Electric Fields and Forces

Description: Electric forces and electric fields.

Learning Goal: To understand Coulomb's law, electric fields, and the connection between the electric field and the electric force.

Coulomb's law gives the electrostatic force $F$ acting between two charges. The magnitude $F$ of the force between two charges $q_1$ and $q_2$ depends on the product of the charges and the square of the distance $r$ between the charges:

$$F = \frac{k|q_1 q_2|}{r^2}$$

where $k = 1/(4\pi\varepsilon_0) = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$. The direction of the force is along the line connecting the two charges. If the charges have the same sign, the force will be repulsive. If the charges have opposite signs, the force will be attractive. In other words, opposite charges attract and like charges repel.

Because the charges are not in contact with each other, there must be an intermediate mechanism to cause the force. This mechanism is the electric field. The electric field at any location is equal to the force per unit charge experienced by a charge placed at that location. In other words, if a charge $q$ experiences a force $F$, the electric field $E$ at that point is

$$E = \frac{F}{q}$$

The electric field vector has the same direction as the force vector on a positive charge and the opposite direction to that of the force vector on a negative charge.

An electric field can be created by a single charge or a distribution of charges. The electric field a distance $r$ from a point charge $q'$ has magnitude

$$E = \frac{k|q'|}{r^2}$$

The electric field points away from positive charges and toward negative charges. A distribution of charges creates an electric field that can be found by taking the vector sum of the fields created by individual point charges. Note that if a charge $q$ is placed in an electric field created by $q'$, $q$ will not significantly affect the electric field if it is small compared to $q'$.

Imagine an isolated positive point charge with a charge $Q$ (many times larger than the charge on a single electron).

Part A

There is a single electron at a distance from the point charge. On which of the following quantities does the force on the electron depend?

Check all that apply.

**ANSWER:**

- the distance between the positive charge and the electron
- the charge on the electron
- the mass of the electron
- the charge of the positive charge
- the mass of the positive charge
According to Coulomb's law, the force between two particles depends on the charge on each of them and the distance between them.

**Part B**

For the same situation as in Part A, on which of the following quantities does the electric field at the electron's position depend? Check all that apply.

**ANSWER:**
- the distance between the positive charge and the electron
- the charge on the electron
- the mass of the electron
- the charge of the positive charge
- the mass of the positive charge
- the radius of the positive charge
- the radius of the electron

The electrostatic force cannot exist unless two charges are present. The electric field, on the other hand, can be created by only one charge. The value of the electric field depends only on the charge producing the electric field and the distance from that charge.

**Part C**

If the total positive charge is \( Q = 1.62 \times 10^{-6} \) C, what is the magnitude of the electric field caused by this charge at point \( P \), a distance \( d = 1.53 \) m from the charge?

Enter your answer numerically in newtons per coulomb.
Part D

What is the direction of the electric field at point P?

Enter the letter of the vector that represents the direction of \( \vec{E}_P \).

**Answer:** G

Part E

Now find the magnitude of the force on an electron placed at point P. Recall that the charge on an electron has magnitude \( e = 1.60 \times 10^{-19} \text{ C} \).

**Hint E.1 Determine how to approach the problem**

What strategy can you use to calculate the force between the positive charge and the electron?

**Answer:** Use Coulomb's law.

**Answer:** Multiply the electric field due to the positive charge by the charge on the electron.

**Answer:** Do either of the above.
Do neither of the above.

Enter your answer numerically in newtons.

**ANSWER:**

\[ F = \frac{(8.99 \cdot 10^9)(1.6 \cdot 10^{-19})Q}{d^2} \text{ N} \]

Part F

What is the direction of the force on an electron placed at point P?

Enter the letter of the vector that represents the direction of \( F \).

**ANSWER:** C

**Electric Field due to Multiple Point Charges**

**Description:** Calculate the electric field due to two charges of the same sign. Then add a third charge at a new point and find its charge (including sign) to cancel the field of the first two charges at a particular point.

Two point charges are placed on the \( x \) axis.
The first charge, \( q_1 = 8.00 \text{ nC} \), is placed a distance 16.0 m from the origin along the positive \( x \) axis; the second charge, \( q_2 = 6.00 \text{ nC} \), is placed a distance 9.00 m from the origin along the negative \( x \) axis.

**Part A**

Calculate the electric field at point A, located at coordinates (0 m, 12.0 m).

**Hint A.1**  **How to approach the problem**

Find the contributions to the electric field at point A separately for \( q_1 \) and \( q_2 \), then add them together (using vector addition) to find the total electric field at that point. You will need to use the Pythagorean theorem to find the distance of each charge from point A.

**Hint A.2**  **Calculate the distance from each charge to point A**

Calculate the distance from each charge to point A.

Enter the two distances, separated by a comma, in meters to three significant figures.

**ANSWER:**

\[
\sqrt{x_1^2 + y_a^2}, \sqrt{x_2^2 + y_a^2}
\]

**Hint A.3**  **Determine the directions of the electric fields**

Which of the following describes the directions of the electric fields \( E_1 \) and \( E_2 \) created by charges \( q_1 \) and \( q_2 \) at point A?

**ANSWER:**

\( \vec{E}_{A1} \) points up and left and \( \vec{E}_{A2} \) points up and right.
In this case, the electric fields due to the two charges have both $x$ and $y$ components that are nonzero. To find the total field, add these two components separately.

**Hint A.4 Calculate the components of $E_1$**

Calculate the $x$ and $y$ components of the electric field $E_1$ at point A due to charge $q_1$.

**Hint A.4.1 Calculate the magnitude of the total field**

Calculate the magnitude of the field $E_1$ at point A due to charge $q_1$ only. Express your answer in newtons per coulomb to three significant figures.

**ANSWER:**

$$E_{A1} = \frac{kq_1}{x_1^2 + y_1^2} \text{ N/C}$$

**Hint A.4.2 How to find the components of the total field**

Once you have found the magnitude of the field, use trigonometry to determine the $x$ and $y$ components of the field. The electric field of a positive point charge points directly away from the charge, so the direction of the electric field at point A due to charge $q_1$ will be along the line joining the two. Use the position coordinates of $q_1$ and point A to find the angle that the line joining the two makes with the $x$ or $y$ axis. Then use this angle to resolve the electric field vector into components. Express your answers in newtons per coulomb, separated by a comma, to three significant figures.

**ANSWER:**

$$E_{A1x}, E_{A1y} = \frac{-kq_1x_1}{(x_1^2 + y_1^2)^{3/2}} \text{ N/C}$$

**Hint A.5 Calculate the components of $E_2$**

Calculate the $x$ and $y$ components of the electric field at point A due to charge $q_2$.

**Hint A.5.1 Calculate the magnitude of the total field**

Calculate the magnitude of the field $E_2$ at point A due to charge $q_2$ only. Express your answer in newtons per coulomb to three significant figures.
**Part B**

An unknown additional charge \( q_3 \) is now placed at point B, located at coordinates \((0 \text{ m}, 15.0 \text{ m})\).

Find the magnitude and sign of \( q_3 \) needed to make the total electric field at point A equal to zero.

**Hint B.1  How to approach the problem**

You have already calculated the electric field at point A due to \( q_1 \) and \( q_2 \). Now find the charge \( q_3 \) needed to make an opposite field at point A, so when the two are added together the total field is zero.

**Hint B.2  Determine the sign of the charge**

Which sign of charge \( q_3 \) is needed to create an electric field \( E_3 \) that points in the opposite direction of the total field due to the other two charges, \( q_1 \) and \( q_2 \)?

**ANSWER:**

- positive
- negative

**Hint B.3  Calculating the magnitude of the new charge**

**ANSWER:**

\[
E_{A2} = \frac{kq_2}{x_2^2 + y_a^2} \quad \text{N/C}
\]
Keep in mind that the magnitude of the field due to q_3 is equal in magnitude to the field due to charges q_1 and q_2. Express your answer in nanocoulombs to three significant figures.

\[ E_{A3} = \frac{kq_3}{r_{A3}^2} \]

**Electric Field Vector Drawing**

**Description:** Simple conceptual question about determining the electric field vector in a region of space from the change in the path of a charged particle. (vector applet)

Each of the four parts of this problem depicts a motion diagram for a charged particle moving through a region of uniform electric field. For each part, draw a vector representing the direction of the electric field.

**Part A**

**Hint A.1  Relationship between electric field and electric force**

The relationship between the electric force that acts on a particle and the electric field at the location of the particle is

\[ \vec{F} = q\vec{E} \]

This formula indicates that the force and the electric field point in the same direction for a positively charged particle, and in opposite directions for a negatively charged particle.

**Hint A.2  Determining the direction of the electric field**

The acceleration of the particle can be determined from the change in its velocity. By Newton’s 2nd law, the force acting on the particle is parallel to its acceleration. Finally, since this is a positively charged particle, the electric field is parallel to the force. Putting this all together results in an electric field that is parallel to the particle’s acceleration.

Draw a vector representing the direction of the electric field. The orientation of the vector will be graded. The location and length of the vector will not be graded.

**ANSWER:**

View

The motion diagram shows that the particle's acceleration points to the right. Because the particle has positive charge, the electric field should point to the right.

**Part B**

**Hint B.1  Relationship between electric field and electric force**
The relationship between the electric force that acts on a particle and the electric field at the location of the particle is

\[ \vec{F} = q\vec{E} \]

This formula indicates that the force and the electric field point in the same direction for a positively charged particle, and in opposite directions for a negatively charged particle.

**Hint B.2  Determining the direction of the electric field**

The acceleration of the particle can be determined from the change in its velocity. By Newton’s 2nd law, the force acting on the particle is parallel to its acceleration. Finally, since this is a negatively charged particle, the electric field is directed opposite to the force. Putting this all together results in an electric field that is directed opposite to the particle’s acceleration.

Draw a vector representing the direction of the electric field. The orientation of the vector will be graded. The location and length of the vector will not be graded.

**ANSWER:**

View

The motion diagram shows that the particle's acceleration points to the right. Because the particle has negative charge, the electric field should point to the left.

**Part C**

**Hint C.1  Relationship between electric field and electric force**

The relationship between the electric force that acts on a particle and the electric field at the location of the particle is

\[ \vec{F} = q\vec{E} \]

This formula indicates that the force and the electric field point in the same direction for a positively charged particle, and in opposite directions for a negatively charged particle.

**Hint C.2  Determining the direction of the electric field**

The acceleration of the particle can be determined from the change in its velocity. By Newton’s 2nd law, the force acting on the particle is parallel to its acceleration. Finally, since this is a positively charged particle, the electric field is parallel to the force. Putting this all together results in an electric field that is parallel to the particle’s acceleration.

Because the electric field is uniform, you can find the direction of the particle's acceleration by subtracting any two consecutive velocity vectors graphically. If \( v_i \) and \( v_f \) are any two consecutive velocities, you can subtract \( v_i \) from \( v_f \) by placing \( -v_i \) at the tip of \( v_f \). \( v_f - v_i \) is the vector that starts at the tail of \( v_f \) and ends at the tip of \( v_i \).

To find the direction of the particle's acceleration graphically, use two unlabeled vectors to represent \( -v_i \) and \( v_f - v_i \). Pick any two vectors \( v_i \) and \( v_f \) that would make your subtraction easier; you can verify your result by subtracting any other pair of consecutive vectors.

Draw a vector representing the direction of the electric field. The orientation of the vector will be graded. The location and length of the vector will not be graded.

**ANSWER:**
Part D

Hint D.1  Relationship between electric field and electric force

The relationship between the electric force that acts on a particle and the electric field at the location of the particle is

\[ \vec{F} = q\vec{E} \]

This formula indicates that the force and the electric field point in the same direction for a positively charged particle, and in opposite directions for a negatively charged particle.

Hint D.2  Determining the direction of the electric field

The acceleration of the particle can be determined from the change in the illustrated velocity vectors. By Newton’s 2nd law, the force acting on the particle is parallel to its acceleration. Finally, since this is a negatively charged particle, the electric field is directed opposite to the electric force. Putting this all together results in an electric field that is directed opposite to the particle’s acceleration.

Because the electric field is uniform, you can find the direction of the particle's acceleration by subtracting any two consecutive velocity vectors graphically. If \( v_i \) and \( v_f \) are any two consecutive velocities, you can subtract \( v_i \) from \( v_f \) by placing \( -v_i \) at the tip of \( v_f \). \( v_f - v_i \) is the vector that starts at the tail of \( v_f \) and ends at the tip of \( v_i \).

To find the direction of the particle's acceleration graphically, use two unlabeled vectors to represent \( -v_i \) and \( v_f - v_i \). Pick any two vectors \( v_i \) and \( v_f \) that would make your subtraction easier; you can verify your result by subtracting any other pair of consecutive vectors.

Draw a vector representing the direction of the electric field. The orientation of the vector will be graded. The location and length of the vector will not be graded.

ANSWER:

Video Tutor: Charged Rod and Aluminum Can

Description: A PVC rod is charged and brought near an aluminum can that is free to roll. What does the can do?

First, launch the video below. You will be asked to use your knowledge of physics to predict the outcome of an experiment. Then, close the video window and answer the questions at right. You can watch the video again at any point.
Part A
Consider the situation in the figure below, where two charged rods are placed a distance \( d \) on either side of an aluminum can. What does the can do?

\[ 
\begin{align*}
  &+q \\
  &\text{Can} \\
  &-q \\
\end{align*}
\]

**Hint A.1  How to approach the problem.**
This problem asks you to think about *induced* charge on the surface of an object and the resulting *polarization force*.
To get started, draw a diagram. Draw the induced *surface* charges on the outside of the can. Next, draw a force diagram (free-body diagram) to show the forces exerted on the can. Aluminum is a conductor.

**ANSWER:**
- Roll to the left
- Stays still
- Roll to the right
The positively charged rod induces a negative charge on the left side of the can, creating an attractive force between the rod and the can. However, the negatively charged rod induces an equal positive charge on the right side of the can, which creates an attractive force between the can and that rod. The net force acting on the can is zero.

**Part B**

Now, consider the situation shown in the figure below. What does the can do?

![Figure showing two positively charged rods and a can](image)

**ANSWER:**
- Stays still
- Rolls to the left
- Rolls to the right

The polarization force is always attractive, so the can does not move.

**Part C**

Using the setup from the first question, imagine that you briefly touch the negatively charged rod to the can. You then hold the two rods at equal distances on either side of the can. What does the can do?

**Hint C.1 How to approach the problem.**

This problem asks you to consider what happens to a conductor after being touched by a charged object. What charge will the can have after being touched?

**ANSWER:**
- Rolls away from the positively charged rod
- Does not move
- Rolls toward the positively charged rod

The can acquires a net negative charge after being touched, so it is then attracted to the positively charged rod.

**Video Tutor: Electroscope in Conducting Shell**
Description: An electroscope needle is suspended by a conducting support inside one hemisphere of a conducting shell. If the needle is charged and the two halves of the shell are then brought together, what happens to the charge on the needle? First, launch the video below. You will be asked to use your knowledge of physics to predict the outcome of an experiment. Then, close the video window and answer the question at right. You can watch the video again at any point.

Part A
As in the video, we apply a charge +Q to the half-shell that carries the electroscope. This time, we also apply a charge −Q to the other half-shell. When we bring the two halves together, we observe that the electroscope discharges, just as in the video. What does the electroscope needle do when you separate the two half-shells again?

Hint A.1 How to approach the problem.
The half-shell with a charge of −Q has an excess of electrons, and the half-shell with a charge of +Q has an exactly equal deficit of electrons. What happens when these two charged, conducting half-shells are brought together? What is the net charge of the resulting whole sphere? (Recall that electrons can move through a conductor.) Will the half-shells have a net charge when you separate them again? If the half-shell with the electroscope needle carries a net charge, the needle will deflect.

ANSWER:  
- It does not deflect at all.
- It deflects more than it did at the end of the video.
- It deflects the same amount as at end of the video.
- It deflects less than it did at the end of the video.

The spherical surface has zero net charge after the two halves are brought together. The two half-spheres remain electrically neutral after they are separated.
P20.34. Prepare: As shown in the following figure, the maximum possible torque will occur when the dipole is oriented perpendicular to the electric field.

\[ \vec{E} = 7.4 \times 10^5 \text{ N/C} \]

Solve: The torque due to the force on both the positive and negative charge is clockwise. We can write the total torque as the sum of the torque on the positive and negative charges.

\[ \tau_{\text{total}} = \tau_+ + \tau_- = (L/2)F_+ + (L/2)F_- = (L/2)qE + (L/2)qE = LqE = 3.1 \times 10^{-4} \text{ N} \cdot \text{m} \]

Assess: From the figure it is apparent that the torques are clockwise and that they will add. The units of the calculation are correct and we should expect a small torque for such small charges.

P20.36. Prepare: Equation 20.8 tells us the force on a charged object in an electric field: \( \vec{F}_{\text{ext}} = q\vec{E} \).

We are given \( q = e \) and \( E = 1.0 \times 10^7 \text{ N/C} \).

Solve: \( F_{\text{ext}} = qE = (1.6 \times 10^{-19} \text{ C})(1.0 \times 10^7 \text{ N/C}) = 1.6 \times 10^{-12} \text{ N} \)

Assess: Notice the C’s cancel out leaving units of N. The answer is very small, but that is what we expect for such a small charge.

P20.58. Prepare: We have sufficient information to determine the electric force on the charged bee and the weight of the bee. Knowing these two forces we can determine their ratio. When these two forces are equal in magnitude and opposite in direction, the bee will hang suspended in air.

Solve: (a) The ratio of the electric force to the weight is determined by:

\[ \frac{F_e}{w} = \frac{qE}{mg} = 2.3 \times 10^{-6} \]

(b) The bee will hang suspended when the electric force is equal to the weight: \( F_e = w \) or \( qE = mg \).

This gives an electric field of \( E = mg/q = 4.3 \times 10^5 \text{ N/C} \).

Assess: Table 20.2 informs us that an electric field of \( 10^6 \text{ N/C} \) will create a spark in air. Note that the required electric field is greater than the air breakdown electric field. As a result we don’t expect the bees to just hang in the air due to the charge acquired while flying; they will have provide some of the lift.
P20.68. Prepare: Since the bead is in equilibrium the sum of the forces on it is zero. Assume the rod is frictionless so the only forces are gravity and the electric force.

Solve: Solve for $r$ in the application of Newton’s second law.

$$\Sigma F = K \frac{q_1 q_2}{r^2} - mg = 0 \quad \Rightarrow \quad r^2 = K \frac{q_1 q_2}{mg}$$

$$r = \sqrt{K \frac{q_1 q_2}{mg}} = \sqrt{\frac{(9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) (15 \text{ nC})(10 \text{ nC})}{(5.0 \times 10^{-7} \text{ kg})(9.8 \text{ m/s}^2)}} = 5.2 \text{ cm}$$

Assess: 5.2 cm seems to be a reasonable distance.