There are 8 problems for a total of 100 points. Show your work in order to receive credit.

(24 pts.) 1. Let \( p \): Jack is nimble, \( q \): Jill is quick, and \( r \): Jose jumped over the candlestick.

(a) Write each of the following in terms of \( p, q, r \), and appropriate logical connections.
   i. Jack is nimble and Jill is not quick.
   ii. It is not the case that (Jose jumped over the candlestick and Jack is nimble).
   iii. Jack is nimble and Jill is quick, or Jose didn’t jump over the candlestick.

(b) Write an English sentence that corresponds to each of the following.
   i. \( \overline{p} \land r \)
   ii. \( q \lor (p \land q) \)
   iii. \( \overline{p} \lor r \)

(10 pts.) 2. If \( p, r \) are true and \( q \) is false, find the truth value of the following proposition.
   \[
   (p \rightarrow q) \lor [(\overline{p} \land r) \rightarrow (p \land r)]
   \]

(16 pts.) 3. Make a truth table for \( (p \land (p \lor q)) \).

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>(p \land (p \lor q))</th>
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<tbody>
<tr>
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(10 pts.) 4. Restate the proposition “A sufficient condition for Katrina to take the algorithms course is that she pass discrete mathematics.” in the form of a conditional proposition.
5. Write the converse and the contrapositive of the proposition of problem 4.
converse:

contrapositive:

6. Let \( p(x) \): if \( x > 1 \), then \( x^2 > x \).
Write the following statement in words and tell whether it is true or false.
For every \( x \), \( p(x) \).

7. Which of the followings is logically equivalent to \( \exists x \, \forall y \, P(x, y) \).
   (a) \( \forall x \, \forall y \, P(x, y) \)
   (b) \( \exists x \, \forall y \, P(x, y) \)
   (c) \( \forall x \, \exists y \, P(x, y) \)
   (d) \( \forall x \, \exists y \, P(x, y) \)

8. Use mathematical induction to prove that the following statement is true for every positive integer \( n \).
\[
5 + 10 + 15 + \cdots + 5n = \frac{5n(n + 1)}{2}
\]