Vectors – Solutions

Q3.3. **Reason:** Consider two vectors \( \vec{A} \) and \( \vec{B} \). Their sum can be found using the method of algebraic addition. In Question 3.2 we found that the components of the zero vector are both zero. The components of the resultant of \( \vec{A} \) and \( \vec{B} \) must then be zero also. So

\[
R_x = A_x + B_x = 0 \\
R_y = A_y + B_y = 0
\]

Solving for the components of \( \vec{B} \) in terms of \( \vec{A} \) gives \( B_x = -A_x \) and \( B_y = -A_y \). Then the magnitude of \( \vec{B} \) is

\[
\sqrt{(B_x)^2 + (B_y)^2} = \sqrt{(-A_x)^2 + (-A_y)^2} = \sqrt{(A_x)^2 + (A_y)^2}.
\]

So then the magnitude of \( \vec{B} \) is exactly equal to the magnitude of \( \vec{A} \).

**Assess:** For two vectors to add to zero, the vectors must have exactly the same magnitude and point in opposite directions.

P3.7. **Prepare:** The figure below shows the components \( v_x \) and \( v_y \), and the angle \( \theta \). We will use Tactics Box 3.3 to find the sign attached to the components of a vector.

![Diagram](image)

**Solve:** We have,

\[
v_x = -v \sin 40^\circ, \quad \text{or} \quad -10 \text{ m/s} = -v \sin 40^\circ, \quad \text{or} \quad v = 15.56 \text{ m/s}.
\]

Thus the \( x \)-component is \( v_x = v \cos 40^\circ = (15.56 \text{ m/s}) \cos 40^\circ = 12 \text{ m/s} \).

**Assess:** Note that we had to insert the minus sign manually with \( v_y \) since the vector is in the fourth quadrant.

P3.11. **Prepare:** We will follow rules given in the Tactics Box 3.3.

![Diagram](image)

**Solve:** (a) Vector \( \vec{d} \) points to the right and down, so the components \( d_x \) and \( d_y \) are positive and negative, respectively:

\[
d_x = d \cos \theta = (100 \text{ m}) \cos 45^\circ = 70.7 \text{ m} \quad \text{and} \quad d_y = -d \sin \theta = -(100 \text{ m}) \sin 45^\circ = -71 \text{ m}
\]

(b) Vector \( \vec{v} \) points to the right and up, so the components \( v_x \) and \( v_y \) are both positive:
(c) Vector \( \vec{a} \) has the following components:

\[ a_x = -a \cos \theta = -(5.0 \text{ m/s}^2) \cos 90^\circ = 0 \text{ m/s}^2 \]
\[ a_y = -a \sin \theta = -(5.0 \text{ m/s}^2) \sin 90^\circ = -5.0 \text{ m/s}^2 \]

**Assess:** The components have same units as the vectors. Note the minus signs we have manually inserted according to the Tactics Box 3.3.

**P3.14. Prepare:** We can use Equations 3.11 and 3.12 to find the magnitude and direction of a vector given its components.

**Solve:**  (a) See the following diagram.

Using Equation 3.11,

\[ v = \sqrt{(v_x)^2 + (v_y)^2} = \sqrt{(10 \text{ m/s})^2 + (-30 \text{ m/s})^2} = 32 \text{ m/s} \]

Using Equation 3.12,

\[ \theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{30 \text{ m/s}}{10 \text{ m/s}}\right) = 72^\circ \]

(b) See the following diagram.

Using Equation 3.11,

\[ a = \sqrt{(a_x)^2 + (a_y)^2} = \sqrt{(20 \text{ m/s}^2)^2 + (10 \text{ m/s}^2)^2} = 22 \text{ m/s}^2 \]

Using Equation 3.12,

\[ \theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{10 \text{ m/s}^2}{20 \text{ m/s}^2}\right) = 27^\circ \]

**Assess:** Comparing the vector diagrams to the calculations, both these answers make sense.

**P3.15. Prepare:** Assume you start at the spot labeled home and that Strawberry Fields and Penny Lane are perpendicular. We will not assume that the lengths in the figure are to scale. We will write each vector in component form for easy addition. We will then need to take the sum and compute the magnitude and direction to report the final answer.

The Strawberry Fields vector is \( \vec{S} = (S_x, S_y) = (2.0 \text{ km}, 0.0 \text{ km}) \).

The Penny Lane vector is \( \vec{P} = (P_x, P_y) = (0.0 \text{ km}, -1.0 \text{ km}) \).
The Abbey Road vector is

\[ \vec{A} = (A_x, A_y) \]
\[ = (A \cos \phi, A \sin \phi) \]
\[ = ((4.0 \text{ km}) \sin (40^\circ), (4.0 \text{ km}) \cos (40^\circ)) \]
\[ = (2.57 \text{ km}, 3.06 \text{ km}) \]

(The cos and sin are interchanged from Equation 3.10 because \( \phi \) is measured from the \( y \)-axis. We have used an extra significant figure for extra accuracy.)

**Solve:** Now add the respective components of the three vectors to get the components of the total displacement.

\[
\vec{S} = (2.0 \text{ km}, 0.0 \text{ km})
\]
\[
\vec{P} = (0.0 \text{ km}, -1.0 \text{ km})
\]
\[
\vec{A} = (2.57 \text{ km}, 3.06 \text{ km})
\]
\[
\vec{D} = (4.57 \text{ km}, 2.06 \text{ km})
\]

Now use Equations 3.11 and 3.13.

\[
D = \sqrt{(D_x)^2 + (D_y)^2} = \sqrt{(4.57 \text{ km})^2 + (2.06 \text{ km})^2} = 5.1 \text{ km}
\]
\[
\theta = \tan^{-1}\left(\frac{D_y}{D_x}\right) = \tan^{-1}\left(\frac{2.06 \text{ km}}{4.57 \text{ km}}\right) = 24^\circ
\]

where \( \theta \) is measured ccw from the positive \( x \)-axis.

**Assess:** Even though the figure may not be precisely to scale, it, or one you draw, would convince you that the answers for the magnitude and direction are both reasonable.

**P3.46. Prepare:** The vectors \( \vec{A}, \vec{B}, \) and \( \vec{C} \) are shown. We will first calculate the \( x \)- and \( y \)-components of each vector and then obtain the magnitude and the direction of the vector \( \vec{D} \).

<table>
<thead>
<tr>
<th>( \vec{A} )</th>
<th>( \vec{B} )</th>
<th>( \vec{C} )</th>
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<tbody>
<tr>
<td>( A_x )</td>
<td>( B_x )</td>
<td>( C_x )</td>
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<tr>
<td>( A_y )</td>
<td>( B_y )</td>
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Find \( D_x, D_y, \theta \) relative to \( +x \)

Solve: (a) The vectors \( \vec{A}, \vec{B}, \) and \( \vec{C} \) are drawn above.

(b) The components of the vectors \( \vec{A}, \vec{B}, \) and \( \vec{C} \) are \( A_x = (3 \text{ m}) \cos 20^\circ = 2.82 \text{ m} \) and \( A_y = -(3 \text{ m}) \sin 20^\circ = -1.03 \text{ m} \);
\( B_x = 0 \text{ m} \) and \( B_y = 2 \text{ m} \); \( C_x = -(5 \text{ m}) \cos 70^\circ = -1.71 \text{ m} \) and \( C_y = -(5 \text{ m}) \sin 70^\circ = -4.70 \text{ m} \).

(c) We have \( \vec{D} = \vec{A} + \vec{B} + \vec{C} = \vec{D}_x + \vec{D}_y \), which means \( D_x = 1.11 \) and \( D_y = -3.73 \).

\[
D = \sqrt{(1.11 \text{ m})^2 + (-3.73 \text{ m})^2} = 3.9 \text{ m} \quad \theta = \tan^{-1}\left(\frac{3.73}{1.11}\right) = \tan^{-1}3.36 = 73^\circ
\]

The direction of \( \vec{D} \) is south of east, 73\(^\circ\) below the positive \( x \)-axis.
**P3.48. Prepare:** The minute hand of the watch is shown in the figure.

\[ S_{8:00} = 2.0 \text{ cm, north} \quad \text{and} \quad S_{8:20} = (2.0 \cos 30^\circ, \text{east}) + (2.0 \sin 30^\circ, \text{south}). \]

The displacement vector is

\[ \Delta \vec{r} = S_{8:20} - S_{8:00} = (1.74 \text{ cm, east}) + (3.00 \text{ cm, south}) \]

(b) We have \( S_{8:00} = 2.0 \text{ cm, north} \) and \( S_{9:00} = 2.0 \text{ cm, north} \). The displacement vector is \( \Delta \vec{r} = S_{9:00} - S_{8:00} = 0 \).

**Assess:** The displacement vector in part (a) has positive \( x \)-component (toward east) and negative \( y \)-component (toward south). The vector thus is to the right and points down, in the IV quadrant. This is what the vector drawn from the tip of the 8:00 A.M. arm to the tip of the 8:20 A.M. arm will be.