MTH231 FINAL December 5, 2001

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There are 9 problems for a total of 150 points. Show your work in order to receive credit.

(10 pts.) 1. Make a truth table for \((p \rightarrow q) \lor [(\neg q \land r) \rightarrow (p \land r)]\).


(15 pts.) 2. Prove that the following statement is true by using mathematical induction.

\[
1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + n(n + 1) = \frac{n(n + 1)(n + 2)}{3}
\]
3. Let \( A = \{ x \in \mathbb{R} \mid x^3 + 1 = 0 \}, \ B = \{ x \in \mathbb{Z} \mid x = 3m \}, \ C = \{ 0, 2, 4, 6, \ldots \}, \) and \( D = \{ x \in \mathbb{Z} \mid x^2 \leq 25 \}. \) Determine if each of the following is true or false.

(a) \( C \subseteq D \)  (b) \( \{ 4, 16 \} \subseteq C \)  (c) \( \{ 4, 16 \} \subseteq E \)  
(d) \( D \subseteq D \)  (e) \( B \subseteq \emptyset \)

4. Write the negation of each of the following sentences it could be read as an English sentence.

(a) The weather is bad and I will not go to work.

(b) If Carol goes to the picnic, then she will have a good time.

(c) I will not win the game or I will not enter the contest.

5. For the sequence \( a \) defined by \( a_n = 3n + 4, \) find

(a) \( a_5 \)

(b) \( \sum_{i=1}^{4} a_i \)

(c) \( \prod_{i=1}^{4} a_i \)

6. Let \( b_n = \sum_{i=1}^{n} (i + 2)^2 - i^2. \)

(a) Find \( b_3 \) and \( b_4. \)

(b) Is \( b \) increasing or decreasing?

7. For the following, use the hashing function \( h, \) which takes the first three digits of the account number as one number and the last four digits as another number, adds them, and then applies the mod 59 function.

(a) Assume there are 8250 customer records to be stored using this hashing function.

i. How many linked lists will be required for the storage of these records?

ii. If an approximately even distribution is achieved, roughly how many records will be stored by each linked list?

(b) Determine to which list the given customer account should be attached.

i. 2561384

ii. 6082376

iii. 4984620
8. Use the Euclidean algorithm to compute GCD(4296, 7260). Show all your work.

9. Recall that the Fibonacci sequence is defined as follows.
   \[ a_1 = a_2 = 1; \ a_n = a_{n-1} + a_{n-2} \text{ for } n \geq 3 \]

   (5 pts.) (a) Find \( a_6 \) and \( a_7 \).

   (10 pts.) (b) Write a recursive algorithm to compute \( a_n \).

   (10 pts.) (c) Give a proof using mathematical induction that your algorithm is correct.