1. You forgot to study for your Statistics test. The test consists of 20 problems all multiple-choice. Each question has 5 possible answers. You randomly guess the answer to each question.
   a. What is the expected number you will answer correctly?

   X is binomial with \( n=20, p=0.20 \) so \( E[X] = np = (20)(0.20) = 4 \) problems.

   b. What is the probability that you will get at most 2 correct?

   \[ P(X \leq 2) = P(0) + P(1) + P(2) = 0.012 + 0.058 + 0.0137 = 0.207 \]

   I used the binomial table in the back of the book with \( n=20; \ p=0.20; \ x=0,1,2 \)

   c. What is the probability that you will get at least 3 correct?

   \[ P(X \geq 3) = P(3)+P(4)+...+P(20) \OR \]

   \[ P(X \geq 3) = 1 - P(X \leq 2) = 1 - 0.207 = 0.793 \] (I used the result from part b)

   d. Would it be unusual to get 15 correct?

   I need to find the z-score. I already know the mean is 4 from part a). The standard deviation is \( \sigma = \sqrt{20(0.2)(0.8)} = 1.79 \)

   \[ z = \frac{15 - 4}{1.79} = 6.15 \] which is greater than 2 so yes, this is unusual.
2. Let $X$ be the amount of orange juice (in ounces per day) consumed by an American. Suppose that it is known that the standard deviation of $X$ is $\sigma = 3.4$. The Orange juice Growers Association asserted that Americans consumed, on average, 15 ounces of orange juice per day. To check this assertion a random sample of 50 people were asked how much orange juice they drink per day and the average was found to be $\bar{x} = 14$ ounces of orange juice.

a. Find a 90% confidence interval for $\mu$, the true average number of ounces per day of orange juice consumed by an American. Does your confidence interval support the claim of the Orange juice Growers Association? Explain.

$$z_{\frac{\alpha}{2}} = 1.645; \ E = 1.645 \frac{3.4}{\sqrt{50}} = 0.79$$

Confidence interval: $(14 - 0.79, 14 + 0.79) = (13.21 \text{ ounces}, 14.79 \text{ ounces})$. NO, the confidence interval does not support the claim that the average is 15 ounces because 15 is not in our confidence interval. Our interval suggests the average is less.

b. Find a 99% confidence interval for $\mu$, the true average number of ounces per day of orange juice consumed by an American. Does your confidence interval support the claim of the Orange juice Growers Association? Explain.

$$z_{\frac{\alpha}{2}} = 2.58; \ E = 2.58 \frac{3.4}{\sqrt{50}} = 1.24$$

Confidence interval: $(14 - 1.24, 14 + 1.24) = (12.76 \text{ ounces}, 15.24 \text{ ounces})$. YES, this confidence interval does support the claim since 15 ounces is in the confidence interval.

c. Suppose you want to do an additional test and find a 99% confidence interval with margin of error only 0.5. How many people should be in your sample?

$$n = \left( \frac{2.58(3.4)}{0.5} \right)^2 = 307.79$$

The sample size should be 308 people.
3. In a casino game called HIGH-LOW, there are three possible bets. Assume that $1 is the size of the bet. A pair of fair, 6-sided dice is rolled and their sum is calculated. If you bet LOW, you win $1 if the sum of the dice is \{2,3,4,5,6\}. If you bet HIGH, you win $1 if the sum of the dice is \{8,9,10,11,12\}. If you bet on \{7\}, you win $4 if a sum of 7 is rolled. Otherwise you lose on each of the three bets. In all three cases, your original dollar is returned if you win. Find the expected value of the game to the bettor for each of these three bets.

For your convenience a sample space of two rolled dice is given:
{ (1,1), (1,2), (1,3), (1,4), (1,5), (1,6),
  (2,1), (2,2), (2,3), (2,4), (2,5), (2,6),
  (3,1), (3,2), (3,3), (3,4), (3,5), (3,6),
  (4,1), (4,2), (4,3), (4,4), (4,5), (4,6),
  (5,1), (5,2), (5,3), (5,4), (5,5), (5,6),
  (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}

Bet LOW:

We will either win (X=$1) or lose (X=-$1)
P(1) = 15/36; P(-1) = 21/36
so E[x] = 1(15/36) + (-1)(21/36) = - $0.17

Bet HIGH:

We will either win (X=$1) or lose (X=-$1)
P(1) = 15/36; P(-1) = 21/36
so E[x] = 1(15/36) + (-1)(21/36) = - $0.17

Bet 7:
We will either win (X=$4) or lose (X=-$1)
P(1) = 6/36; P(-1) = 30/36
so E[x] = 4(6/36) + (-1)(30/36) = - $0.17
4. Let X be the number of points scored by the WOU Wolves in a football game. Suppose that X has a normal distribution with mean $\mu = 26$ with standard deviation $\sigma = 16$.

a. Draw a well-labeled sketch of this normal curve. Label at least 5 points on the x-axis

   label the x-axis at the mean (26) and at the points +/- 1,2,3 standard deviations above/below.

b. Find the probability that the Wolves will score less than 20 points in their next game.

\[
P(X < 20) = P(Z < \frac{20 - 26}{16}) = P(Z < -0.38) = 0.3520 \text{ (normal table)}
\]

c. Find the probability that in the next 5 games the average number of points scored will be at least 30 points.

I see the word “average” so this is a CLT problem:

\[
P(\bar{x} \geq 30) = P \left( Z \geq \frac{30 - 26}{\frac{16}{\sqrt{5}}} \right) = P(Z \geq 0.56) = 1 - 0.7123 = 0.2877
\]

Note – even though our sample size was only 5, we could still use the CLT since our underlying distribution was assumed to be normal.