Chapter 1 – Mastering Physics - Solutions

The Trip Abroad

Learning Goal:
To apply Problem-Solving Strategy 1.1 Unit conversions.
While driving in an exotic foreign land, you see a speed limit sign on a highway that reads 1.50×10^5 furlongs /fortnight. How many miles per hour is this? (One furlong is 1/8 mile and a fortnight is 14 days. A furlong originally referred to the length of a plowed furrow.)

Problem-Solving Strategy

Units are multiplied and divided just like ordinary algebraic symbols. A ratio of units, such as 1 min /60 sec (or 1 min = 60 sec), is called a conversion factor and forms an equality. Here we don’t mean that the number 1 is equal to the number 60 but rather that 1 min represents the same physical time interval as 60 s. In a physical sense, multiplying by the quantity 1 min/ 60 sec is really multiplying by unity, which doesn't change the physical meaning of the quantity. To find the number of seconds in 3 min, we write

\[ 3 \text{ min} = (3 \text{ min}) \left( \frac{60 \text{ s}}{1 \text{ min}} \right) = 180 \text{ s} \]

This expression makes sense. There are more seconds than minutes in the same time interval.

SET UP
Before writing any equations, organize your information and draw appropriate diagrams. To convert units, you must know the correct conversion factors.

Part A
How many hours are in one fortnight?
Enter your answer in hours to four significant figures.
Hint 1. How to find hours from fortnights
Recall from the problem introduction that 1 fortnight= 14 days. Find how many hours are in 14 days using the conversion

\[ 1 \text{ fortnight} = (14 \text{ days}) \left( \frac{24 \text{ hours}}{1 \text{ day}} \right) \]

ANSWER:
1 fortnight= 336.0 hours

Part B
How many miles are in 1.50×10^5 furlongs?
Enter your answer in miles to four significant figures.

SOLVE
Now that you've set up the problem, choose appropriate equations and solve for your unknowns.

Part C
The highway sign reads 1.50×10^5 furlongs / fortnight. How many miles per hour is this?
Enter your answer in miles per hour to three significant figures.

ANSWER:

\[ D/ (8 \times 14 \times 24) = 55.8 \text{ Miles / hour} \]

Converting units is a useful skill to have. However, you do not have to break down a conversion into little parts as this problem had you do. The conversion you just completed could also be accomplished by carrying out the following multiplication with \( D = 1.50 \times 10^5 \) furlongs:

\[ \text{Consistency of Units} \]

In physics, every physical quantity is measured with respect to a unit. Time is measured in seconds, length is measured in meters, and mass is measured in kilograms. Knowing the units of physical quantities will help you solve problems in physics.

Part A

Gravity causes objects to be attracted to one another. This attraction keeps our feet firmly planted on the ground and causes the moon to orbit the earth. The force of gravitational attraction is represented by the equation

\[ F = \frac{Gm_1m_2}{r^2}, \]

where \( F \) is the magnitude of the gravitational attraction on either body, \( m_1 \) and \( m_2 \) are the masses of the bodies, \( r \) is the distance between them, and \( G \) is the gravitational constant. In SI units, the units of force are \( \text{kg m/s}^2 \), the units of mass are \( \text{kg} \), and the units of distance are \( \text{m} \). For this equation to have consistent units, the units of \( G \) must be which of the following?

Hint 1. How to approach the problem
To solve this problem, we start with the equation

\[ F = \frac{Gm_1m_2}{r^2}. \]

For each symbol whose units we know, we replace the symbol with those units. For example, we replace \( m_1 \) with \( \text{kg} \). We now solve this equation for \( G \).

Part B

One consequence of Einstein's theory of special relativity is that mass is a form of energy. This mass-energy relationship is perhaps the most famous of all physics equations:

\[ E = mc^2, \]

where \( m \) is mass, \( c \) is the speed of the light, and \( E \) is the energy. In SI units, the units of speed are \( \text{m/s} \). For the preceding equation to have consistent units (the same units on both sides of the equation), the units of \( E \) must be which of the following?

Hint 1. How to approach the problem
To solve this problem, we start with the equation
For each symbol whose units we know, we replace the symbol with those units. For example, we replace m with kg. We now solve this equation for E.

To solve the types of problems typified by these examples, we start with the given equation. For each symbol whose units we know, we replace the symbol with those units. For example, we replace m with kg. We now solve this equation for the units of the unknown variable.

± A Trip to Europe

Learning Goal:
To understand how to use dimensional analysis to solve problems.

Dimensional analysis is a useful tool for solving problems that involve unit conversions. Since unit conversion is not limited to physics problems but is part of our everyday life, correct use of conversion factors is essential to working through problems of practical importance.

For example, dimensional analysis could be used in problems involving currency exchange. Say you want to calculate how many euros you get if you exchange 3600 US dollars (USD), given the exchange rate 1 EUR = 1.20 USD, that is, 1 euro to 1.20 US dollars. Begin by writing down the starting value, 3600 USD. This can also be written as a fraction:

\[
\frac{3600 \text{ USD}}{1}
\]

Next, convert dollars to euros. This conversion involves multiplying by a simple conversion factor derived from the exchange rate:

\[
\frac{1.00 \text{ EUR}}{1.20 \text{ USD}}
\]

Note that the "dollar" unit, USD, should appear on the bottom of this conversion factor, since USD appears on the top of the starting value.

Finally, since dollars are divided by dollars, the units can be canceled and the final result is

Currency exchange is only one example of many practical situations where dimensional analysis may help you to work through problems. Remember that dimensional analysis involves multiplying a given value by a conversion factor, resulting in a value in the new units. The conversion factor can be the ratio of any two quantities, as long as the ratio is equal to one.

You and your friends are organizing a trip to Europe. Your plan is to rent a car and drive through the major European capitals. By consulting a map you estimate that you will cover a total
distance of 5000 km. Consider the euro-dollar exchange rate given in the introduction and use dimensional analysis to work through these simple problems.

Part A
You select a rental package that includes a car with an average consumption of 6.00 liters of fuel per 100 km. Considering that in Europe the average fuel cost is 1.063 euros/liter, how much (in US dollars) will you spend in fuel on your trip?
Express your answer numerically in US dollars to three significant figures.

Hint 1. How to approach the problem
Begin with writing the total distance in kilometers. Then multiply this by the consumption rate [in liters/ (100 km)] of your rental car to calculate how many liters of fuel are needed. Then compute the total cost of fuel (in euros) by multiplying your expression by the average fuel cost. Finally, convert the total cost of fuel to dollars.

Hint 2. Find the unit factor to express the cost of fuel in euros
Which of these unit factors can be used to calculate the cost of fuel in euros?

Hint 1. Conversion factor
When calculating the total cost of fuel in euros, you divide kilometers by kilometers (and cancel out the kilometer units) and multiply the resulting value in liters by a conversion factor that has the liter unit at the bottom, so that again you can cancel out units.

Hint 3. Find the unit factor to convert euros to dollars
Which of these unit factors can be used to convert the total cost of fuel to dollars?

Hint 1. Conversion factor
Since you are converting the cost of fuel from euros to dollars, you need to multiply by a conversion factor that has the euro unit, EUR, on the bottom, so that you can cancel out units.

Hint 4. Canceling units
You can avoid many mistakes in unit conversion problems if you are careful to cancel units.

ANSWER:
Cost of fuel = 383 USD

Part B
How many gallons of fuel would the rental car consume per mile?
Express your answer numerically in gallons per mile to three significant figures.

Hint 1. How to approach the problem

Begin with writing the consumption rate in liters per kilometer. Then multiply this by the unit factor needed to convert kilometers to miles. Finally convert liters to gallons. Be sure to cancel units so that your answer is dimensionally consistent.

Hint 2. Find the unit factor to convert kilometers to miles
Which of these unit factors can be used to convert kilometers to miles?

Hint 1. Conversion factor
Since you are converting the car's fuel consumption per kilometer to the car's fuel consumption per mile you need to multiply by a conversion factor that has the kilometer unit on the top, so that you can cancel out units.

Hint 3. Find the unit factor to convert liters to gallons
Which of these unit factors can be used to convert liters to gallons?

Hint 1. Conversion factor
Since you are converting the car's consumption in liters to gallons, you need to multiply by a conversion factor that has the liter unit at the bottom, so that you can cancel out units.

**ANSWER:**

\[2.55 \times 10^{-2} \text{ gallons/mile}\]

**Part C**

What is the average cost, in dollars per gallon, of fuel in Europe? Express your answer numerically in dollars per gallon to three significant figures.

**Hint 1. How to approach the problem**

Begin with writing the cost of fuel in euros per liter. Then multiply this by the unit factor needed to convert euros to dollars. Finally convert liters to gallons. Be sure to cancel units, to make sure that your answer is dimensionally consistent.

**Hint 2. Find the unit factor to convert euros to dollars**

Which of these unit factors can be used to convert the price of fuel from euros to dollars?

**Hint 3. Find the unit factor to convert liters to gallons**

Which of these unit factors can be used to convert the price of fuel per liter to a price per gallon?

**ANSWER:**

\[4.83 \text{ USD/Gallon}\]

### Adding Scalar Multiples of Vectors Graphically

Draw the vectors indicated. You may use any extra (unlabeled) vectors that are helpful; but, keep in mind that the unlabeled vectors should not be part of your submission.

**Part A**

Draw the vector \( \mathbf{C} = \mathbf{A} + 2 \mathbf{B} \).

The length and orientation of the vector will be graded. The location of the vector is not important.

![Diagram of vectors](image)

\[\mathbf{C} = \mathbf{A} + 2 \mathbf{B}\]

**Part B**

Draw the vector \( \mathbf{C} = 1.5 \mathbf{A} - 3 \mathbf{B} \).

The length and orientation of the vector will be graded. The location of the vector is not important.
Part C
Draw the vector \( \vec{C} = 0.5 \vec{A} + 2 \vec{B} \).
The length and orientation of the vector will be graded. The location of the vector is not important.

Answers to Multiple-Choice Problems

Solutions to Problems

*1.1. Set Up: We know the following equalities: \( 1 \text{ mg} = 10^2 \text{ g} \); \( 1 \mu \text{g} = 10^6 \text{ g} \); \( 1 \text{ kilohms} = 1000 \text{ ohms} \); and \( 1 \text{ milliamp} = 10^2 \text{ amp} \).

Solve: In each case multiply the quantity to be converted by unity, expressed in different units. Construct an expression for unity so that the units to be changed cancel and we are left with the new desired units.

(a) \( (2400 \text{ mg/day}) \left( \frac{10^2 \text{ g}}{1 \text{ mg}} \right) = 2.40 \text{ g/day} \sqrt{b^2 - 4ac} \)

(b) \( (120 \mu \text{g/day}) \left( \frac{10^6 \text{ g}}{1 \mu \text{g}} \right) = 1.20 \times 10^2 \text{ g/day} \)

(c) \( (500 \text{ mg/day}) \left( \frac{10^2 \text{ g}}{1 \text{ mg}} \right) = 0.500 \text{ g/day} \)
(d) \( (1500 \text{ ohms}) \left( \frac{1 \text{ kilohm}}{10^3 \text{ ohms}} \right) = 1.50 \text{ kilohms} \)

(e) \( (0.020 \text{ amp}) = \left( \frac{1 \text{ milliamp}}{10^{-3} \text{ amp}} \right) = 20 \text{ milliamps} \)

**Reflect:** In each case, the number representing the quantity is larger when expressed in the smaller unit. For example, it takes more milligrams to express a mass than to express the mass in grams.

*1.5. Set Up:* We need to apply the following conversion equalities: \( 1000 \text{ g} = 1.00 \text{ kg} \), \( 100 \text{ cm} = 1.00 \text{ m} \), and \( 1.00 \text{ L} = 1000 \text{ cm}^3 \).

**Solve:**
(a) \( (1.00 \text{ g/cm}^3) \left( \frac{1.00 \text{ kg}}{1000 \text{ g}} \right) \left( \frac{100 \text{ cm}}{1.00 \text{ m}} \right)^3 = 1000 \text{ kg/m}^3 \)

(b) \( (1050 \text{ kg/m}^3) \left( \frac{1000 \text{ g}}{1.00 \text{ kg}} \right) \left( \frac{1.00 m}{1000 \text{ cm}} \right)^3 = 1.05 \text{ g/cm}^3 \)

(c) \( (1.00 \text{ L}) \left( \frac{1000 \text{ cm}^3}{1.00 \text{ L}} \right) \left( \frac{1.00 \text{ g}}{1.00 \text{ cm}^3} \right) \left( \frac{1.00 \text{ kg}}{1000 \text{ g}} \right) = 1.00 \text{ kg}; \quad (1.00 \text{ kg}) \left( \frac{2.205 \text{ lb}}{1.00 \text{ kg}} \right) = 2.20 \text{ lb} \)

**Reflect:** We could express the density of water as \( 1.00 \text{ kg/L} \).

1.11. **Set Up:** We know: 1 euro = $1.25; and 1 gal = 3.788 L.

**Solve:** \( (1.35 \text{ euros/L}) \left( \frac{$1.25}{1 \text{ euro}} \right) \left( \frac{3.788 \text{ L}}{1 \text{ gal}} \right) = $6.39 \text{ per gallon.} \) Currently, in 2005, gasoline in the U.S. costs about $2 per gallon so the price in Europe is about three times higher.

1.12. **Set Up:** From Appendix A, the volume \( V \) of a sphere is given in terms of its radius as \( V = \frac{4}{3} \pi r^3 \) while its surface area \( A \) is given as \( A = 4 \pi r^2 \). Also, by definition, the radius is one-half the diameter or \( r = d/2 = 1.0 \mu \text{m} \). Finally, the necessary equalities for this problem are: \( 1 \mu \text{m} = 10^{-6} \text{ m} \); \( 1 \text{ cm} = 10^{-2} \text{ m} \); and \( 1 \text{ mm} = 10^{-3} \text{ m} \).

**Solve:** \( V = \frac{4}{3} \pi r^3 = \frac{4}{3} \pi (1.0 \mu \text{m})^3 \left( \frac{10^{-6} \text{ m}}{1 \mu \text{m}} \right)^3 \left( \frac{1 \text{ cm}}{10^{-2} \text{ m}} \right)^3 = 4.2 \times 10^{-12} \text{ cm}^3 \)

and
\[
A = 4 \pi r^2 = 4 \pi (1.0 \mu \text{m})^2 \left( \frac{10^{-6} \text{ m}}{1 \mu \text{m}} \right)^2 \left( \frac{1 \text{ mm}}{10^{-3} \text{ m}} \right)^2 = 1.3 \times 10^{-5} \text{ mm}^2
\]

1.20. **Set Up:** To calculate the densities, we need to find the spherical volume, \( V = \frac{4}{3} \pi r^3 \), and the mass of the atom or nucleus as the sum of the masses of its constituent particles. For the atom, \( m = 2(m_p + m_n + m_e) \) while for the nucleus \( m = 2(m_p + m_n) \). We thus need mass data from Appendix F: \( m_p = 1.673 \times 10^{-27} \text{ kg}; \quad m_n = 1.675 \times 10^{-27} \text{ kg}; \quad \text{and} \quad m_e = 9.109 \times 10^{-31} \text{ kg}. \) The unit conversion factor \( 1 \text{ g/cm}^3 = 1000 \text{ kg/m}^3 \) is also needed.
**Solve:** (a) Given \( r = 0.050 \text{ nm} = 0.050 \times 10^{-9} \text{ m} \), \( V = \frac{4}{3} \pi r^3 = \frac{4}{3} \pi (0.050 \times 10^{-9} \text{ m})^3 = 5.24 \times 10^{-31} \text{ m}^3 \).

\[
m = 2(m_p + m_n + m_e) = 6.70 \times 10^{-27} \text{ kg}
\]

\[
density = \frac{m}{V} = \frac{6.70 \times 10^{-27} \text{ kg}}{5.24 \times 10^{-31} \text{ m}^3} = 1.3 \times 10^4 \text{ kg/m}^3 = 13 \text{ g/cm}^3
\]

The density of the helium atom is 13 times larger than the density of pure water.

(b) Given \( r = 1.0 \text{ fm} = 1.0 \times 10^{-15} \text{ m} \), \( V = \frac{4}{3} \pi r^3 = \frac{4}{3} \pi (1.0 \times 10^{-15} \text{ m})^3 = 4.19 \times 10^{-45} \text{ m}^3 \).

\[
m = 2(m_p + m_n) = 6.70 \times 10^{-27} \text{ kg}
\]

\[
density = \frac{m}{V} = \frac{6.70 \times 10^{-27} \text{ kg}}{4.19 \times 10^{-45} \text{ m}^3} = 1.6 \times 10^{18} \text{ kg/m}^3 = 1.6 \times 10^{15} \text{ g/cm}^3
\]

In Problem 1.19 we found the density of a neutron star to be \( 4.7 \times 10^{14} \text{ g/cm}^3 \). By comparison, the density of the helium nucleus is 3 times larger than the density of a neutron star.

**1.31. Set Up:** An average middle-aged (40 year-old) adult at rest has a heart rate of roughly 75 beats per minute. To calculate the number of beats in a lifetime, use the current average lifespan of 80 years. The volume of blood pumped during this interval is then the volume per beat multiplied by the total beats.

**Solve:**

\[
N_{\text{beats}} = (75 \text{ beats/min}) \left(\frac{60 \text{ min}}{1 \text{ h}}\right) \left(\frac{24 \text{ h}}{1 \text{ day}}\right) \left(\frac{365 \text{ days}}{1 \text{ yr}}\right) \left(\frac{80 \text{ yr}}{\text{ lifespan}}\right) = 3 \times 10^9 \text{ beats/lifespan}
\]

\[
V_{\text{blood}} = (50 \text{ cm}^3/\text{beat}) \left(\frac{1 \text{ L}}{1000 \text{ cm}^3}\right) \left(\frac{1 \text{ gal}}{3.788 \text{ L}}\right) \left(\frac{3 \times 10^9 \text{ beats}}{\text{ lifespan}}\right) = 4 \times 10^7 \text{ gal/lifespan}
\]

**1.32. Set Up:** Estimate that one step is \( \frac{1}{3} \text{ m} \). Estimate that you walk 100 m in 1 minute. The distance to the Moon is \( 3.8 \times 10^8 \text{ m} \).

**Solve:** The time it takes is

\[
t = \left(\frac{1 \text{ minute}}{100 \text{ m}}\right) (3.8 \times 10^8 \text{ m}) = 3.8 \times 10^6 \text{ minutes},
\]

which is about 7 years. The number of steps would be:

\[
\frac{3.8 \times 10^8 \text{ m}}{\frac{1}{3} \text{ m/step}} = 1 \times 10^9 \text{ steps}.
\]

**1.31. Set Up:** An average middle-aged (40 year-old) adult at rest has a heart rate of roughly 75 beats per minute. To calculate the number of beats in a lifetime, use the current average lifespan of 80 years. The volume of blood pumped during this interval is then the volume per beat multiplied by the total beats.

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\]

\[
V_{\text{blood}} = (50 \text{ cm}^3/\text{beat}) \left(\frac{1 \text{ L}}{1000 \text{ cm}^3}\right) \left(\frac{1 \text{ gal}}{3.788 \text{ L}}\right) \left(\frac{3 \times 10^9 \text{ beats}}{\text{ lifespan}}\right) = 4 \times 10^7 \text{ gal/lifespan}
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**Solve:** The time it takes is

\[
t = \left(\frac{1 \text{ minute}}{100 \text{ m}}\right) (3.8 \times 10^8 \text{ m}) = 3.8 \times 10^6 \text{ minutes},
\]
which is about 7 years. The number of steps would be:
\[
\frac{3.8 \times 10^8 \text{ m}}{4 \text{ m/step}} = 1 \times 10^9 \text{ steps.}
\]

1.48. Set Up: The counterclockwise angles each vector makes with the +x axis are: \( \theta_A = 30^\circ \), \( \theta_B = 120^\circ \), and \( \theta_C = 233^\circ \). The components of each vector are shown in Figure (a) below.

\[\begin{align*}
A_x &= A \cos 30^\circ = 87 \text{ N}; & A_y &= A \sin 30^\circ = 50 \text{ N}; & B_x &= B \cos 120^\circ = -40 \text{ N}; & B_y &= B \sin 120^\circ = 69 \text{ N}; & C_x &= C \cos 233^\circ = -24 \text{ N}; & C_y &= C \sin 233^\circ = -32 \text{ N}.
\end{align*}\]

(a) \( \sum \vec{F}_x = \vec{A} + \vec{B} + \vec{C} \) is the resultant pull.

\[\begin{align*}
R_x &= A_x + B_x + C_x = 87 \text{ N} + (-40 \text{ N}) + (-24 \text{ N}) = +23 \text{ N} \\
R_y &= A_y + B_y + C_y = 50 \text{ N} + 69 \text{ N} + (-32 \text{ N}) = +87 \text{ N}
\end{align*}\]

(c) \( R_x \), \( R_y \), and \( \vec{R} \) are shown in Figure (b) above.

\[R = \sqrt{R_x^2 + R_y^2} = 90 \text{ N} \quad \text{and} \quad \tan \theta = \frac{R_y}{R_x} = \frac{87 \text{ N}}{23 \text{ N}} \quad \text{so} \quad \theta = 75^\circ\]
(d) The vector addition diagram is given in Figure (c) above. Careful measurement gives an $\mathbf{R}$ value that agrees with our results using components.

**1.50. Set Up:** Use coordinates for which $+x$ is east and $+y$ is north. The driver’s vector displacements are: $\mathbf{A} = 2.6 \text{ km, 0° of north}$; $\mathbf{B} = 4.0 \text{ km, 0° of east}$; $\mathbf{C} = 3.1 \text{ km, 45° north of east}$.

**Solve:**

- $R_x = A_x + B_x + C_x = 0 + 4.0 \text{ km} + (3.1 \text{ km}) \cos (45°) = 6.2 \text{ km}$;
- $R_y = A_y + B_y + C_y = 2.6 \text{ km} + 0 + (3.1 \text{ km}) \sin (45°) = 4.8 \text{ km}$; $R = \sqrt{R_x^2 + R_y^2} = 7.8 \text{ km}$;
- $\theta = \tan^{-1}[(4.8 \text{ km}) / (6.2 \text{ km})] = 38°$; $\mathbf{R} = 7.8 \text{ km, 38° north of east}$. This result is confirmed by the figure below.

![Vector Addition Diagram](image)

**1.68. Set Up:** The volume of a sphere is $V = \frac{4}{3}\pi r^3$. Note that $1 \mu m = 10^{-6} \text{ m} = 10^{-3} \text{ mm}$.

**Solve:** Solve for $r$ to obtain the diameter of a typical alveolus:

$$d = 2r = 2\left(\frac{3V}{4\pi}\right)^{1/3} = 2\left(\frac{3(4.2\times10^6 \mu m^3)}{4\pi}\right)^{1/3}\left(\frac{10^{-3} \text{ mm}}{1 \mu m}\right) = 0.20 \text{ mm}.$$ 

The correct answer is A.