Potential Energy of a Battery

Description: The electric potential and potential energy associated with a battery.

Learning Goal: To understand electrical potential, electrical potential energy, and the relationship between them.

Electric potential and electric potential energy are related but different concepts. Be careful not to confuse the terms. Electrical potential energy $U$ is the potential energy that a charge $q$ has due to its position relative to other charges. The electric potential $V$ at a specific position is a measure of the amount of potential energy per unit charge that a particle of net charge $q$ would have at that position. In other words, if a charge $q$ has an electric potential energy $U$, the electric potential $V$ at the location of $q$ is

$$V = \frac{U_E}{q}.$$

Recall that the gravitational potential energy ($U_g = mg\Delta y$) of an object of mass $m$ depends on where you define $y=0$. The difference $\Delta U$ in gravitational potential energy between two points is the physically relevant quantity. Similarly, for electric potential energy, the important quantity is the change in electric potential energy: $\Delta U = q \Delta V$. This is why we often just measure the potential difference $\Delta V$. When we say that the potential of a car battery is 12 V, we mean that the potential difference between the positive and negative terminals of the battery is 12 V.

Consider dropping a ball from rest. This ball moves from a state of high gravitational potential energy to one of low gravitational potential energy as it falls to the ground. Similarly, charges move from a state of high electric potential energy to one of low electric potential energy.

Part A

Mustang Sally just finished restoring her 1965 Ford Mustang car. To save money, she did not get a new battery. When she tries to start the car, she discovers that the battery is dead and she needs a jump start. While unhooking the jumper cables, the positive and negative cables almost touch and a spark jumps between the ends of the cables. This spark is caused by the movement of electrons through the air between the battery terminals. In what direction are the electrons traveling?

**Hint A.1 Another way to think about the movement of charge**

You can think of the movement of charges in terms of Coulomb's force. A positive (high) potential is created by positive charges and a low (negative) potential is created by negative charges. To understand which way electrons will flow across a potential difference, think about the forces on an electron. An electron will be repelled by a negative charge and attracted to a positive charge. The negative terminal of a battery can be viewed as having a negative charge.

**ANSWER:** The electrons are traveling from terminal.

The positive terminal is at a higher potential than the negative terminal. Unless provided with energy, positive charges will flow from a high to a low potential, and negatively charged electrons will flow from a low to a high potential. The table below summarizes this movement.

<table>
<thead>
<tr>
<th>$\Delta U_E$</th>
<th>$q$</th>
<th>$\Delta V$</th>
<th>Direction of motion</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-$</td>
<td>$+$</td>
<td>$-$</td>
<td>high to low potential</td>
</tr>
<tr>
<td>$-$</td>
<td>$-$</td>
<td>$+$</td>
<td>low to high potential</td>
</tr>
</tbody>
</table>

*Since potential difference is the energy per unit charge, it is measured in units of energy divided by charge. Specifically, potential difference is generally measured in volts (whose symbol is V). One volt is...*
equal to one joule per coulomb: $1 \text{V} = 1 \text{J} / 1 \text{C}$.

Part B

There is a 12 V potential difference between the positive and negative ends of the jumper cables, which are a short distance apart. An electron at the negative end ready to jump to the positive end has a certain amount of potential energy. On what quantities does this electrical potential energy depend?

**Hint B.1**  The expression for electric potential energy

The electric potential energy difference is given by the potential difference times the charge:

$$\Delta U_E = q \Delta V$$

**ANSWER:**
- the distance between the ends of the cables
- the potential difference between the ends of the cables
- the charge on the electron
- the distance and the potential difference
- the distance and the charge
- the potential difference and the charge
- the potential difference, charge, and distance

Part C

Assume that two of the electrons at the negative terminal have attached themselves to a nearby neutral atom. There is now a negative ion with a charge $-2e$ at this terminal. What are the electric potential and electric potential energy of the negative ion relative to the electron?

**ANSWER:**
- The electric potential and the electric potential energy are both twice as much.
- The electric potential is twice as much and the electric potential energy is the same.
- The electric potential is the same and the electric potential energy is twice as much.
- The electric potential and the electric potential energy are both the same.
- The electric potential is the same and the electric potential energy is increased by the mass ratio of the oxygen ion to the electron.
- The electric potential is twice as much and the electric potential energy is increased by the mass ratio of the oxygen ion to the electron.

Part D

What is the electric potential energy of an electron at the negative end of the cable, relative to the positive end of the cable? In other words, assume that the electric potential of the positive terminal is 0 V and that of the negative terminal is -12 V. Recall that $e=1.602 \times 10^{-19} \text{C}$.

Enter your answer numerically in joules.

**ANSWER:**

$$U_E = \frac{1.92 \times 10^{-18}}{12 \times 1.6 \times 10^{-19}} \text{J}$$
Part E

At the negative terminal of the battery the electron has electric potential energy. What happens to this energy as the electron jumps from the negative to the positive terminal?

ANSWER:  
- It disappears.
- It is converted to kinetic energy.
- It heats the battery.
- It increases the potential of the battery.

Just as gravitational potential energy is converted to kinetic energy when something falls, electrical potential energy is converted to kinetic energy when a charge goes from a high potential energy state to a low potential energy state.

Part F

If you wanted to move an electron from the positive to the negative terminal of the battery, how much work would you need to do on the electron?

Hint F.1  Formula for work

The work done on a charge \( q \) is equal to the product \( -q \, \Delta V \).

Enter your answer numerically in joules.

ANSWER:  
\[ W = 1.92 \times 10^{-18} \, \text{J} \]

Because moving a negative charge from the positive to the negative terminal of the battery would increase its electric potential energy, it would take positive work to move the charge. This is similar to lifting a ball upward. You do positive work on the ball to increase its gravitational potential energy.

PSS 21.1 Conservation of energy in charge interactions

Description: Knight/Jones/Field Problem-Solving Strategy 21.1 Conservation of energy in charge interactions.


An alpha particle, which is the same as a helium-4 nucleus, is momentarily at rest in a region of space occupied by an electric field. The particle then begins to move. Find the speed of the alpha particle after it has moved through a potential difference of \( -3.45 \times 10^{-3} \, \text{V} \).

The charge and the mass of an alpha particle are \( q = 3.20 \times 10^{-19} \, \text{C} \) and \( m = 6.68 \times 10^{-27} \, \text{kg} \), respectively.

PROBLEM-SOLVING STRATEGY  Conservation of energy in charge interactions

PREPARE  Draw a before-and-after visual overview. Define symbols that will be used in the problem, list known values, and identify what you're trying to find.

SOLVE  The mathematical representation is based on the law of conservation of mechanical energy:

\[ K_f + qV_f = K_i + qV_i \]

- Find the electric potential at both the initial and final positions. You may need to calculate it from a known expression for the potential, such as that of a point charge.
- \( K_i \) and \( K_f \) are the total kinetic energies of all moving particles.
- Some problems may need additional conservation laws, such as conservation of charge or conservation of momentum.

ASSESS  Check that your result has the correct units, is reasonable, and answers the question.
Prepare

Identify the system involving the alpha particle. Then examine the forces acting within this system to determine whether mechanical energy is conserved. Make certain to draw a before-and-after visual overview and to define your symbols.

Part A

Mechanical energy is conserved in the presence of which of the following types of forces? Select all that apply.

**ANSWER:**
- electrostatic
- magnetic
- frictional
- gravitational

Gravitational, electrostatic, and magnetic forces are the only forces acting on the system involving the alpha particle, so the mechanical energy of the system is conserved. Note that in this problem you can ignore gravitational and magnetic forces because the electric force is much larger.

Part B

Which of the following quantities are unknown? Select all that apply.

**ANSWER:**
- the initial speed of the alpha particle
- the difference in potential between the initial and final positions of the alpha particle
- the value of the electric potential at the initial position of the alpha particle
- the charge of the alpha particle
- the mass of the alpha particle
- the value of the electric potential at the final position of the alpha particle
- the final speed of the alpha particle

The unknown quantity that you have to find is the final speed of the alpha particle, $v_f$. Although you do not know the electric potential at the initial and final positions, you do know the difference between these values, $\Delta V = V_f - V_i = 3.45 \times 10^{-3} \text{ V}$.

Before you start creating the equation you will use to solve for $v_f$, first draw a before-and-after visual overview.

Solve

Use conservation of energy $K_f + qV_f = K_i + qV_i$ to solve for the final kinetic energy. Then use this value to solve for the final velocity of the alpha particle.

Part C

What is the value of the change in potential energy, $\Delta U = U_f - U_i$, of the alpha particle?

**Hint C.1  Relationship between electric potential energy and potential**

At a point where the electric potential is $V$, the electric potential energy $U$ of a charged particle $q$ is $U = qV$. Hence, the change in electric potential energy is related to the change in electric potential:
Express your answer in joules using three significant figures.

\[ \Delta U = q_a \Delta V \text{ J} \]

Part D

What is the final velocity of the alpha particle, \(v_f\)?

**Hint D.1 Identify the initial kinetic energy**

What is the initial kinetic energy \(K_i\) of the alpha particle? Express your answer in joules.

**ANSWER:**

\[ K_i = 0 \text{ J} \]

**Hint D.2 Find the final kinetic energy**

Use the mathematical representation of the law of mechanical energy to find the value of the final kinetic energy \(K_f\) of the alpha particle. Express your answer in joules using three significant figures.

**ANSWER:**

\[ K_f = -q_a \Delta V \text{ J} \]

**Hint D.3 Kinetic energy relationship**

The kinetic energy of a particle with mass \(m\) and speed \(v\) is \(K = \frac{1}{2}mv^2\). Express your answer in meters per second using three significant figures.

**ANSWER:**

\[ (v_f)_\alpha = \sqrt{-\frac{2q_a \Delta V}{m_a}} \text{ m/s} \]

Assess

Check to see that your final result makes sense.

Part E

If you had carried out the algebra using variables before plugging numbers into your expressions, you would have found that
\[ (v_f)_\alpha = \sqrt{-\frac{2q\Delta V}{m_\alpha}} \]

where delta V is measured in volts. To verify that this expression for \(v_f\) has the correct units of velocity, you need to perform some unit analysis. Begin by finding the equivalent of a volt in terms of basic SI units. What is a volt in terms of meters (m), seconds (s), kilograms (kg), and coulombs (C)? Express your answer using the symbols for the units, meters, seconds, kilograms, and coulombs.

**ANSWER:**

\[ \text{volt} = \frac{\text{kg} \cdot \text{m}^2}{\text{C} \cdot \text{s}^2} \]

A volt is equivalent to \( \frac{\text{kg} \cdot \text{m}^2}{\text{C} \cdot \text{s}^2} \). If you had known this before using the expression given here for \(v_f\), you could have carried out a unit analysis to determine that your expression would give you the desired units of velocity (m/s). This process would look like

**Electric Potential and Potential Energy**

**Description:** Short quantitative problem on the motion of a charged particle along a line of varying potential and a line of constant potential. Based on Young/Geller Quantitative Analysis 18.2.

A particle with charge \(1.60 \times 10^{-19} \text{ C}\) is placed on the \(x\) axis in a region where the electric potential due to other charges increases in the +\(x\) direction but does not change in the \(y\) or \(z\) direction.

**Part A**

The particle, initially at rest, is acted upon only by the electric force and moves from point \(a\) to point \(b\) along the \(x\) axis, increasing its kinetic energy by \(3.20 \times 10^{-19} \text{ J}\). In what direction and through what potential difference \(V_b - V_a\) does the particle move?

**Hint A.1  How to approach the problem**

Because no forces other than the electric force act on the particle, the positively charged particle must move in the direction parallel to the electric field, and the field must do positive work on the particle. Recall that when the electric field does positive work on a charged particle, the potential energy of the particle decreases. Thus, the particle must move in the direction in which its potential energy decreases (which is consistent with the fact that the particle's kinetic energy increases as it moves from \(a\) to \(b\)). Moreover, from the definition of potential and the energy conservation equation, you can directly calculate the potential difference \(V_a - V_b\).

**Hint A.2  Electric potential**

The electric potential \(V\) at any point in an electric field is the electric potential energy \(U\) per unit charge associated with a test charge \(q'\) at that point:

\[ V = \frac{U}{q'} \]

**Hint A.3  Find the change in potential energy of the particle**

What is the change in potential energy of the particle, \(U_b - U_a\), as it moves from \(a\) to \(b\)?

**Hint A.3.1  Energy conservation**
Recall that the total mechanical energy (kinetic plus potential) is conserved. That is,

\[ K_a + U_a = K_b + U_b, \]

where the subscripts refer to points \( a \) and \( b \), and \( K \) and \( U \) are the corresponding kinetic and potential energies.

**Hint A.3.2  Find the change in kinetic energy of the particle**

What is the change in kinetic energy of the particle, \( K_b - K_a \), as it moves from \( a \) to \( b \)? Recall that particle is initially at rest, and its kinetic energy at \( b \) is \( 3.20 \times 10^{-19} \) J.

Express your answer in joules.

**ANSWER:**

\[ K_b - K_a = K \text{ J} \]

Express your answer in joules.

**ANSWER:**

\[ U_b - U_a = -K \text{ J} \]

**ANSWER:**

- The particle moves to the left through a potential difference of \( V_b - V_a = 2.00 \) V.
- The particle moves to the left through a potential difference of \( V_b - V_a = -2.00 \) V.
- The particle moves to the right through a potential difference of \( V_b - V_a = 2.00 \) V.
- The particle moves to the right through a potential difference of \( V_b - V_a = -2.00 \) V.
- The particle moves to the left through a potential difference of \( V_b - V_a = 20.0 \) V.
- The particle moves to the right through a potential difference of \( V_b - V_a = -20.0 \) V.

In general, if no forces other than the electric force act on a positively charged particle, the particle always moves toward a point at lower potential.

**Part B**

If the particle moves from point \( b \) to point \( c \) in the \( y \) direction, what is the change in its potential energy, \( U_c - U_b \)?

**Hint B.1  How to approach the problem**

Recall that the electric potential increases in the \(+x\) direction but does not change in the \( y \) or \( z \) direction.

**ANSWER:**

\[ +3.20 \times 10^{-19} \text{ J} \]

\[ -3.20 \times 10^{-19} \text{ J} \]

\[ 0 \]

Every time a charged particle moves along a line of constant potential, its potential energy remains constant and the electric field does no work on the particle.
Electric Potential Ranking Task

Description: Short conceptual problem involving electrical potentials of point charges. (ranking task)

In the figure there are two point charges, +q and -q. There are also six positions, labeled A through F, at various distances from the two point charges. You will be asked about the electric potential at the different points (A through F).

Part A

Rank the locations A to F on the basis of the electric potential at each point. Rank positive electric potentials as higher than negative electric potentials.

**Hint A.1  Definition of electric potential**

The electric potential surrounding a point charge is defined by

\[ V = \frac{kq}{r} \]

where q is the source charge creating the electric potential and r is the distance between the source charge and the point of interest. If more than one source is present, determine the electric potential from each source and sum the results.

**Hint A.2  Conceptualizing electric potential**

Because positive charges create positive electric potentials in their vicinity and negative charges create negative potentials in their vicinity, electric potential is sometimes visualized as a sort of "elevation." Positive charges represent mountain peaks and negative charges deep valleys. In this picture, when you are close to a positive charge, you are "high up" and have a higher positive potential. Conversely, near a negative charge, you are deep in a "valley" and have a negative potential. The utility of this picture becomes clearer when we begin to think of charges moving through a region of space containing an electric potential. Just as particles naturally roll downhill, converting gravitational potential energy into kinetic energy, positively charged particles naturally "roll downhill" as well, toward regions of lower electric potential, converting electrical potential energy into kinetic energy.
Rank the locations from highest to lowest potential. To rank items as equivalent, overlap them.

**ANSWER:**

![Diagram of two charges and six points (a through f)](image)

**Change in Electric Potential Ranking Task**

**Description:** Short conceptual problem related to the electric potential difference between pairs of points. (ranking task)

In the diagram below, there are two charges of +q and –q and six points (a through f) at various distances from the two charges.

You will be asked to rank changes in the electric potential along paths between pairs of points.

**Part A**

Using the diagram to the left, rank each of the given paths on the basis of the change in electric potential. Rank the largest-magnitude positive change (increase in electric potential) as largest and the largest-magnitude negative change (decrease in electric potential) as smallest.

**Hint A.1 Change in electric potential**

Determining the change in electric potential along some path involves determining the electric potential at the two end points of the path, and subtracting:

\[ \Delta V = V_{\text{final}} - V_{\text{initial}}. \]

**Hint A.2 Determine the algebraic sign of the change in potential**

The path from point d to point a results in a positive change in electric potential. Which of the other paths also involves a positive change in electric potential (i.e., electric potential that increases along the path)?
**ANSWER:**
- from b to a
- from f to e
- from c to d
- from c to e
- from c to b

**Hint A.3  Conceptualizing changes in electric potential**

Since positive charges create large positive electric potentials in their vicinity and negative charges create negative potentials in their vicinity, electric potential is sometimes visualized as a sort of “elevation.” Positive charges represent mountain peaks and negative charges deep valleys. In this picture, when you are close to a positive charge, you are high “up” and have a large positive potential. Conversely, near a negative charge you are deep in a “valley” and have a negative potential. Thus, changes in electric potential can be thought of as changes in elevation. The change is positive if you are moving “uphill” and the change is negative if you move “downhill.” The farther you travel either uphill or downhill, the larger the magnitude of the change in electric potential.

Rank from largest to smallest. To rank items as equivalent, overlap them.

**ANSWER:**

**Electric Fields and Equipotential Surfaces**

**Description:** Find the work done to move a unit charge from and to given points on a diagram showing equipotential surfaces, and compare the magnitude of the electric field at these points.

The dashed lines in the diagram represent cross sections of equipotential surfaces drawn in 1 V
Part A

What is the work $W_{ab}$ done by the electric force to move a 1 C charge from A to B?

**Hint A.1  Find the potential difference between A and B**

What is the potential difference $V_a - V_b$ between point A and point B?

**Hint A.1.1  Equipotential surfaces**

Recall that an equipotential surface is a surface on which the electric potential is the same at every point.

Express your answer in volts.

**ANSWER:**

$$V_a - V_b = 0 \text{ V}$$

**Hint A.2  Potential difference and work**

Recall that the potential difference (in volts) between a point a and a point b equals the work (in joules) done by the electric force to move a 1 C charge from a to b.

Express your answer in joules.

**ANSWER:**

$$W_{ab} = 0 \text{ J}$$

Part B

What is the work $W_{ad}$ done by the electric force to move a 1 C charge from A to D?

**Hint B.1  Find the potential difference between A and D**

What is the potential difference $V_d - V_a$ between point A and point D?
Express your answer in volts.

**ANSWER:**

\[ V_D - V_A = -1 \text{ V} \]

**Hint B.2 Potential difference and work**

Recall that the electric potential energy difference between any two points is equal to the negative of the work done by the electric force as a charged object moves between those two points. If we combine this with the relationship between electric potential energy and electric potential we have:

\[ W_{AD}, \text{ by electric force} = -\Delta V_{AD}Q \]

Express your answer in joules.

**ANSWER:**

\[ W_{AD} = 1 \text{ J} \]

**Part C**

The magnitude of the electric field at point C is

**Hint C.1 Electric field and equipotential surfaces**

Since the diagram shows equal potential differences between adjacent surfaces, equal amounts of work are done to move a particular charge from one surface to the next adjacent one. It follows then that if the equipotentials are closer together, the electric force does the same amount of work in a smaller displacement than if the equipotentials were farther apart. Therefore, the electric force, as well as the corresponding electric field, has a larger magnitude.

**ANSWER:** greater than the magnitude of the electric field at point B.

less than the magnitude of the electric field at point B.

equal to the magnitude of the electric field at point B.

unknown because the value of the electric potential at point C is unknown.

**Capacitance: A Review**

**Description:** Multiple-choice questions on concept of capacitance. Use after Introduction to Capacitance. Some questions are repetitions of those in other problems. This fact may get some students frustrated.

**Learning Goal:** To review the meaning of capacitance and ways of changing the capacitance of a parallel-plate capacitor. Capacitance is one of the central concepts in electrostatics. Understanding its meaning and the difference between its definition and the ways of calculating capacitance can be challenging at first. This tutorial is meant to help you become more comfortable with capacitance. Recall the fundamental formula for capacitance:

\[ C = \frac{Q}{V} \]

where \( C \) is the capacitance in farads, \( Q \) is the charge stored on the plates in coulombs, and \( V \) is the potential difference (or voltage) between the plates. In the following problems it may help to keep in
mind that the voltage is related to the strength of the electric field $E$ and the distance between the plates, $d$, by

$$V = Ed.$$ 

**Part A**

What property of objects is best measured by their capacitance?

**ANSWER:**
- the ability to conduct electric current
- the ability to distort an external electrostatic field
- the ability to store charge

Capacitance is a measure of the ability of a system of two conductors to store electric charge and energy. It is defined as $C = Q/V$. This ratio remains constant as long as the system retains its geometry and the amount of dielectric does not change. Capacitors are special devices designed to combine a large capacitance with a small size. However, any pair of conductors separated by a dielectric (or vacuum) has some capacitance. Even an isolated electrode has a small capacitance. That is, if a charge $Q$ is placed on it, its potential $V$ with respect to ground would change, and the ratio $Q/V$ is its capacitance $C$.

**Part B**

Consider an air-filled charged capacitor. How can its capacitance be increased?

**Hint B.1  What does capacitance depend on?**

Capacitance depends on the inherent properties of the system of conductors, such as its geometry and the presence of dielectric, not on the charge placed on the conductors. Specifically, capacitance depends on the area $A$ of the conducting plates and the distance $d$ between the plates and is given by

$$C' = \varepsilon_0 \frac{A}{d},$$

where $\varepsilon_0$ is a constant called the permittivity of free space.

**ANSWER:**
- Increase the charge on the capacitor.
- Decrease the charge on the capacitor.
- Increase the spacing between the plates of the capacitor.
- Decrease the spacing between the plates of the capacitor.
- Increase the length of the wires leading to the capacitor plates.

**Part C**

Consider a charged parallel-plate capacitor. How can its capacitance be halved? Check all that apply.

**ANSWER:**
- Double the charge.
- Double the plate area.
- Double the plate separation.
- Halve the charge.
- Halve the plate area.
- Halve the plate separation.
Part D

Consider a charged parallel-plate capacitor. Which combination of changes would quadruple its capacitance?

**ANSWER:**
- Double the charge and double the plate area.
- Double the charge and double the plate separation.
- Halve the charge and double the plate separation.
- Halve the charge and double the plate area.
- Halve the plate separation and double the plate area.
- Double the plate separation and halve the plate area.

Video Tutor: Charged Conductor with Teardrop Shape

**Description:** A teardrop-shaped conductor is charged, and its blunt and pointed ends are touched by electrodes of equal surface area. Which electrode acquires a greater charge? First, launch the video below. You will be asked to use your knowledge of physics to predict the outcome of an experiment. Then, close the video window and answer the question at right. You can watch the video again at any point.

Part A

Two conducting spheres are each given a charge Q. The radius of the larger sphere is three times greater than that of the smaller sphere. If the electric field just outside of the smaller sphere is \( E_0 \), then the electric field just outside of the larger sphere is

**Hint A.1 How to approach the problem.**

The electric field just outside of a conductor is proportional to the conductor's surface charge density. The surface charge density of a sphere is calculated by dividing the total charge on the sphere by the sphere's surface area. Think about how changing the radius of a sphere changes the sphere's surface area.

**ANSWER:** \( \frac{1}{9} E_0 \)
The larger sphere has nine times the surface area of the smaller one, and this reduces the surface charge density by a factor of nine.

Questions

**Q21.21. Reason:** The entire top plate is at the same potential as the positive terminal of the battery and the bottom plate is at the same potential as the negative terminal of the battery (assuming ideal wires). Each capacitor plate is an equipotential surface, so the potential is the same anywhere on the top plate as it is at a. The potential difference is the same between any point on the top plate and any point on the bottom plate: 6V.
The correct choice is A.
Assess: The angle of the dotted line is completely irrelevant. The conclusion to this question would be valid even if the plates were not flat or parallel.

**Q21.22. Reason:** Since the direction of the electric field is in the direction of decreasing electric potential, the electric field is directed from the 300 V to the −100 V electric potential. Since positive charges move in the direction of the electric field, the charge will move from the site of the 300 V to the site of the −100 V electric potential. The correct choice is A.
Assess. This question points out two important things: First, the relationship between electric potential difference and the electric field and second, the relationship between the charge on an object and the force it experiences due to an electric field.

**Q21.23. Reason:** By examining the 0 V equipotential line and the 100 V equipotential line we see that each line is 50 V different from an adjacent equipotential line.
To get from the 0 V equipotential line to point C requires us to go in the opposite direction from the positive equipotentials, so our answer must be negative. Point C is two lines over from the −300 V equipotential line, so the potential at C must be −400 V.
The correct choice is A.
Assess: This can be thought of as a contour map showing elevation and the reasoning would lead to the same conclusion.

**Q21.24. Reason:** Knowing the difference in electric potential $\Delta V$ and the change in position $d$ for two equipotential lines, we can determine the electric field near the midpoint between these two equipotential lines by $E = \Delta V/d$, (if the field is nonuniform, the smaller $d$, the more accurate the value of $E$). Using the equipotential lines on either side of the point of interest, we get for each case that $\Delta V_1 = \Delta V_2 = \Delta V_3 = 100$ V.
We can do an eyeball comparison of each $d$ (the separation between the two equipotential lines on either side of the point of interest) to get $d_1 > d_2 > d_3$, which allows us to write $E_1 > E_2 > E_3$. The correct choice is C.
Assess: This question emphasizes the relationship between electric potential difference and electric field.

**Q21.25. Reason:** While the spacing of the equipotential lines varies over the entire figure, the spacing is reasonably constant near point C. The electric field vector will point “down the hill” from higher potential to lower potential, and it will be perpendicular to the equipotential line.
The strength of the electric field is indicated by how close the lines are together. We must make a scale measurement on the figure. Do this with a ruler, or simply take a piece of paper, put it next to either the x- or y-axis, and make a couple of marks on it showing a scaled distance of, say, 0.5 m. Then put the paper over point C perpendicular to the equipotential line with the two marks straddling it and count the number of equipotential lines. There are about eight lines (corresponding to 400 V) in the 0.5 m distance. The field strength is \( E = \frac{\Delta V}{d} = \frac{400 \text{ V}}{0.5 \text{ m}} = 800 \text{ V/m} \).

The correct choice is C. 

Assess: The equipotentials above point C are spaced farther apart than those below point C, but the average around C is easily obtained this way, especially since we aren’t looking for a lot of significant figures.

Q21.27. Reason: Looking at the equipotential lines on either side of point B tells us that each equipotential line is 50 V different from an adjacent equipotential line. Point A is one equipotential line on the negative side of the 0 V equipotential, so the potential at A is –50 V. From Question 23 we know the potential at C is –400 V, so \( \Delta V_{AC} = V_A - V_C = -50 \text{ V} - (-400 \text{ V}) = 350 \text{ V} \).

The work done is equal to the change in electric potential energy.

\[
W = \Delta U_{\text{elec}} = q\Delta V = (+10 \text{ nC})(350 \text{ V}) = 3.5 \times 10^{-4} \text{ J}
\]

The correct choice is A.

Assess: 3.5 \times 10^{-4} \text{ J} is not very much but the charge is small as well, so this is a reasonable answer. Notice the actual distance between points A and C is not important as long as we know the potential difference between them. Also note the units: \( \text{C} \cdot \text{V} = \text{J} \).

PROBLEMS

P21.1. Prepare: The work done is equal to the change in electric potential energy: \( W = \Delta U_{\text{elec}} = q\Delta V \).

We are given that \( \Delta V = 150 \text{ V} - 300 \text{ V} = -150 \text{ V} \).

Solve: Solve the first equation for \( q \):

\[
q = \frac{U}{kq} = (-4.0 \times 10^{-6} \text{ J})(10.0 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(4.4 \times 10^{-10} \text{ C}) = -1.0 \times 10^{-8} \text{ C}
\]

Assess: The answer is negative because it requires positive work to move a negative charge to a lower potential. As for units, \( \text{J} / \text{V} = \text{C} \).

P21.3. Prepare: The work done is equal to the change in electric potential energy: \( W = \Delta U_{\text{elec}} = q\Delta V \).

Solve the equation for \( \Delta V \):

\[
\Delta V = \Delta U_{\text{elec}} / q.
\]

Solve:

As the charge is moved from A to B, its electric potential changes by

\[
\Delta V_{A\rightarrow B} = \frac{(\Delta U_{\text{elec}})_{A\rightarrow B}}{q} = \frac{3.0 \text{ J}}{15 \text{ nC}} = 200 \text{ V}
\]

Thus B is at a higher potential than A.

When the charges is moved from C to B, its electric potential changes by

\[
\Delta V_{C\rightarrow B} = \frac{(\Delta U_{\text{elec}})_{C\rightarrow B}}{q} = \frac{-5.0 \text{ J}}{15 \text{ nC}} = -330 \text{ V}
\]

This means B is at a lower potential than C. Thus if the charge were moved in the opposite direction, from B to C, its electric potential would increase, so that \( \Delta V_{B\rightarrow C} = \Delta V_{C\rightarrow B} = -330 \text{ V} \). The total change in electric potential in moving from A to C is then \( V_C - V_A = \Delta V_{AC} = \Delta V_{AB} + \Delta V_{BC} = 200 \text{ V} - (-330 \text{ V}) = 530 \text{ V} \).

Assess: The potential at B is between the potential at A and the potential at C. So if we move the charge from A to B and then to C, it will take a total work of \( 3.0 \mu \text{J} - (-5.0 \mu \text{J}) = 8.0 \mu \text{J} \).
\[
\Delta V_{AC} = \frac{(U_{\text{elec}})_{AC}}{q} = \frac{8.0 \, \mu J}{15 \, \text{nC}} = 530 \, \text{V}
\]

P21.8. Prepare: Energy is conserved. The potential energy is determined by the electric potential. The figure shows a before-and-after pictorial representation of a He\(^{+}\) ion moving through a potential difference. The ion’s initial speed is zero and its final speed is \(1.0 \times 10^6 \, \text{m/s}\). A positive charge speeds up as it moves into a region of lower potential (\(U \rightarrow K\)).

\[
\begin{align*}
\text{Before} & \quad v_i = 0 \, \text{m/s} \\
\text{After} & \quad v_f = 1.0 \times 10^6 \, \text{m/s}
\end{align*}
\]

\[
\Delta V = V_f - V_i
\]

Solve: The potential energy of charge \(q\) is \(U = qV\). Conservation of energy, expressed in terms of the electric potential \(V\), is

\[
K_i + qV_i = K_i + qV_i \Rightarrow q(V_f - V_i) = K_i - K_f \Rightarrow \Delta V = \frac{K_f - K_i}{q} = \frac{0 \, \text{J} - \frac{1}{2}mv_f^2}{q}
\]

\[
= \frac{4(1.67 \times 10^{-33} \, \text{kg})(1.0 \times 10^6 \, \text{m/s})^2}{2(1.60 \times 10^{-19} \, \text{C})} = -2.1 \times 10^4 \, \text{V}
\]

Assess: This result implies that the helium ion moves from a higher potential toward a lower potential.

P21.9. Prepare: Energy is conserved. The potential energy is determined by the electric potential. The figure shows a before-and-after pictorial representation of an electron moving through a potential difference.

\[
\begin{align*}
\text{Before} & \quad v_i = 500,000 \, \text{m/s} \\
\text{After} & \quad v_f = 0 \, \text{m/s}
\end{align*}
\]

\[
\Delta V = V_f - V_i
\]

Solve: (a) Because the electron is a negative charge and it slows down as it travels, it must be moving from a region of higher potential to a region of lower potential.

(b) Using the conservation of energy equation,

\[
K_i + U_i = K_i + U_i \Rightarrow K_i + qV_i = K_i + qV_i \Rightarrow V_i - V_i = \frac{1}{q}(K_i - K_i) = \frac{1}{(-e)} \left( \frac{1}{2}mv_i^2 - 0 \, \text{J} \right)
\]

\[
\Rightarrow \Delta V = -\frac{mv_i^2}{2e} = -\frac{(9.11 \times 10^{-31} \, \text{kg})(5.0 \times 10^3 \, \text{m/s})^2}{2(1.60 \times 10^{-19} \, \text{C})} = -0.712 \, \text{V} = -0.7 \, \text{V}
\]

(c)

\[
K_i = \frac{1}{2}mv_i^2 = q\Delta V = (-e)(-0.712 \, \text{V}) = 0.7 \, \text{eV}
\]

Assess: The negative sign with \(\Delta V\) verifies that the electron moves from a higher potential region to a lower potential region.

P21.18. Prepare: Please refer to Figure P21.18. The net potential is the sum of the potentials due to each charge given by Equation 21.10.

Solve: The potential at the dot is

\[
V = \frac{1}{4\pi \varepsilon_0} \frac{q_1}{r_1} + \frac{1}{4\pi \varepsilon_0} \frac{q_2}{r_2} + \frac{1}{4\pi \varepsilon_0} \frac{q_3}{r_3} = (9.0 \times 10^9 \, \text{N} \cdot \text{m}^2/\text{C}^2) \left[ 2.0 \times 10^{-4} \, \text{C} \right] \left[ \frac{2.0 \times 10^{-4} \, \text{C}}{0.040 \, \text{m}} + \frac{2.0 \times 10^{-4} \, \text{C}}{0.050 \, \text{m}} + \frac{2.0 \times 10^{-4} \, \text{C}}{0.030 \, \text{m}} \right] = 1400 \, \text{V}
\]
Assess: Potential is a scalar quantity, so we found the net potential by adding three scalar quantities.

P21.19. Prepare: The electric potential difference between the plates is determined by the uniform electric field in the parallel-plate capacitor as given by Equation 21.6.
Solve: (a) The potential of an ordinary AA or AAA battery is 1.5 V. Actually, this is the potential difference between the two terminals of the battery. If the electric potential of the negative terminal is taken to be zero, then the positive terminal is at a potential of 1.5 V.
(b) If a battery with a potential difference of 1.5 V is connected to a parallel-plate capacitor, the potential difference between the two capacitor plates is also 1.5 V. Thus, \( \Delta V_C = 1.5 \, V = V_p - V_n = Ed \)
where \( d \) is the separation between the two plates. The electric field inside a parallel-plate capacitor is

\[
E = \frac{Q}{AE_0} \Rightarrow 1.5 \, V = \frac{Q}{AE_0} d \Rightarrow Q = \frac{(1.5 \, V)(AE_0)}{d} = \frac{(1.5 \, V)(\pi(2.0 \times 10^{-2} \, m)^2)(8.85 \times 10^{-12} \, C^2/N \cdot m^2)}{2.0 \times 10^{-3} \, m} = 8.3 \times 10^{-12} \, C
\]

Thus, the battery moves \( 8.3 \times 10^{-12} \, C \) of electron charge from the positive to the negative plate of the capacitor.
Assess: This is the charge on the positive plate. The other plate has a charge of \(-8.3 \times 10^{-12} \, C\).

P21.20. Prepare: Please refer to Figure P21.20. In a region that has a uniform electric field, Equation 21.17 gives the magnitude of the potential difference between two points.
Solve: (a) The electric field points “downhill.” So, point A is at a higher potential than point B.
(b) The magnitude of the potential difference between points A and B is

\[ \Delta V = Ed = (1000 \, V/m)(0.07 \, m) = 70 \, V \]

That is, the potential at point A is 70 V higher than the potential at point B.
Assess: Electric field points from higher potential to lower potential.

P21.28. Prepare: Knowing that the relationship between the capacitance of the capacitor, the charge on each plate of the capacitor, and the electric potential difference across the plates of the capacitor is \( C = \frac{Q}{V} \), we can determine the capacitance of the capacitor (which will not change), the charge on the plates for the second case, and then the difference in charge.
Solve:

\[
C = \frac{Q}{V_1} = 6.0 \times 10^{-6} \, C/3.0 \, V = 2.0 \times 10^{-6} \, F
\]
\[ Q_2 = CV_2 = (2.0 \times 10^{-6} \, F)(5.0 \, V) = 10 \times 10^{-6} \, C \]
\[ \Delta Q = Q_2 - Q_1 = 10 \times 10^{-6} \, C - 6.0 \times 10^{-6} \, C = 4.0 \times 10^{-6} \, C = 4.0 \, \mu C \]
Assess: Considering the other charges, this is a reasonable value.

P21.35. Prepare: Equation 21.22 gives the capacitance of a parallel-plate capacitor with a dielectric (the paper).

\[
C = \frac{k\varepsilon_0 A}{d}
\]
We are given \( A = (0.35 \, m)^2 = 0.1225 \, m^2 \) and \( d = 0.25 \times 10^{-3} \, m \). Assuming the science fair is at 20°C, we also look up the dielectric constant of paper in Table 21.2: \( \kappa_{\text{paper}} = 3.0 \).

Solve:

\[
C = \frac{k\varepsilon_0 A}{d} = \frac{(3.0)(8.85 \times 10^{-12} \, F/m)(0.1225 \, m^2)}{0.25 \times 10^{-3} \, m} = 13 \, nF
\]
Assess: The answer could be expressed in scientific notation as \( 1.3 \times 10^{-8} \, F \) but capacitances are usually given in pF (sometimes pronounced “puff”), nF, or \( \mu F \). In our calculation the \( m^2 \) cancels, leaving F.
P21.39. Solve: From Equation 21.23, the energy stored in a capacitor is
\[ U_C = \frac{1}{2} C (\Delta V_C)^2 \Rightarrow \Delta V_C = \sqrt{\frac{2 U_C}{C}} = \sqrt{\frac{2(1.0 \text{ J})}{1.0 \times 10^{-6} \text{ F}}} = 1400 \text{ V} \]
Assess: This potential difference is not unusual for capacitors.

(P21.57. Prepare: Outside a charged sphere the electric potential is identical to that of a point charge at the center and is given by Equation 21.11.
Solve: (a) For a proton, assumed to be a point charge, the electric potential is
\[ V = \frac{1}{4\pi \varepsilon_0} \frac{(+e)}{r} = \left(9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right) \frac{1.60 \times 10^{-19} \text{ C}}{0.053 \times 10^{-9} \text{ m}} = 27.2 \text{ V} = 27 \text{ V} \]
(b) The potential energy of a charge \( q \) at a point where the potential is \( V \) is \( U = qV \). The potential energy of the electron in the proton’s potential is
\[ U = (-1.60 \times 10^{-19} \text{ C}) (27.2 \text{ V}) = -4.4 \times 10^{-18} \text{ J} \]

P21.62. Prepare: Equation 21.17 gives the relation between the electric field and the potential difference: \( E = \Delta V/d \).
We are given in the text that \( \Delta V = 0.070 \text{ V} \) and in the problem that \( d = 8 \times 10^{-9} \text{ m} \).
Solve:
\[ E = \frac{\Delta V}{d} = \frac{0.070 \text{ V}}{8 \times 10^{-9} \text{ m}} = 8.75 \times 10^6 \text{ V/m} \approx 8.8 \times 10^6 \text{ V/m} \]
Assess: This rounds to \( 10^7 \text{ V/m} \) to one significant figure. This is very large considering that a spark in air requires a field of only about \( 3 \times 10^6 \text{ V/m} \)

P21.63. Prepare: Equation 21.1 gives the relation between the electric potential energy and the potential difference: \( \Delta U_{\text{elec}} = q \Delta V \).
For a \( \text{Na}^+ \) ion \( q = +e \), \( \Delta V = (0 \text{ V}) - (-70 \text{ mV}) = 70 \text{ mV} \).
Solve:
\[ \Delta U_{\text{elec}} = q \Delta V = (+1e)(70 \text{ mV}) = 0.070 \text{ eV} \]
Because \( \Delta U_{\text{elec}} \) is positive the energy increases.
Assess: The ion is pushed “uphill” in the ion pump by energy derived from ATP.

P21.64. Prepare: If there is an electric potential difference \( \Delta V \) across a cell membrane of thickness \( d \), the electric field in the membrane is \( E = \Delta V/d \). This electric field exerts a force on a charged particle of magnitude \( F = qE = q \Delta V/d \).
Solve: The force on the molecular ion is
\[ F = q \Delta V/d = (10e)(70 \text{ mV}) = (10)(1.6 \times 10^{-19} \text{ C})(7 \times 10^{-2} \text{ V})/5.0 \times 10^{-9} \text{ m} = 2.2 \times 10^{-11} \text{ N} \]
Assess: We expect the force to be small, because the electric field and charge are both small.

P21.66. Prepare: Since the only forces in this problem are the forces between the alpha particle and the antiproton, the total momentum of these two is conserved as they accelerate toward one another. In addition, the sum of their kinetic energies and the potential energy must be constant.
Solve: The conservation of momentum may be written as:

\[ m_\alpha (v_\alpha)_i + m_\bar{\beta} (v_{\bar{\beta}})_i = m_\alpha (v_\alpha)_f + m_\bar{\beta} (v_{\bar{\beta}})_f \]

and the conservation of energy may be written as:

\[ \frac{1}{2} m_\alpha (v_\alpha)_i^2 + \frac{1}{2} m_\bar{\beta} (v_{\bar{\beta}})_i^2 + \frac{K(-e)(2e)}{r_i} = \frac{1}{2} m_\alpha (v_\alpha)_f^2 + \frac{1}{2} m_\bar{\beta} (v_{\bar{\beta}})_f^2 + \frac{K(-e)(2e)}{r_f} \]

The left hand sides of both equations are zero because the initial velocities are zero and the initial separation is great which means that the potential energy term is practically zero. If we solve the momentum equation, 

\[ 0 = m_\alpha (v_\alpha)_i + m_\bar{\beta} (v_{\bar{\beta}})_i \], for \((v_{\bar{\beta}})_i\), we get \((v_{\bar{\beta}})_i = \frac{-m_\bar{\beta} (v_{\bar{\beta}})_f}{m_\alpha}\). Now if this expression is plugged into the energy equation, with the left hand side set to zero, we get:

\[ 0 = \frac{1}{2} m_\alpha \left( \frac{-m_\bar{\beta} (v_{\bar{\beta}})_f}{m_\alpha} \right)^2 + \frac{1}{2} m_\bar{\beta} (v_{\bar{\beta}})_f^2 - \frac{2Ke^2}{r_i} \]

\[ (v_{\bar{\beta}})_i = \frac{4Ke^2 (m_\alpha)}{m_\bar{\beta} (m_\alpha + m_\bar{\beta})r_i} = \frac{4(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(1.60 \times 10^{-19} \text{ C})^2(4)}{5(1.67 \times 10^{-27} \text{ kg})(2.5 \times 10^{-3} \text{ m})} = 13.3 \text{ km/s} \]

In the last equation, we used the fact that \(m_\alpha = 4m_\bar{\beta}\). Now we can find the velocity of the alpha particle using \((v_{\alpha})_i = \frac{-m_\bar{\beta} (v_{\bar{\beta}})_f}{m_\alpha}\):

\[ (v_{\alpha})_i = \frac{-1(13.3 \text{ km/s})}{4} = -3.3 \text{ km/s} \]

The minus sign tells us that the alpha particle and antiproton are traveling in opposite directions.

Assess: It makes sense that the alpha particle is traveling slower than the antiproton, considering that it is more massive.

P21.71. Prepare: From conservation of energy, the increase in kinetic energy of the protons equals their decrease in potential energy. In other words, \(\Delta K = -\Delta U = -q\Delta V\). In the following figure, the source of the potential difference is a capacitor, with the negatively charged plate having a hole cut in the center to allow protons through. Notice that while the protons are between the plates, they are accelerating, but once they pass through the plate on the right, they maintain a constant speed. This is because there is no electric field outside an ideal capacitor.

Solve: If we solve the equation for \(q\), we get:
\[ q = \frac{-\Delta K}{\Delta V} = \frac{\Delta K}{|\Delta V|} = \frac{0.10 \text{ J}}{10 \times 10^6 \text{ V}} = 1.0 \times 10^{-8} \text{ C} = 10 \text{ nC} \]

Here we have used the fact that since the protons are accelerating, they must be traveling through a decreasing potential. Consequently, \( \Delta V \) is negative and \(-\Delta V = |\Delta V|\).

**Assess:** Since the protons have a charge of \( e \), their kinetic energy after passing the potential difference is

\[
10 \text{ MeV} = \left( \frac{1.60 \times 10^{-19} \text{ J}}{1 \text{ eV}} \right) (10 \times 10^6 \text{ eV}) = 1.60 \times 10^{-12} \text{ J}.
\]

Since the total energy was 0.10 J, we can determine the number of protons, \( N \):

\[
N = \frac{(0.10 \text{ J})/(1.60 \times 10^{-12} \text{ J})}{6.3 \times 10^9} = 6.3 \times 10^{10}.
\]

As a check, the charge of this quantity of protons is

\[
(6.3 \times 10^{10})(1.60 \times 10^{-19} \text{ C}) = 10 \text{ nC},
\]

as we find in the statement of the problem.

**P21.73. Prepare:** The proton is fired from a distance much greater than the nuclear diameter, so \( r_i = \infty \) and \( U_i = 0 \text{ J} \). Because the nucleus is so small, a proton that is even a few atoms away is, for all practical purposes, at infinity. As the proton approaches the nucleus, it is slowed by the repulsive electric force. At the end point, the proton has just reached the surface of the nucleus (\( r_f = \text{nuclear diameter} \)) with \( v_f = 0 \text{ m/s} \). (The proton won’t remain at this point but will be pushed back out again, but the subsequent motion is not part of this problem.) Initially, the proton has kinetic energy but no potential energy. At the point of closest approach, where \( v_f = 0 \text{ m/s} \), the proton has potential energy but no kinetic energy. Energy is conserved. Because the iron nucleus is very large compared to the proton, we will assume that the nucleus does not move (no recoil) and that the proton is essentially a point particle with no diameter.

\[
\begin{aligned}
\text{Before} & \quad \text{Fe} \quad q = +26e \\
& \quad v_i = 42 \text{ m/s} \quad r_i = \infty \\
\text{After} & \quad \text{Fe} \quad q = +26e \\
& \quad v_f = 0 \\
\end{aligned}
\]

**Solve:** Because energy is conserved, \( K_i + U_i = K_f + U_f \). This equation is

\[
0 \text{ J} + \frac{(e)(26e)}{4\pi\varepsilon_0 r_i} = \frac{1}{2} m_{\text{proton}} v_i^2 + 0 \text{ J}
\]

where \( r_i \) is half the nuclear diameter. The initial speed of the proton is

\[
v_i = \sqrt{\frac{2(e)(26e)}{4\pi\varepsilon_0 r_i m_{\text{proton}}}} = \frac{2(1.6 \times 10^{-19} \text{ C})(26 \times 1.6 \times 10^{-19} \text{ C})(9.0 \times 10^7 \text{ N} \cdot \text{m}^2/\text{C}^2)}{(1.67 \times 10^{-27} \text{ kg})(4.5 \times 10^{-3} \text{ m})} = 4.0 \times 10^7 \text{ m/s}
\]

**Assess:** Extremely large deceleration of the proton occurs as the proton is brought to rest momentarily.

**P21.80. Prepare:** The energy stored in a capacitor is related to the physical size of the capacitor and the electric field in the capacitor by \( U = \varepsilon_0 AdE^2/2 \).

**Solve:** The energy stored in the air-filled capacitor may be determined by

\[
U_{\text{air}} = \varepsilon_0 AdE^2/2 = (8.85 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2))(0.15 \text{ m})^2(5.0 \times 10^{-4} \text{ m})(3.0 \times 10^6 \text{ V/m})^2/2 = 4.5 \times 10^{-4} \text{ J}
\]

The energy stored in the Teflon-filled capacitor may be determined by

\[
U_{\text{Teflon}} = \varepsilon_0 AdE^2/2 = (2 \times 8.85 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2))(0.15 \text{ m})^2(5.0 \times 10^{-4} \text{ m})(60 \times 10^6 \text{ V/m})^2/2 = 0.36 \text{ J}
\]

**Assess:** The Teflon-filled capacitor can store 800 times the amount of energy as the air-filled capacitor.