MTH231 MIDTERM II November 16, 2001

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There are 9 problems for a total of 100 points. Show your work in order to receive credit.

1. Let $A = \{a, \{b, c\}, d\}$. Identify each of the following as true or false. (2 pts. each)
   
   (a) $c \in A$
   
   (b) $\{c, d\} \in A$
   
   (c) $\{b, c\} \in A$
   
   (d) $\{b, c\} \subseteq A$
   
   (e) $\{d\} \subseteq A$
   
   (f) $\{a, d\} \subseteq A$

2. Let $U = \{m, a, t, h, i, s, f, u, n\}$ and the sets $J = \{m, a, n\}, K = \{f, a, u, s, t\}$, and $L = \{s, i, a, m\}$.
   
   (a) Find $(J \cup K) \cap L =$

   (b) Draw a Venn diagram that shows the relationships among the three sets.

3. For the sequence $a$ defined by $a_n = 4n - 3$, find (4 pts. each)
   
   (a) $a_6$
   
   (b) $\sum_{i=1}^{4} a_i$
   
   (c) $\prod_{i=1}^{4} a_i$

4. Let $b_n = \sum_{i=1}^{n} (i - 1)^2 + i^2$.
   
   (a) Find $b_2$ and $b_4$.

   (b) Is $b$ increasing or decreasing?
Let $\alpha = aacbdd$ and $\beta = a^2b^3cd$. Find

(a) $\alpha \beta$
(b) $\beta^2$
(c) $|\beta|$
(d) $|\alpha \beta^2 \alpha|$

Determine whether the relation $R$ on the set $X$ is an equivalence relation.

(a) $X = \{a, b, c, d\}$, $R = \{(a, a), (b, a), (b, b), (c, c), (d, d), (d, c)\}$
(b) $X = \{1, 2, 3, 4, 5\}$, $R = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3, 1), (2, 3), (3, 3), (4, 4), (3, 2), (5, 5)\}$

In the following, determine whether the relation defined on the set of positive integers is reflexive, symmetric, antisymmetric, and/or transitive.

(a) $(x, y) \in R$ if $x + y$ is even.
(b) $(x, y) \in R$ if $x = y^k$ for some positive integer $k$.
(c) Determine whether the above relations are equivalence relation, partial order, or neither.

Compute each of the following.

(a) $\lfloor 2.78 \rfloor =$
(b) $\lceil -17.3 \rceil =$
(c) $\lfloor 2.78 \rfloor =$
(d) $\lceil -17.3 \rceil =$

For the following, use the hashing function $h$, which takes the first three digits of the account number as one number and the last four digits as another number, adds them, and then applies the mod 59 function.

(a) Assume there are 7500 customer records to be stored using this hashing function.
   i. How many linked lists will be required for the storage of these records?
   ii. If an approximately even distribution is achieved, roughly how many records will be stored by each linked list?
(b) Determine to which list the given customer account should be attached.
   i. 3759273
   ii. 7149021
   iii. 5167249