of oxygen. The proton has a positive charge of magnitude 1.6 × 10⁻¹⁹ C.
Solve: The amount of positive charge in 1.0 mole of oxygen is
\[
6.02 \times 10^{23} \times (16 \times 1.6 \times 10^{-19} C) = 1.5 \times 10^{6} C
\]
Assess: Coulomb is a “big” unit of charge, so 1 mole of oxygen has a lot of positive charge.

P20.6. Prepare: We will use the charge model and the model of a conductor as a material through which electrons move. An electron has a negative charge of magnitude 1.6 × 10⁻¹⁹ C.
Solve: Because the metal spheres are identical, the total charge is split equally between the two spheres. That is, \(q_A = q_B = 5 \times 10^{11}\) electrons. Thus, the charge on metal sphere A and B is \((5 \times 10^{11})(-1.6 \times 10^{-19} C) = -80 nC\).
Assess: Flow of charge from one charged conductor to another occurs when they come into contact.

P20.8. Prepare: When two identical conducting spheres are in contact the charge is evenly distributed between them.
Solve: (a) The following grid shows the initial charge on each of the spheres and the charge after each event.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial charge</td>
<td>(q)</td>
<td>(-q/2)</td>
<td>O</td>
</tr>
<tr>
<td>C is touched to B</td>
<td>(q)</td>
<td>(-q/4)</td>
<td>(-q/4)</td>
</tr>
<tr>
<td>C is touched to A</td>
<td>(3q/8)</td>
<td>(-q/4)</td>
<td>(3q/8)</td>
</tr>
</tbody>
</table>
(b) The following grid shows the initial charge on each of the spheres and the charge after each event.
<table>
<thead>
<tr>
<th>Initial charge</th>
<th>( q )</th>
<th>(-q/2)</th>
<th>( O )</th>
</tr>
</thead>
<tbody>
<tr>
<td>C is touched to A</td>
<td>( q/2 )</td>
<td>(-q/2)</td>
<td>( q/2 )</td>
</tr>
<tr>
<td>C is touched to B</td>
<td>( q/2 )</td>
<td>( O )</td>
<td>( O )</td>
</tr>
</tbody>
</table>

**Assess:** The end result depends on the order in which the various events occur.

**P20.15. Prepare:** Please refer to Figure P20.15. The charged particles are point charges. The charge \( q_2 \) is in static equilibrium, so the net force on \( q_2 \) is zero. If \( q_2 \) is positive, \( q_1 \) will have to be positive to make the net force zero on \( q_2 \). And, if \( q_2 \) is negative, \( q_1 \) will still have to be positive for \( q_2 \) to be in equilibrium. We will assume that the charge \( q_2 \) is positive. For this situation, the force on \( q_2 \) by the \(-2 \text{nC}\) charge is to the left and by \( q_1 \) is to the right.

**Solve:** We have

\[
\vec{F}_{\text{net on } q_2} = \vec{F}_{q_1 \text{ on } q_2} + \vec{F}_{-2 \text{nC on } q_2} = \left( \frac{1}{4\pi \varepsilon_0} \frac{|q||q_2|}{(0.2 \text{ m})^2}, +\text{x-direction} \right) + \left( \frac{1}{4\pi \varepsilon_0} \frac{(2 \times 10^{-9} \text{ C})|q_2|}{(0.1 \text{ m})^2}, -\text{x-direction} \right) = 0 \text{ N/C}
\]

Thus,

\[
\frac{q_1}{(0.2 \text{ m})^2} - 2 \times 10^{-9} \text{ C} \frac{(0.1 \text{ m})^2}{(0.05 \text{ m})^2} = 0 \text{ N/C} \Rightarrow q_1 = 8.0 \text{ nC}
\]

**Assess:** If the charge \( q_2 \) is assumed negative, the force on \( q_2 \) by the \(-2 \text{nC}\) charge is to the right and by \( q_1 \) is to the left. The magnitude of \( q_1 \) remains unchanged.

**P20.20. Prepare:** We could compute the original distance between the charges, but it is not necessary. The field at the position of the 50 nC particle is the force between the two divided by that charge: \( F = qE \Rightarrow E = F/q \).

**Solve:**

\[
E_{\text{due to } 50 \text{nC}} = k \frac{30 \text{nC}}{r^2} = \frac{F_{\text{between } 30 \text{nC and } 50 \text{nC}}}{50 \text{nC}} = \frac{0.035 \text{N}}{50 \text{nC}} = 7.0 \times 10^5 \text{N/C}
\]

**Assess:** This method is shorter than computing the original distance and then computing the field, and demonstrates deeper understanding. The answer is a typical field strength.

**P20.25. Prepare:** The electric field is that of the two charges placed on the \( y \)-axis. Please refer to Figure P20.25. We denote the upper charge by \( q_1 \) and the lower charge by \( q_2 \). The electric field at the dot due to the positive charge is directed away from the charge and making an angle of 45° below the \(+x\) axis, but the electric field due to the negative charge is directed toward it making an angle of 45° below the \(-x\) axis.

**Solve:** The electric field strength of \( q_1 \) is

\[
E_1 = k \frac{|q_1|}{r_1^2} = \frac{(9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1 \times 10^{-9} \text{ C})}{(0.05 \text{ m})^2 + (0.05 \text{ m})^2} = 1800 \text{ N/C}
\]

Similarly, the electric field strength of \( q_2 \) is

\[
E_2 = k \frac{|q_2|}{r_2^2} = \frac{(9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1 \times 10^{-9} \text{ C})}{(0.05 \text{ m})^2 + (0.05 \text{ m})^2} = 1800 \text{ N/C}
\]

We will now calculate the components of these electric fields. The electric field due to \( q_1 \) is away from \( q_1 \) in the fourth quadrant and that due to \( q_2 \) is toward \( q_2 \) in the third quadrant. Their components are

\[
E_{1x} = E_1 \cos 45° \\
E_{1y} = -E_1 \sin 45° \\
E_{2x} = -E_2 \cos 45° \\
E_{2y} = -E_2 \sin 45°
\]
The $x$ and $y$ components of the net electric field are:

$$ (E_{\text{net}})_x = E_{1x} + E_{2x} = E_x \cos 45^\circ - E_y \cos 45^\circ = 0 \, \text{N/C} $$
$$ (E_{\text{net}})_y = E_{1y} + E_{2y} = -E_x \sin 45^\circ - E_y \sin 45^\circ = -2500 \, \text{N/C} $$

Thus, the strength of the electric field is 2500 N/C and its direction is vertically downward.

**Assess:** A quick visualization of the components of the two electric fields shows that the horizontal components cancel.

**P20.26. Prepare:** A field is the agent that exerts an electric force on a charge. Because the weight of the plastic ball acts downward, the electric force must act upward.

![Diagram of electric field and force](attachment:image.png)

**Solve:** Newton’s second law on the plastic ball is \( \Sigma (\vec{F}_{\text{net}})_y = \vec{F}_{\text{on q}} - \vec{w} \). To balance the weight with the electric force,

$$ F_{\text{on q}} = w \Rightarrow |q| E = mg \Rightarrow E = \frac{mg}{|q|} = \frac{(1.0 \times 10^{-3} \, \text{kg})(9.8 \, \text{N/kg})}{3.0 \times 10^{-8} \, \text{C}} = 3.3 \times 10^6 \, \text{N/C} $$

Because \( F_{\text{on q}} \) must be upward and the charge is negative, the electric field at the location of the plastic ball must be pointing downward. Thus \( \vec{E} = (3.3 \times 10^6 \, \text{N/C}, \text{downward}) \).

**Assess:** \( \vec{F} = q\vec{E} \) means the sign of the charge \( q \) determines the direction of \( \vec{F} \) or \( \vec{E} \). For positive \( q \), \( \vec{E} \) and \( \vec{F} \) are pointing in the same direction. But \( \vec{E} \) and \( \vec{F} \) point in opposite directions when \( q \) is negative.

**P20.29. Prepare:** The electric field magnitude between the two parallel plates of the capacitor is given by Equation 20.7.

**Solve:** The electric field is \( E = Q/(\varepsilon_o A) \). So,

$$ \frac{E_f}{E_i} = \frac{Q_f/A_f}{Q_i/A_i} = \left( \frac{Q_f}{Q_i} \right) \left( \frac{A_i}{A_f} \right) $$

(a) If \( Q \) is doubled (with area remaining the same), the ratio of the final and initial electric field strengths will be doubled.

(b) If the length \( L \) of the plates is doubled, the area increases by a factor of 4. Thus, with the charge on the plates remaining the same, a doubling of length will decrease the ratio by a factor of 4.

(c) The electric field does not depend on the separation between the plates, so the ratio will be 1.

**P20.42. Prepare:** The two charged spheres are point charges. The electric force on one charged sphere due to the other charged sphere is equal to the sphere’s mass multiplied by its acceleration. Because the spheres are identical and equally charged, \( m_1 = m_2 = m \) and \( q_1 = q_2 = q \).

**Solve:** We have
\[ F_{2 \text{ on } 1} = F_{1 \text{ on } 2} = \frac{Kq_1q_2}{r^2} = \frac{Kq^2}{r^2} = ma \]
\[ \Rightarrow q^2 = \frac{ma}{K} = \frac{(1.0 \times 10^{-3} \text{ kg})(225 \text{ m/s}^2)(2.0 \times 10^{-2} \text{ m})^2}{9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2} = 1.0 \times 10^{-14} \text{ C}^2 \]
\[ \Rightarrow q = 1.0 \times 10^{-7} \text{ C} = 100 \text{ nC} \]

**P20.45. Prepare:** The electric field is that of the two 1 nC charges located on the y-axis. Please refer to Figure P20.45. We denote the top 1 nC charge by \( q_1 \) and the bottom 1 nC charge by \( q_2 \). The electric fields \( (\vec{E}_1 \text{ and } \vec{E}_2) \) of both the positive charges are directed away from their respective charges. With vector addition, they yield the net electric field \( \vec{E}_{\text{net}} \) at the point P indicated by the dot.

**Solve:** The electric fields from \( q_1 \) and \( q_2 \) are
\[ \vec{E}_1 = \left( \frac{K|q_1|}{r_1^2}, \text{ along } +x\text{-axis} \right) = \left( \frac{(9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1 \times 10^{-9} \text{ C})}{(0.05 \text{ m})^2}, \text{ along } +x\text{-axis} \right) = (3600 \text{ N/C, along } +x\text{-axis}) \]
\[ \vec{E}_2 = \left( \frac{1}{4\pi\varepsilon_0} \frac{|q_2|}{r_2^2}, \theta \text{ above } +x\text{-axis} \right) = (720 \text{ N/C, } \theta \text{ above } +x\text{-axis}) \]

Because \( \tan \theta = \frac{10 \text{ cm}/5 \text{ cm}}{9.3} \approx 63.43^\circ \).
We will now calculate the components of these electric fields. The electric field due to \( q_1 \) is away from \( q_1 \) along +x and that due to \( q_2 \) is away from \( q_2 \) in the first quadrant. Their components are
\[ E_{1x} = E_1 \]
\[ E_{1y} = 0 \]
\[ E_{2x} = E_1 \cos 63.45^\circ \]
\[ E_{2y} = E_2 \sin 63.45^\circ \]

The x and y components of the net electric field are:
\[ (E_{\text{net}})_x = E_{1x} + E_{2x} = E_1 + E_1 \cos 63.45^\circ = 3922 \text{ N/C} \]
\[ (E_{\text{net}})_y = E_{1y} + E_{2y} = 0 + E_2 \sin 63.45^\circ = 644 \text{ N/C} \]

Thus, the strength of the electric field at P is
\[ E_{\text{net}} = \sqrt{(3922 \text{ N/C})^2 + (644 \text{ N/C})^2} = 3975 \text{ N/C} \]
which will be reported as 4000 N/C.
To find the angle this net vector makes with the x-axis, we calculate
\[ \tan \phi = \frac{644 \text{ N/C}}{3922 \text{ N/C}} \Rightarrow \phi = 9.3^\circ \]

**Assess:** Because of the inverse square dependence on distance, \( E_1 < E_2 \). Additionally, because the point P has no special symmetry relative to the charges, we expected the net field to be at an angle relative to the x-axis.

**P20.49. Prepare:** The charges are point charges. Please refer to Figure P20.49. Placing the 1 nC charge at the origin and calling it \( q_1 \), the \( q_2 \) charge is in the first quadrant, the \( q_3 \) charge is in the fourth quadrant, the \( q_4 \) charge is in the third quadrant, and the \( q_5 \) charge is in the second quadrant. The electric force on \( q_1 \) is the vector sum of the forces \( \vec{F}_{2 \text{ on } 1}, \vec{F}_{3 \text{ on } 1}, \vec{F}_{4 \text{ on } 1}, \text{ and } \vec{F}_{5 \text{ on } 1} \).

**Solve:** The magnitude of the four forces is the same because all four charges are equal and equidistant from \( q_1 \). So,
Thus, $F_{\text{net}} = (3.6 \times 10^{-4} \text{ N}, \text{ toward } q_2) + (3.6 \times 10^{-4} \text{ N}, \text{ toward } q_3) + (3.6 \times 10^{-4} \text{ N}, \text{ toward } q_4) + (3.6 \times 10^{-4} \text{ N}, \text{ toward } q_5)$.

It is now easy to see that the net force on $q_1$ is zero.

**Assess:** Look at the symmetry of the charges. It is no surprise that the net force on charge $q_1$ is zero.

**P20.50.** Prepare: The charges are point charges. Please refer to Figure P20.50.

Solve: Placing the 1 nC charge at the origin and calling it $q_1$, the $q_2$ charge is in the first quadrant, the $q_3$ charge is in the third quadrant, and the $q_4$ charge is in the second quadrant. The electric force on $q_1$ is the vector sum of the electric forces from the other four charges $q_2$, $q_3$, $q_4$, and $q_5$. The magnitude of these four forces is the same because all four charges are equal in magnitude and are equidistant from $q_1$. So,

$$F_{\text{net}} = F_{2 \text{ on } 1} = F_{3 \text{ on } 1} = F_{4 \text{ on } 1} = F_{5 \text{ on } 1} = \frac{(9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(2 \times 10^{-9} \text{ C})(1 \times 10^{-9} \text{ C})}{(0.5 \times 10^{-2} \text{ m})^2 + (0.5 \times 10^{-2} \text{ m})^2} = 3.6 \times 10^{-4} \text{ N}$$

Thus, $F_{\text{net}} = (3.6 \times 10^{-4} \text{ N}, \text{ away from } q_2) + (3.6 \times 10^{-4} \text{ N}, \text{ away from } q_3) + (3.6 \times 10^{-4} \text{ N}, \text{ toward } q_4) + (3.6 \times 10^{-4} \text{ N}, \text{ toward } q_5)$.

So the force on the 1 nC charge is $1.02 \times 10^{-3} \text{ N}$ directed to the left.

**P20.65.** Prepare: The charged ball attached to the string is a point charge. The ball is in static equilibrium in the external electric field when the string makes an angle $\theta = 20^\circ$ with the vertical. The three forces acting on the charged ball are the electric force due to the field, the weight of the ball, and the tension force.

**Solve:** In static equilibrium, Newton’s second law for the ball is $F_{\text{net}} = T + \vec{w} + \vec{F}_e = \vec{0}$. In component form,

$$F_{\text{net}} = T_x + 0 \text{ N} + qE = 0 \text{ N} \quad (F_{\text{net}})_y = T_y - mg + 0 \text{ N} = 0 \text{ N}$$
The two previous equations simplify to

\[ T \sin \theta = qE \quad T \cos \theta = mg \]

Divide the first equation by the second to get

\[ \tan \theta = \frac{qE}{mg} \Rightarrow q = \frac{mg \tan \theta}{E} = \frac{(5.0 \times 10^{-3} \text{ kg})(9.8 \text{ N/kg}) \tan 20^\circ}{100,000 \text{ N/C}} = 1.78 \times 10^{-7} \text{ C} = 180 \text{ nC} \]

to two significant figures.