Mathematics 396

Elementary Problem Solving

Coursepack

Spring Term 2011

Kruczek

Required

Table of Contents

MTH 396 Syllabus for Spring 2011

Klay Kruczek **Phone**: 503-838-8829 **Office**: MNB 125 **Email**: kruczekk@wou.edu

OFFICE HOURS & SCHEDULE					
Time	**M**	**T**	**W**	**R**	**F**
10:00 – 11:00	***Office***		***Office***		
11:00 – 12:00	MTH 358	MTH 358	MTH 358	MTH 358	
12:00 – 1:00	MTH 396	Lunch	MTH 396	Lunch	
1:00 – 2:00					
2:00 – 2:30	Lunch	***Office***			
2:30 – 3:30	***Office***				
3:30 – 5:00		MEETINGS		***Office***	
5:00 – 6:50		MTH 495		MTH 495	

PREREQUISITES: Mathematics 211, 212, and 213 with a grade of C- or better.

TEXT: "Problem Solving Strategies Crossing the River with Dogs", Johnson and Herr

COURSE CONTENT: This course is designed for students planning to be elementary or middle school teachers. Our goals for this term are to:

- Improve problem solving skills by building upon and extending knowledge acquired in the foundational sequence;
- Learn to write non-routine problems that introduce, enhance or illustrate important mathematical themes or ideas;
- Become proficient in using the state problem solving scoring guide;
- Use the Math Forum Resource to experience reading, replying, and evaluating young problem solvers' solutions.

ATTENDANCE: Daily attendance is required for your success in this course. If you miss class, it is your responsibility to ask a classmate for notes on the material you have missed.

READING THE TEXT: It is a good idea to carefully read the section before you start your homework. I encourage you to ask questions about the examples presented in the book. You may ask questions about the text both in class and during office hours.

TEXT PROBLEMS HOMEWORK: The homework assignments will usually assessed by a peer review process. Exams will be based on homework problems and in-class activities. I will check random homework problems to find out how everybody is progressing.

JOURNAL HOMEWORK: There will be four journal quizzes throughout the quarter. Please see the Journal Assignment sheet (in your course pack) for more details. The class schedule will give you the quiz dates.

PROBLEMS OF THE WEEK (POWs): Throughout the course, you will be assigned three special problems to help you focus on clear and precise explanations. More detailed information is provided on the POW Assignments sheet. The class schedule will give you the quiz dates.

PORTFOLIO PROBLEMS: You will be asked to write three problems geared toward elelmentary and middle school students. More detailed information is provided on the Portfolio Assignments sheet.

MATH FORUM: In the latter portion of the course, we will work with the Math Forum's Online Mentoring Program. In this portion of the course, you will each mentor elementary school students across the country answering the Problem of the Week.

EXAMS: There will be two mid-term exams in this course and one final. All exams are timed, and the Final Exam is an in-class group final (on Tuesday 6/7 2 – 3:50p.m.) **Students with a 95% going into the final exam will be exempt.**

COURSE NOTEBOOK: Please file your work into a 3-ring binder divided into these sections: Class Notes & Activities; Text HW; Portfolio; Journal Assignments; POWs; Math Forum

COURSE GRADING: Each day your participation in class will be noted. Documented excuses for illness will be accepted for late work. Notification must be prompt and in advance.

	Exams (Midterms and Final)	300 pts.
	POWs	4 x 40 pts = 160 pts.
	Journal Assignments	3 x 15 pts = 45 pts.
	Text Problems Homework	7 x 20 pts = 140 pts.
	Portfolio Problems	3 x 40 pts=120 pts.
	Math Forum Mentoring and Training	235 pts.

STANDARD GRADING SCALE FOR THIS COURSE

% Range	Grade	% Range	Grade	% Range	Grade	% Range	Grade	% Range
93 –100	A	87 – 89	B+	77 – 79	C+	67 – 69	D+	< 60
90 – 92	A-	83 – 86	B	73 – 76	C	63 – 66	D	**Grade**
		80 – 82	B-	70 – 72	C-	60 – 62	D-	F

LATE WORK POLICY: All work is due by 4:00 p.m. on the due date. All due items may be turned in, unexcused, 1 class day late (by 4 p.m.) for 80% credit or 2 class days (by 4 p.m.) late for 60% credit. There will be NO credit for assignments more than 2 class days late.

EXCUSED LATE WORK / MAKE UP EXAMS: Excused late work or make up exams will only be accepted in the case of documented emergency or a documented university sanctioned absence from class. **Prior** notification and my agreement are required. Ordinary illness of a day or two does not count as a documented emergency, even with a note from a doctor.

APPROPRIATE CLASSROOM BEHAVIOR: Proscribed Conduct for all students is described in the University Catalog. In particular for this course any student found cheating on an exam or copying from another student's exam paper will receive a zero score on that exam.

LEARNING DISABILITIES: If you have a documented learning disability, please talk to me during the first few days of class. I will accommodate you in any way that I can. If you have a documented disability, which requires any academic accommodations, you must go to the Office of Disability Services (ODS) for appropriate coordination of your accommodations. You can drop by APSC 405 or call ODS (503) 838-8250 (VTTY) to schedule an appointment.

INCOMPLETE POLICY: An Incomplete can only be granted for a student who is passing a class and has a documented emergency that prevents him/her from completing the course.

CELL PHONES: Turn off your cell phones before class. DO NOT TEXT IN CLASS!

Mathematics 396 Spring 2011

ASSIGNMENTS (DUE DATES Listed!)

MONDAY			WEDNESDAY	
3/28			3/30	Email assignment
4/4	Ch.1: pp.19-23 Set A: 1, 4, 7, 15 Ch.2: pp. 38-41 Set A: 6, 9 Ch 3: pp. 61-66 Set A: 3, 14		4/6	Journal 1 Quiz POW 1
4/11	Ch 3: pp. 61-66 Set A: 10, 11, 13 Ch.4: pp.94-105 Set A: 9, 12 Ch 5: pp.126-31 Set A: 3, 5, Set B: 3		4/13	Portfolio 1
4/18	Ch 5: pp.126-31 Set A: 12, 13 Ch. 6: pp.155-60 Set A: 8, 11, 16 Ch. 7: pp. 178-181 Set A: 3, 9, 11		4/20	Study for OMG Exam POW 2
4/25	Ch. 9: pp.244-247 Set A: 2, 7, Set B: 5 Ch.10: pp. 253-4 Set A1: 1 Ch.10: pp. 268-72 SetA2: 8, 9, 12, 20 Journal 2 Quiz		4/27	Online Mentoring (Begin 1st) Portfolio 2
5/2	Online Mentoring		5/4	Online Mentoring POW 3
5/9	Ch. 11: pp. 293-296 Set A: 3, 7, 12, 15 Ch. 12: pp. 318 - 323 Set A: 8, 9, 10, 14 Journal 3 Quiz Mentoring (Wrap-up of 1st problem)		5/11	Online Mentoring (Begin 2nd) Portfolio 3
5/16	Online Mentoring		5/18	Online Mentoring
5/23	Ch: 8: pp. 211-4 Set A: 3(f – j), 8, 10, 11 Ch. 15: pp. 405-7 Set A: 13,14,17;SetB: 1 Mentoring (Wrap-up of 2nd problem)		5/25	POW 4 Study for Problem Solving Exam
5/30			6/1	Ch: 13 pp. 345-9 Set A: 2, 6, 7, 17 Ch. 16: pp 427- 9: 1, 2, 7 Ch: 17: p. 456 Set A: 4

CLASS SCHEDULE

MONDAY			WEDNESDAY	
3/28	Intro. to Problem Solving; Polya's Framework; Assessing Problem Solving (Activities 1 – 3)		3/30	Finish Glass is Half Full Visual Methods (Activity 4)
4/4	Text HW: Ch. 1 – 3 More Visual Methods (Activity 5)		4/6	Horsin' Around Journal 1 Quiz
4/11	Text HW: Ch. 3 – 5 Mentor training (7 Congruent Rectangles)		4/13	Online Mentoring Guide
4/18	Text HW: Ch. 5 - 7 OMG: Wrap-up of lessons & Review		4/20	**Exam on Online Mentoring Guide**
4/25	Text HW: Ch. 9, 10 Journal 2 Quiz Review of Solns to FunPOW 1		4/27	Begin Mentoring of FunPOW 1
5/2	Venn Diagrams (Activity 8) Live Mentoring		5/4	Venn Diagrams Continued (Activity 8) Live Mentoring
5/9	Journal 3 Quiz Text HW: Ch. 11, 12 Review of Solns to FunPOW 2		5/11	Begin Mentoring of FunPOW 2
5/16	Algebra (Activity 9, 10) Live Mentoring		5/18	Algebra (Activity 11) Live Mentoring
5/23	Text HW: Ch. 8, 15 Counting (Activity 6, 7)		5/25	**Exam on Problem Solving**
5/30	No School Memorial Day		6/1	Text HW: Ch. 13, 16, 17 Number Theory (Activity 12)

MATH 396 SCORE SHEET SPRING 2011

TEXT HOMEWORK	
Text HW 1	/20
Text HW 2	/20
Text HW 3	/20
Text HW 4	/20
Text HW 5	/20
Text HW 6	/20
Text HW 7	/20
Total Text Homework	**/140**

POW	
POW 1	/40
POW 2	/40
POW 3	/40
POW 4	/40
Total POW	/160

PORTFOLIO	
Portfolio 1	/40
Portfolio 2	/40
Portfolio 3	/40
Total Portfolio	**/120**

JOURNAL QUIZ	
Journal 1	/15
Journal 2	/15
Journal 3	/15
Total Journal	**/45**

MENTOR TRAINING / FOLLOW UP	
In-class train 1 (Horsin')	/15
In-class train 2 (rectangles)	/15
Online training (OMG)	/25
Total Mentor Training	**/55**

MENTOR THREADS	
MP 1 –Thread 1	/30
MP 1 –Thread 2	/30
MP 1 –Thread 3	/30
MP 2 –Thread 1	/30
MP 2 –Thread 2	/30
MP 2 –Thread 3	/30
Total Mentor Threads	**/180**

EXAMS	
Mentor Exam	/50
Problem Solving Exam	/100
Final Exam	/150
Total Exams	/300

COURSE TOTALS	
Text Homework	**/140**
POW	**/160**
Portfolio	**/120**
Journal	**/45**
Mentor Training	**/55**
Mentor Threads	**/180**
Exams	**/300**
TOTAL	**/1000**

Activity 1: Word Problems

1. List the steps you need to perform in order to solve a mathematics word problem.

2. In your cooperative groups discuss what makes a good word problem. List your ideas below.

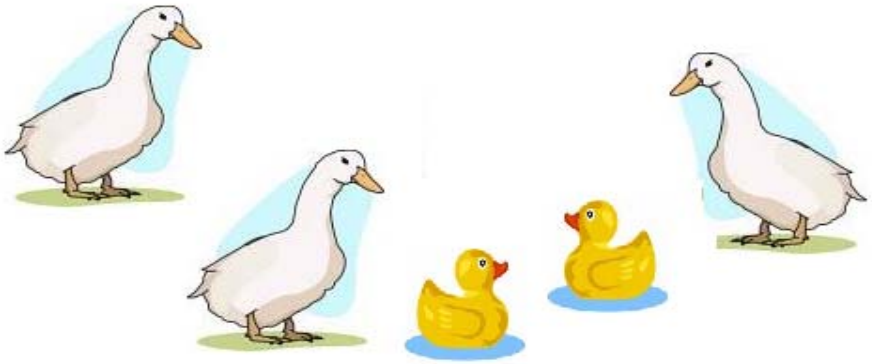
3. Discuss what *teachers* need to know when doing a word problem with their class. List your ideas below.

4. Where do you find good math problems for your classroom?

5. How do you go about modifying a problem to make it more appropriate for your classroom?

Activity 2: Ducky Promenade

Ducky Promenade



Three big ducks went out one day
With two ducklings behind.
Fourteen kilograms was their weight
When all five were combined.

Add one duck and one duckling
And their weight increased.
Nineteen kilograms was the total
Of the seven with webbed feet.

If all ducklings have one weight,
And all ducks have another,
Then you should find the total weight
Of three – one child, two mothers.

Activity 3: Juicy Juice (ideas by Sandy Kravolec)

1. In your cooperative groups, spend time brainstorming some ideas of how to assess problem solving at the K-8 level. Record your ideas below.

2. Construct a simple rubric, worth 21 points, with four components to be used in assessing the solution to a rich word problem. Write a brief one sentence description for each score in each category.

   Category I)

   Category II)

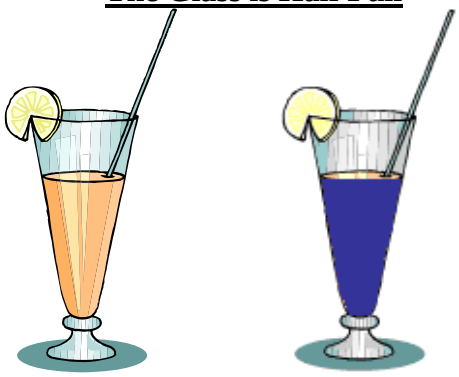
   Category III)

   Category IV)

3. Record the class scoring rubric the class agrees to use for this activity.

4. In your cooperative groups, solve the problem below and make a poster of your solution.

The Glass is Half Full



Two glasses are each filled with the same amount of juice – one with grape juice, one with orange juice. A spoonful of the grape juice is removed from the grape juice and put into the glass with the orange juice. The juices are mixed completely. Then the same spoonful (carefully measured) is removed from the orange/grape mixture and put into the glass with the grape juice.

Question: After the juices have been exchanged, is there more grape in the orange juice, or orange in the grape juice?

5. When every group has completed the problem, go around the classroom with your group and grade your assigned posters with the class rubric.

6. Open up all the assessments your group has received from other groups. What is your reaction?

Activity 4: Visual Methods 1—Percentage and Ratio Problems

Example 1:
At a clearance sale, the price of a DVD player was discounted 40%. The discounted price was $120. What was the original price?

Example 2:
Two numbers are in the ratio of 3 to 4. One number is 17 less than the other. What are the numbers?

Example 3:
In a collection of dimes and nickels worth $6.40, the ratio of the number of dimes to the number of nickels is 3 to 2. Find the number of each type of coin.

- Your group will be assigned two problems from the problems on the following page.
- You are required to solve these problems using pictures or diagrams. A table, just algebra or guess and check are not allowed.
- Be sure that every member of your group can do both problems, as you will be explaining how to do these problems to your fellow classmates.
- Once everyone knows how to do their assigned problems, each member of your group (except one) will switch tables. Make sure that no member of your group is at the table you moved to.
- Each person should explain their problems to their new group.

1. The price of a large chocolate bar was increased by 20%. It now costs $1.50. What did it cost before the increase?

2. A telethon for a local charity raised $45,000, which is 125% of the goal. What was the goal?

3. The ratio of two numbers is 8 to 5. Their difference is 45. What are the numbers?

4. Sandra sold 40% more Campfire Girls candy than her sister. Together they sold $408 worth. How much did each girl sell?

5. Three numbers are in the ratio of 5 to 6 to 7 and their total is 720. Find the numbers.

6. A length of string that is 180cm long is cut into 3 pieces. The second piece is 25% longer than the first, and the third piece is 25% shorter than the first. How long is each piece?

7. Jane paid $17.49 for a book, including tax. If the sales tax is 6%, what was the price without tax.

8. If two gallons of mixture with a 30% concentration of ammonia is added to 20 gallons of water, what is the concentration of ammonia in the new concoction?

9. The ratio of the length of a rectangle to its width is 4 to 3. Its area is 300 square inches. What are its length and width?

10. The ratio of Sue's age to her father's age is 2 to 7. In three years, their ages will total 60 years. How old is each now?

11. Clara is 31 years old. Her sister Molly is 47. In how many years, will their total ages be in the ratio of 4 to 5?

Activity 5: Visual Methods 2—Still More Percentage and Ratio Problems & Puzzle Problems

Your group will be assigned two problems below. Make sure that you truly understand how to solve these problems (by picture or diagram) and can clearly explain what you used to solve the problem, as you will have to present them to the class on the board.

1. During its January clearance sale, a gift shop offered all its merchandise at a 20% discount. Arnold bought an item for $33.39, which included a 5% sales tax. What was the original price of the item, without tax?

2. A car is sold with a $1000 rebate and a 15% discount. Which gives the better price for the buyer: (a) rebate and then discount the price after rebate, (b) discount the price and then rebate?

3. A baseball team has played ¾ of its season and won 45% of its games. What percent of its remaining games must it win to have a break-even season (i.e. win half its games)?

4. Separate 43 people into 2 groups so that the first group has 5 less than 3 times the number in the second group.

5. The sum of 2 numbers is 40. Their difference is 14. What are the numbers?

6. The sides of one square are 2 inches longer than the sides of another square, and its area is 48 square inches greater. What is the length of the side of the smaller square?

7. Two sisters together have 20 books. If the younger sister lost 3 books and the older sister doubled the number she has, they would have a total of 30 books. How many books does each have?

8. Of the children in a room, 3/5 are girls. There would be an equal number of boys and girls if the number of boys is doubled and 6 more girls were added. How many children are in the room?

9. One pump can fill a tank in 6 hours. Another pump can fill it in 4 hours. If both pumps are used, how long will it take to fill the tank?

Activity 6: Counting 1--Twelve Days of Christmas

In the song "The Twelve Days of Christmas," on the second day of Christmas, you get two turtle doves AND a partridge in a pear tree. Therefore, you get a total of three gifts on the second day of Christmas alone. Fill in both of the following tables. These tables have been partially been filled in to get you started. Does your first table total match your second table total?

Day	Number of Gifts Given on this Day	Total Number of Gifts Given to Date	
1	1	1	
2	3	4	
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			**Total gifts given**

Day	New Present Given	Number of Days Given	Number of this Present Needed for all 12 Days
1	Partridge in a Pear Tree	12	12
2	Turtle Doves	11	
3	French Hens		
4	Calling Birds		
5	Golden Rings		
6	Geese a Layin'		
7	Swans a Swimmin'		
8	Maids a Milkin'		
9	Ladies Dancin'		
10	Lords a' Leapin'		
11	Pipers Pipin'		
12	Drummers Drummin'		
		Total gifts given	

Activity 7: Counting 2

Part I: Conference Questions

In 1991, there were 9 teams in the ***Atlantic Coast Conference (ACC)***
{Maryland, UVA, NC St, Duke, UNC, Wake Forest, Clemson, Georgia Tech, Florida State}

There were also 8 teams in the ***Ivy League***
{Harvard, Dartmouth, Cornell, Columbia, Princeton, Penn, Yale, Brown}

1. The play-in round of the ACC basketball tournament had the two teams with the worst records play each other. How many possible match-ups are there for this game?

2. In how many ways can you choose the ACC champion and the ACC runner-up? In how many ways, can you choose who finishes first, second and third in the ACC conference?

3. In the ***East Coast Bowl***, the conference champions of the Ivy League and ACC play each other. How many different match-ups are possible for this bowl?

4. How many possible winners do you have for the ***East Coast Bowl***?

5. Can you extend each of these problems? For example, if you have 8 shirts, 6 pairs of pants and 2 pairs of shoes, how many different outfits can you form (wearing one of each article of clothing)? Try to answer the question WITHOUT listing all the possible outfits.

Part 2: Pizza Questions

6. Naples Pizza has 5 different toppings (Bacon, Mushroom, Onions, Pepperoni and Sausage). How many different pizzas can you make using these toppings?

7. Suppose the pigs went on strike and Naples has a bacon shortage, so they only have 4 toppings for pizzas now. How many different pizzas can they make now?

8. What happens if they only had 3 toppings to work with (i.e. say the mushrooms went bad)?

9. For pizzas with 5 available toppings to work with, how many of those pizzas had no toppings? How many had exactly 1 topping? 2 toppings? 3 toppings? 4 toppings? 5 toppings?

10. For pizzas with 4 available toppings to work with, how many of those pizzas had no toppings? How many had exactly 1 topping? 2 toppings? 3 toppings? 4 toppings?

11. For pizzas with 3 available toppings to work with, how many of those pizzas had no toppings? How many had exactly 1 topping? 2 toppings? 3 toppings?

12. When there are n toppings to work with, how many different types of pizza can be made?

13. When there are n toppings available, how many k topping pizzas can be made ($0 < k \leq n$)?

Activity 8: Venn Diagrams

Example

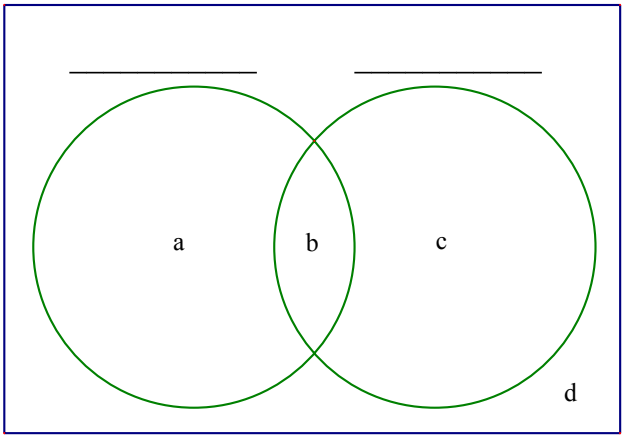
Use a Venn diagram to sole the following problem:

There were 700 people asked the following two questions:

1) Do you like hot dogs?

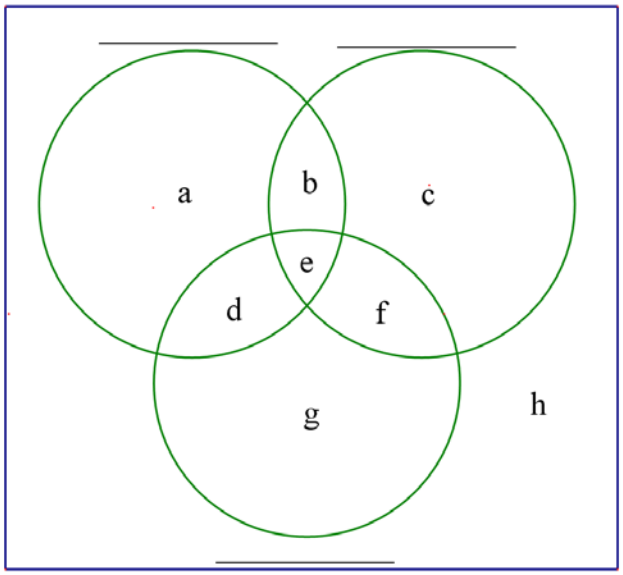
2) Do you like hamburgers?

Of those people interviewed: 254 said they liked hamburgers, 461 said they liked hot dogs, and 140 people answered yes to both questions. How many of the people interviewed didn't like either?

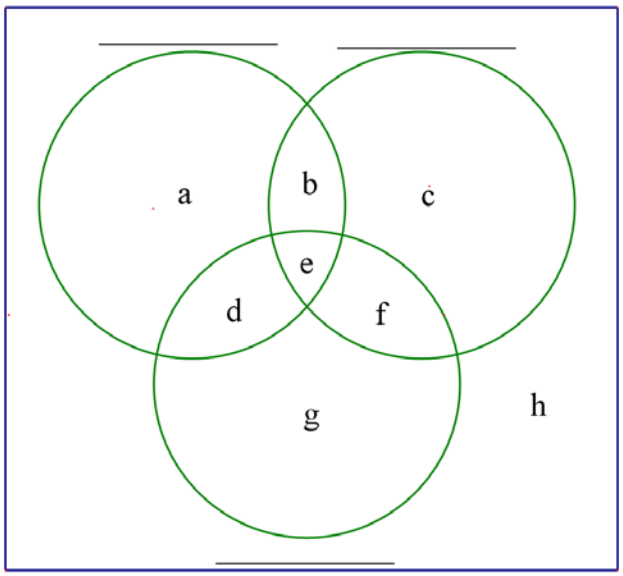


When solving a problem using Venn Diagrams, what are the steps you take?

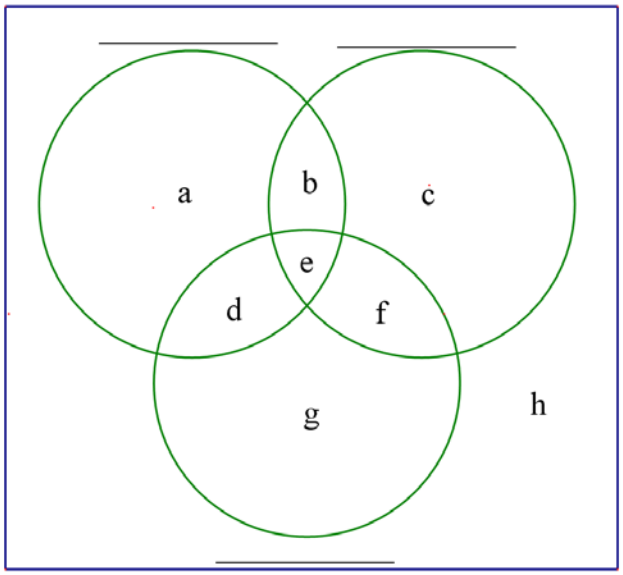
1. In a recent customer interview, 800 people took taste tests. Of those, 723 said they like either cola, root beer or orange soda. Of the people surveyed, 316 like both root beer and cola, while another 87 said they like cola but do not like root beer. Furthermore, a total of 427 in the survey like root beer. How many people in this survey like orange soda, but do not like either root beer or cola?



2. Ten students are taking neither psychology nor Spanish. Five students are taking neither algebra nor Spanish. Seven students are taking neither algebra nor psychology. All of the students are taking at least one of the three subjects. No student is taking all three subjects. Twenty-one students are not taking psychology. Seventeen students are taking algebra. Seventeen students are taking Spanish. How many students are taking psychology?



3. Twenty-four dogs are in a kennel. Twelve of the dogs are black, six of the dogs have short tails, and fifteen of the dogs have long hair. There is only one dog that is black with a short tail and long hair. Two of the dogs are black with short tails. Two of the dogs have short tails and long hair. If all of the dogs in the kennel have at least one of the mentioned characteristics, how many dogs are black with long hair but do not have short tails?



Activity 9: Algebra Techniques 1

1. When solving a problem which requires you to solve the problem by algebra, what are the steps you need to take?

2. The price of Mountain Dew at Kwik-i-Mart increased by 10% because of a dew shortage. If the price is now $1.65, what was the original price?

   a. Solve the problem using visual methods (draw and use a picture).

   b. Find the answer a second way by writing and solving an algebraic equation.

3. Two-thirds of Sally's Halloween candies were chocolaty and the rest were fruity. If the number of fruity candies were tripled and the number of chocolaty candies increased by eight, then she would have an equal number of chocolaty and fruity candies. How many chocolaty candies does Sally have?

   a. Solve the problem using visual methods (draw and use a picture).

   b. Find the answer a second way by writing and solving an algebraic equation.

4. Jesse's uncle is four more than three times his age. Together Jesse's age and his uncle's age add up to 52.

   a. Solve the problem by using a guess and check table.

   b. Find the answer a second way by writing and solving an algebraic equation.

5. One day you see 7 cycle riders and 19 wheels ride by your house. How many bikes and how many tricycles were there?

a. Solve the problem by using a guess and check table.

b. Find the answer a second way by writing and solving an algebraic equation.

Solve the following problems using ALGEBRA.

6. The sum of three consecutive numbers is 54. What are the numbers?

7. Monty lived one-fourth of his life in New York, one-fifth of his life in St. Louis, one-third of his life in Santa Fe, and has spent the last 13 years in Tucson. How old is Monty?

8. Cici and Amantina have lots of stickers. Cici had one third as many as Amantina had, but then Amantina gave Cici six of her stickers, and now Cici has half as many as Amantina. How many stickers did each girl start with?

9. A group of students was transported to the championship basketball game on buses and vans. When one bus and two vans unloaded, there were 55 students. A few minutes later, two more buses and one van unloaded 89 students. In all, three buses and eight vans drove students to the game. How many students went to the game?

Activity 10: Algebra 2 – Rate Problems
Solve each of the problems below by ALGEBRA.

Rate Problems: Distance = rate x time $(\mathbf{d} = \mathbf{r} \times \mathbf{t})$

1. A train starts from Portland and heads for NYC at 40 mph. Two hours later a passenger train leaves the same station for NYC traveling 60 mph. How long before the trains are the same distance from Portland?

2. Bob owns a house on a river. The river flows at 3 miles per hour. Bob decides to start rowing downstream, with the current, at noon. He wants to return to his house at 5 p.m. At what time should Bob turn around and row home if he normally rows 5 miles per hour in water that has no current?

3. Blaise rode his bike to his friend Elroy's house, which was 15 miles away. After he had been riding for half an hour, he got a flat tire. He walked his bike the rest of the way. The total trip took him 3 hours. If his walking rate was one-fourth as fast as his riding rate, how fast did he ride?

4. (from MATHCOUNTS) A man is running through a train tunnel. When he is $\frac{2}{5}$ of the way through, he hears a train that is approaching the tunnel from behind him at a speed of 60mph. Whether he runs ahead or runs back, he will reach an end of the tunnel at the same time the train reaches that end. At what rate, in mph, is he running? (Hint: Draw a diagram.)

Activity 11: Algebra 3 – Work Problems

Many problems are related to the "rate × time = distance" formula. "Work" problems can often be solved by considering the rate of work: amount of work each worker does per some unit of time. It's important to pay attention to units.

$$\text{Rate of work} \times \text{Time worked} = \text{Amount of work done}$$

"Amount of work done" can be a fraction where 1 represents completion of the whole job. For example if it takes Kim 2 hours to clean her apartment, then her rate of work can be though of as $\frac{1}{2}$ apartment per hour . If she works 2 hours she gets 1 whole apartment cleaned:

$$\frac{1}{2}\frac{\text{apartment}}{\text{hour}} \times 2 \text{ hours} = \frac{2 \text{ apartments}}{2} = 1 \text{ apartment}$$

1. It takes Pat 2 hours to paint all the lamp-posts on the street. It takes Tim 3 hours to paint all the lamp-posts on the street. How long will it take the two men if they work together?

   a. Pat's rate of work: What fraction of the whole job does Pat get done in one hour? (This is Pat's rate in terms of work/hour.)

   b. Tim's rate of work: What fraction of the whole job does Tim get done in one hour?

   c. What is their rate per hour if the two men work together?

   d. How long does it take the two men to complete the job working together?

2. Mr. Jones has a blueberry field and 20 children – 12 girls and 8 boys. If the children work together, they can pick all of the blueberries in 10 hours. Assuming that each child works at the same rate, how long would it take just the 8 boys to pick the blueberries?

3. Together Leo and Stan can sort the mail in 3 hours. If Leo works alone, it takes him 7 hours to sort the mail. How long would it take Stan, working alone, to sort the mail?

4. Susan, Amy and Kristin are sisters and are supposed to pick up the toy room every night before bed. If Susan is not helping, Amy and Kristin take 36 minutes to pick up the toy room. If Amy is not helping, Susan and Kristin take an hour. If Kristin is not helping, Susan and Amy take half an hour. If they all work together, how long will it take them to pick up the toy room?

Activity 12: Number Theory

Yellow	1	2	3	4	**5**	6	7	8	9	**10**	11	12	13	14	**15**	16	17	18	19	**20**	**…**
Blue	2	4	6	8	**10**	12	14	16	18	**20**	22	24	26	28	**30**	32	34	36	38	**40**	**…**
Turquoise	3	6	9	12	**15**	18	21	24	27	**30**	33	36	39	42	**45**	48	51	54	57	**60**	**…**
Green	4	8	12	16	**20**	24	28	32	36	**40**	44	48	52	56	**60**	64	68	72	76	**80**	**…**
Red	5	10	15	20	**25**	30	35	40	45	**50**	55	60	65	70	**75**	80	85	90	95	**100**	**…**
Purple	6	12	18	24	**30**	36	42	48	54	**60**	66	72	78	84	**90**	96	102	108	114	**120**	**…**
Black	7	14	21	28	**35**	42	49	56	63	**70**	77	84	91	98	**105**	112	119	126	133	**140**	**…**
Orange	8	16	24	32	**40**	48	56	64	72	**80**	88	96	104	112	**120**	128	136	144	152	**160**	**…**
Grey	9	18	27	36	**45**	54	63	72	81	**90**	99	108	117	126	**135**	144	153	162	171	**180**	**…**

Part I. COLOR ACTIVITY MATRIX (check one column for each phrase)	**Never**	**Only once**	**Occasionally**	**Half the time**	**Usually**	**All but once**	**Always**
1. Green numbers are also blue.							
2. Blue numbers are also green.							
3. Numbers which are purple are both blue and turquoise.							
4. Numbers which are black are both blue and turquoise.							
5. Red numbers are also blue.							
6. Purple numbers are also green.							
7. Grey numbers are also black.							
8. A number is only a single color.							
9. Some number is blue, turquoise, green, red, purple, black, orange, and grey.							
10. 5 is the last digit of a red number.							
11. 5 is the last digit of a black number.							
12. Multiplying orange numbers gives green numbers.							
13. Subtracting green numbers gives blue numbers.							
14. Adding two numbers of the same color gives an answer in that same color.							
15. Multiplying two numbers of the same color gives an answer in that same color.							
16. Squaring a number gives an answer in that same color.							
17. Subtracting red numbers from larger black numbers gives blue numbers.							
18. A green times a turquoise is green.							
19. A green times a turquoise is turquoise.							

Part II: Pink, Blue, Green and Red numbers

Define the following sets of numbers and for questions 1 – 9, either answer the question or explain why there is no solution to the question.

Pink 2 3 5 7 **11** 13 17 19 **23** 29 31 37 41 43 **47** 53 59 61 67 **71** 73 79 83 89 **97** ⋯

Blue 2 4 8 16 **32** 64 128 256 512 **1024** 2048 4096 8192 16384 **32768** 65536 **131072** **262144** **524288** **1048576** ⋯

Green 1 8 27 64 **125** 216 343 512 729 **1000** 1331 1728 2197 2744 **3375** 4096 4913 5832 6859 **8000** …

Red 1 4 9 16 **25** 36 49 64 81 **100** 121 144 169 196 **225** 256 289 324 361 **400** 441 484 529 576 **625** ⋯

1. Find two nonconsecutive red numbers whose sum is red. _____ and _____ (sum is _____)

2. Red numbers squared are still red numbers. Which other colors stay the same color when squared? _____and _____

3. Find two red numbers (each with at least 2 digits) whose product is red. _____ and _____

4. Find two blue numbers (each with at least 3 digits) whose product is blue: _____ and _____

5. Find two green numbers (each with at least 2 digits) whose sum is green. _____ and _____

6. Find two red numbers whose difference is pink. _____ and _____ (It's harder if they are not consecutive.)

7. Find two green numbers whose product is the 12th green number. _____ and _____

8. Find two blue numbers whose product is the 15th blue number: _____ and _____

9. Find a red that is double another red. _____

Challenge Problems
Solve the following using Algebra.

1. Jill is three times as old as Jack was when Jill was as old as Jack is now. When Jack is as old as Jill is now, Jill will be 56. How old are Jack and Jill now?

2. I am thinking of a two-digit number. If the digits of my number are reversed, the new number is 9 greater than my original number. If the tens digit of my original number is doubled and the units digit is halved, the new number is 28 greater than my original number. What is my original number?

3. The absentminded professor walked up the escalator in the department store at a constant rate of one step per second. Just as he reached the top, he realized that he had left his briefcase on the counter where he had just purchased a pocket calculator. Fortunately, no one else was on the up escalator, so the professor decided to run down. He managed three steps per second on his descent (note that he's not skipping steps). After he retrieved his briefcase, the professor decided to repeat the procedure. He noted that he took 18 steps on the way up and 90 steps on the way down. How many steps of the escalator are visible at one time?[1]

[1] From *Problem Solving Through Recreational Mathematics,* Averbach and Chein

Horsin' Around

posted October 20, 2008 (http://mathforum.org/pows)

Zachary travels on a journey of 50 miles. He spends half of his time riding his horse and half of his time walking. When he rides his horse, he covers 9 miles every hour. When he walks, he covers 3 1/2 miles every hour. How long does it take him to complete the journey?

Be sure to explain your strategy and how you know your answer is correct.

Extra: How many miles does he travel by horseback? How many miles does he walk?

Solution #1: Mel (age 14)
SOLUTION
it will take five hours
EXPLANATION
i just divided the two.

Solution #2: Camille (age 15)
SOLUTION
15 hours
EXPLANATION
he walked half of the time at a rate of 3.5 mph which would take 14 hours to do the whole thing

Solution #3: Sarah (age 11)
SOLUTION:
It would take him about 9 hours to complete his journey.
EXPLANATION:
What I did was I added 9, 9, and 9 and got 27 miles and that's 3 hours. The I added 3 1/2, 3 1/2, 3 1/2, 3 1/2, 3 1/2, 3 1/2 and got 24 miles which is 6 hours. And if you add 27 and 24 you get 51 miles. Which is a little over 50 miles.

Solution #4: Lili (age 16)
SOLUTION
45 minutes
EXPLANATION
45 minutes

Solution #5: Heidi (age 13)

SOLUTION

50mi divided by 2 =25 divided 9=2.7 hr to go 25 miles

25 divided 3.5=7.14 hr walk

7.14+2.7=9 hr and 84 min to complete his journey

EXPLANATION

Since he spent half of his time riding on horseback and half walking I divided the amount of miles he travled 50 mi by 2 because its half and got 25. then divided it by 9 because it would give the amount of miles travled on horseback and got 2.7 hr. Then i tock the other half and divided it by 3 and a half to get how many hours. Finally i added 2.7+7.14 and got 9 hours and 84 minutes.

Solution #6: Sara (age 10)
SOLUTION
The answer that i got was 8 hours.
EXPLANATION
First i saw how many miles they went in one hour and i got six and one quarter of a mile. Therefore i got 8 hours.

Solution #7: Alex (age 10)

SOLTUTION

Zachary traveled 25 miles on horseback at 9 miles per hour. It took him 3 hours. He traveled 25 miles walking at 3 and half miles per hour. It took him 7 hours. The journey took him 10 hours.

EXPLANATION

I divided 50 miles by 2 = 25 miles (walked) 25 miles (horseback)

I divided 25 by 3 and half = 7

I divided 25 by 9 = 3

Added together = 10

Names of Group Members:_____

Turn in one clearly written sheet for your group _____/15 points
(mentor training)

1. What do you notice about the students' solution: What does the student understand? What are the student's main problems? List at least 5 items.

2. Is there anything you're wondering about?

3. Create a list of questions that might help the student to learn something past what s/he already knows. If the answer is wrong, ask questions that will lead the student in the correct direction using their own strategy if possible. If the answer is correct, ask questions that will help improve their explanation or extend their knowledge. List *at least* 5 questions.

4. Of the questions listed in 3, choose two that you think would most help move your student along. Explain what you hope those questions would lead the student to do.

The Math Fundamentals Problem of the Week Scoring Rubric — **Horsin' Around** (posted 17 December 2007)
For each category, choose the level that *best describes* the student's work.

	Novice	Apprentice	Practitioner	Expert
Problem Solving				
Interpretation	Does not show much understanding of the problem.	Shows some understanding of the math in the problem. Completes part of the problem.	Understands that Zachary spends equal time riding his horse and walking. Understands that the total trip is 50 miles. Answers the question: How long does it take Zachary to complete the journey?	Solves the main problem correctly. Understands the Extra and answers the questions: How many miles does Zachary ride? How many miles does he walk? Achieves at least Practitioner in Strategy.
Strategy *(NB: based on their interpretation of the problem)*	Does not know how to set up the problem. OR Shows no evidence of strategy. OR Strategy didn't work.	Tries a strategy that makes sense, but isn't enough to solve the whole problem, OR doesn't apply it systematically. OR Verifies a correct answer, but fails to explain how they found it.	Picks a sound strategy. Approaches the problem systematically, achieving success through skill, not luck. Chosen strategy accounts for any answer(s) that changed after checking our answers. Guess-and-Check approach must involve good reasoning and informed guessing.	Does **one or more** of these: Uses two different strategies. Uses a good Extra strategy. Uses an unusual or sophisticated strategy, e.g., effective and appropriate use of technology or algebra.
Accuracy *(NB: based on chosen strategy)*	Has made many errors. OR Shows no math.	Some work is accurate. May have one or two errors. OR Shows very little arithmetic.	Work on main problem is accurate and contains no arithmetic or record keeping mistakes.	Not available for this problem.
Communication				
Completeness *(NB: an incorrect solution can be complete)*	Writes very little to explain how the answer was achieved.	Describes the steps but does not include calculations or numbers. OR Shows calculations without rationale or explanation.	Explains most of the steps taken to solve the problem and the rationale for them. Includes key calculations with rationale. Verifies the answer ("explain … how you know your answer is correct."), especially when using an algebraic approach. Explanation accounts for any answer(s) that changed after checking our answers.	Explains strategy for Extra. Does **one or more** of these: Includes useful extensions and further explanation of concepts or patterns. Provides exceptional insight into the problem. Includes a table of data.
Clarity *(NB: incomplete and incorrect solutions can be explained clearly*	Explanation is very difficult to read and follow.	Explanation isn't totally unclear, but another student wouldn't be able to follow it easily. Spelling errors/typos make it hard to understand.	Attempts to make explanation readable by a peer. Uses level-appropriate math language, including correct units: miles, hours, miles per hour. Shows effort to use good formatting, spelling, grammar, typing. Errors don't interfere with readability.	Formatting makes ideas exceptionally clear. Answer is very readable and appealing, might include a helpful diagram. (A diagram alone doesn't qualify for Expert status.)
Reflection (See list below.)	Does nothing reflective.	Includes one reflective thing.	Includes two reflective things.	Includes 3 or more reflective things or does an exceptional job with 2 of them.
	The items to the right are considered reflective, and could be in the solution OR in the comment they leave after viewing our answer:	• **Revises and improves the submission**. • Checks the answer using a different method. • Explains a hint she/he would give someone.	• Reflects on the reasonableness of the answer. • Connects the problem to prior knowledge/experience. • Describes any errors made and how she/he found and corrected them.	• Comments on AND explains the ease or difficulty of the problem. • Explains where she/he is stuck. • Summarizes the process used.

Practice Scoring Student submissions

Student:	**Age:**	**Rationale / Evidence**

	novice	apprentice	practitioner	expert
Problem Solving				
interpretation				
strategy				
accuracy				
Communication				
completeness				
clarity				
reflection				

Rationale / Evidence
Interpretation
Strategy
Accuracy
Completeness
Clarity
Reflection

Student:	**Age:**	**Rationale / Evidence**

	novice	apprentice	practitioner	expert
Problem Solving				
interpretation				
strategy				
accuracy				
Communication				
completeness				
clarity				
reflection				

Rationale / Evidence
Interpretation
Strategy
Accuracy
Completeness
Clarity
Reflection

Student:	**Age:**	**Rationale / Evidence**

	novice	apprentice	practitioner	expert
Problem Solving				
interpretation				
strategy				
accuracy				
Communication				
completeness				
clarity				
reflection				

Rationale / Evidence
Interpretation
Strategy
Accuracy
Completeness
Clarity
Reflection

Practice Scoring Student submissions

Student:	**Age:**	**Rationale / Evidence**

	novice	**apprentice**	**practitioner**	**expert**
Problem Solving				
interpretation				
strategy				
accuracy				
Communication				
completeness				
clarity				
reflection				

Rationale / Evidence
Interpretation
Strategy
Accuracy
Completeness
Clarity
Reflection

Student:	**Age:**	**Rationale / Evidence**

	novice	**apprentice**	**practitioner**	**expert**
Problem Solving				
interpretation				
strategy				
accuracy				
Communication				
completeness				
clarity				
reflection				

Rationale / Evidence
Interpretation
Strategy
Accuracy
Completeness
Clarity
Reflection

Student:	**Age:**	**Rationale / Evidence**

	novice	**apprentice**	**practitioner**	**expert**
Problem Solving				
interpretation				
strategy				
accuracy				
Communication				
completeness				
clarity				
reflection				

Rationale / Evidence
Interpretation
Strategy
Accuracy
Completeness
Clarity
Reflection

Practice Scoring Student submissions

Student:	Age:	Rationale / Evidence

	novice	apprentice	practitioner	expert
Problem Solving				
interpretation				
strategy				
accuracy				
Communication				
completeness				
clarity				
reflection				

Rationale / Evidence
Interpretation
Strategy
Accuracy
Completeness
Clarity
Reflection

Student:	Age:	Rationale / Evidence

	novice	apprentice	practitioner	expert
Problem Solving				
interpretation				
strategy				
accuracy				
Communication				
completeness				
clarity				
reflection				

Rationale / Evidence
Interpretation
Strategy
Accuracy
Completeness
Clarity
Reflection

Student:	Age:	Rationale / Evidence

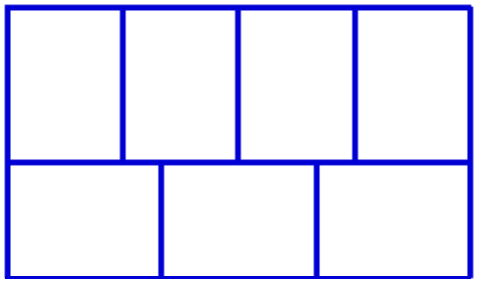
	novice	apprentice	practitioner	expert
Problem Solving				
interpretation				
strategy				
accuracy				
Communication				
completeness				
clarity				
reflection				

Rationale / Evidence
Interpretation
Strategy
Accuracy
Completeness
Clarity
Reflection

Seven Congruent Rectangles

posted October 20, 2002 (http://mathforum.org/pows)

These seven congruent rectangles form a larger rectangle. If the area of the larger rectangle is 756 units^2, what is its perimeter?



Be sure to explain your strategy and how you know your answer is correct.

Problem of the Week Scoring Rubric for *Congruent Rectangles*

For each category, choose the level that ***best describes*** the student's work

	Novice	Apprentice	Practitioner	Expert
Problem Solving				
Interpretation	Understands one or none of the ideas in the Practitioner column	understands only two or three of the ideas listed under Practitioner (for example, they ignore the existence of the small rectangles, or think that the area is 756^2 units)	understands what "756 units^2" represents demonstrates understanding of "congruent" and area and perimeter of a rectangle understands that the small rectangles must be considered attempts to find the perimeter of the large rectangle	as there is no Extra question for this problem, students will not be able to achieve this category
Strategy	has no ideas that will lead them toward a successful solution	relies on luck to get the right answer, or otherwise chooses a weak strategy	uses a strategy that relies on skill, not luck "guess and check" is fine *as long as it did not rely on luck!* — enough information must be included to show that the choices were well-reasoned	uses two different strategies (if they do this *and* compare the two methods, that's consider "reflection")
Accuracy	has made many errors	has made several mistakes or misstatements uses incorrect units	makes few mistakes of consequence and uses largely correct vocabulary uses appropriate units whenever units are used, such as with the final answer	[generally not possible – can't be more accurate than Practitioner]
Communication				
Completeness	has written almost nothing that tells you how they found their answer	explanation without work or work without explanation leaves out enough details that another (somewhat lost) student couldn't follow the explanation doesn't define any variables used	defines any variables used explains all or almost all of the steps taken states any formulas used and explains where most of those formulas come from explains how the dimensions of the small rectangle were found, if that's part of the solution path	adds in useful extensions and further explanation of some of the ideas involved (the additions are helpful, not just "I'll say more to get more credit")
Clarity	explanation is very difficult to read and follow	another student might have trouble following the explanation long and written in one paragraph lots of spelling errors/typos	the steps that *are* explained are explained in such a way that another student would understand (needn't be complete to be clear) makes an effort to check formatting, spelling, and typing (a few errors are okay)	formats things exceptionally clearly answer is very readable and appealing
Reflection	*The items in the columns to the right are considered reflective, and could be in the solution or their comment:*	checks their answer (not the same as viewing our "answer check") reflects on the reasonableness of their answer	connects the problem to prior knowledge or experience explains where they're stuck summarizes the process they used	comments on and explains the ease or difficulty of the problem **revising their answer and improving anything counts as reflection**
	does nothing reflective	does one reflective thing	does two reflective things	does three or more reflective things or great job with two

On Wednesday, next week, we will start our Math Forum Training

One of the things we will each do is to SUBMIT A SOLUTION to the *Seven Congruent Rectangles* problem.

It is more interesting if everyone's solution is not perfect.

DIRECTIONS—1

Create a solution to the *Seven Congruent Rectangles* problem based on the following:

Group 1: Prepare a **perfect** solution to submit on-line
Last name starts with A – E

Group 2: Prepare a solution **with several small errors** to submit on-line
Last name starts with F – M

Group 3: Prepare a solution **with major problems** to submit on-line
Last name starts with N – Z

DIRECTIONS—2

TYPE UP your solution and email it to your instructor; either as a Word attachment or in the body of an email

Send the email *before class starts* on Wednesday, you will also need to access the file; save it in your Sent mail or Cc: yourself when you send the email.

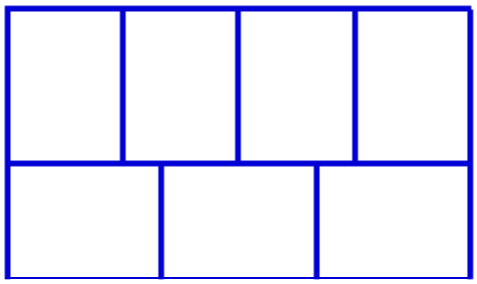
Please use the subject line: Math 396 -- 7 rectangle write up.

15 points (mentor training)

On Wednesday, you will be able to cut and paste your solution during the online training.

7 Congruent Rectangles Student Responses

These seven congruent rectangles form a larger rectangle. If the area of the larger rectangle is 756 units^2, what is its perimeter?



Student 1:

Answer: 144.

Explanation: I divided 756 by 7 and got 108. Then xy=108 and x+y=4x so y=3x so x=9 and y=27. Then my answer is 9*10+2*27=144.

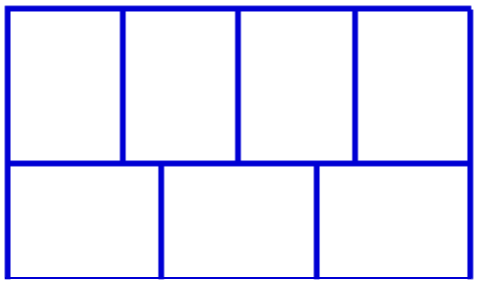
	novice	apprentice	practitioner	expert
Problem Solving				
interpretation				
strategy				
accuracy				
Communication				
completeness				
clarity				
reflection				

SCORE AND WRITE A MENTOR REPLY FOR STUDENT 1:

7 Congruent Rectangles Student Responses

***7 Congruent Rectangles* Student Responses**

These seven congruent rectangles form a larger rectangle. If the area of the larger rectangle is 756 units^2, what is its perimeter?



Student 2:
Answer: I think it can't be solved.

Explanation:
To answer the problem I thought It would be more helpful for me if I would break it down into a easier related problem. I wanted to find the areas of the smaller rectangle and then I could try to find the bigger one. The area of the large rectangle is 756 units squared. I then divided 756 by 7 because there are seven rectangles. I know that congruent means having the same measurment. Therefore, all of the rectangles are the same. 756/7=108

I now know that the area of each of the smaller rectangles is 108 units squared. I already know that area equals length times width. I now need to know what the length and the width of the smaller rectangles are.

The area is 108 so the answer of LxW will have to equal 108.
I first thought I would find the multiples of 108 by the use of the strategy "Guess and check. 108/4=27 L would equal 27 and W would equal 4.To check this I would have to find the L and W for the larger rectangle. The L of the larger rectangle is 58 and W is 15.

L times W of large rectangle should equal 756. 58x15=870. I know now that this is not the right solution.

I will make another guess.108/6=18. L of small rectangle = 18 and W = 6.I will check to see if this is true: L of big is 24 and W of big is 24, 24x24=576.

This is not the right solution either. I will again make another guess:108/12=9.

L of small rectangle = 12 and W = 9

I will check to see if this works:L=21, W: 32 21*32= 672 -No!

I can't seem to find the solution. I'm not sure what I'm doing wrong.

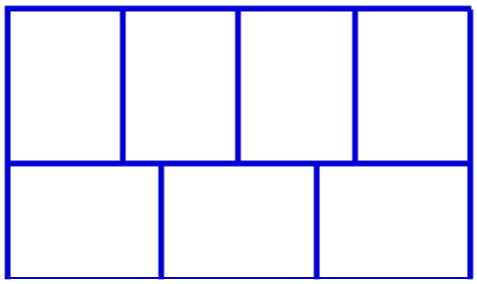
7 Congruent Rectangles Student Responses

	novice	apprentice	practitioner	expert
Problem Solving				
interpretation				
strategy				
accuracy				
Communication				
completeness				
clarity				
reflection				

SCORE AND WRITE A MENTOR REPLY FOR STUDENT 2

***7 Congruent Rectangles* Student Responses**

These seven congruent rectangles form a larger rectangle. If the area of the larger rectangle is 756 units^2, what is its perimeter?



Student 3:

Answer: 36 x 21.

Explanation:
576/7=108. Then 9*12=108 so the small is 9x12 and the big is 36 x 21.

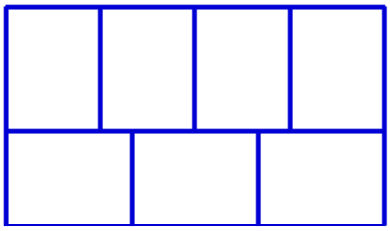
	novice	apprentice	practitioner	expert
Problem Solving				
interpretation				
strategy				
accuracy				
Communication				
completeness				
clarity				
reflection				

SCORE AND WRITE A MENTOR REPLY FOR STUDENT 3

7 Congruent Rectangles Student Responses

***7 Congruent Rectangles* Student Responses**

These seven congruent rectangles form a larger rectangle. If the area of the larger rectangle is 756 units^2, what is its perimeter?



Answer: 150

Explanation:
I know that area = length x width. So I went 12 * 63 = 756. So each side is 12,63,12,63 and the perimeter is when I add them all together: 150.

	novice	apprentice	practitioner	expert
Problem Solving				
interpretation		X		
strategy		X		
accuracy		X		
Communication				
completeness		X		
clarity		X		
reflection		X		

REPLY:
Dear Student,

Thank you for submitting a solution to the Math Forum Seven Congruent Rectangles Problem. I see you understand that area is length times width, but the dimensions that you found were not quite right. I have some hints to help you find the right dimensions.

Notice that the problem states that the seven rectangles inside must be congruent. This means they are each the same size. If the area of the large rectangle is 756 units^2, then what is the area of each of the seven smaller rectangles?

If the large rectangle has sides 12 units x 63 units as you found, then what would be the length of the sides of the smaller rectangles? What would the area of the small rectangles of this dimension be? Does this answer match the area calculation you found above? If not, go back and try some different dimensions for the length and width of the larger rectangle and check and see if those satisfy the conditions of your problem.

ACT AS APPROVER:

Write a reply to the mentor – be sure to include comments on the scores and on their mentor reply

Dear Mentor,

Math Forum @ Drexel

Drexel UNIVERSITY

ONLINE MENTORING GUIDE

Mentoring Guidelines

What to write in a mentor reply:

1. **Read the student's solution with an open mind.** The student may have used a different approach than yours. Try to understand the student's thinking.
2. **Give appropriate and realistic help.** Address one or two of the most critical issues. Start from what the student understands and try to lead her/him from that onto a productive path, rather than pointing in a totally different direction.
3. **Make your scores reflect your suggestions and vice versa.** Your comments should help the student understand what needs to be done to improve scores in each category, but avoid using the scores as the motivation.
4. **Ask probing questions.** Try to find out what the students are thinking, not test them on what you think they should know. For example, it would be better to ask, "How did you get the measure of the interior angles?" than to ask, "Did you use (n-2)180?"
5. **Make specific requests for revisions.** If you can find an error, be specific and point it out without giving the answer. For example, "You're on the right track, but there is a math error in the third line. See if you can correct that and then fix the rest of your answer." or "I get a different product when I multiply 7 times 8."
6. **Help students explain their solutions more clearly and fully.** When students have the correct answer, but have not fully explained how they solved the problem, ask specific questions and offer specific suggestions to move them to higher scores on the rubric. The goal is to move every student forward in problem solving and/or communication.
7. **Stretch students who are practitioners in communication and problem solving.** The problems as stated won't necessarily challenge everyone. See if you can stretch them with challenge questions or invite them to expand their answer in an appropriate way. Let them know what you're asking, and that it's not required.

Techniques

1. **Be personal.** Greet the student by name at the beginning of your reply. Thank them for sending their solution to The Math Forum.
2. **It is not necessary to quote the solution.** Students will be able to see their own work. Quoting is useful if you want to intersperse your comments to address specific parts of their work. This is sometimes helpful if the solution is long, but it is normally not recommended for younger students unless you explain at the top where they'll find your comments.
3. **Be concise.** Say what you have to say, and no more. Writing too much can be confusing. Everything you write should apply to the specifics of the problem.
4. **Avoid asking students to explain misconceptions or flawed strategies** (as you might do in the classroom to gather diagnostic information). In the mentoring context we want to redirect such students toward a more successful approach.
5. **Be mathematically accurate.** Use proper mathematical vocabulary, check your arithmetic, and check their use of both!
6. **Start with something positive, and be specific with your praise.** Criticism is more palatable when it is preceded with something positive. Open your reply by saying something specific about what the student did well. For example, "You explained what you did clearly" or "Good start." If there's nothing obvious, simply thank them for submitting to the Problem of the Week.
7. **Don't praise poor work.** If there are changes that need to be made, don't tell them they did an excellent job.
8. **Write your reply with an aim toward securing a revision.** Handle student errors respectfully. For example, you can say, "Our answers don't agree," or "You should check that part of your answer." We have found that students learn from revising.
9. **Decide how many things to point out.** If there are two main things to do, get them started on both, or at least give them feedback about both. It's better to focus on one or two main points and then help them with more if they revise.
10. **Encourage doing the Extra problem,** especially if the student understands the main problem and is at least very close to the correct answer. (Not all problems have an Extra.)
11. **End with something positive or encouraging and an invitation to revise.** "I'm looking forward to seeing your revision," is useful. Develop your own "voice."
12. **PROOFREAD!** You should model the good communication skills you'd like students to use. Typos and grammatical errors make your writing difficult to read. If you have problems spelling, please use a spell checker and cut-and-paste your reply. Watch out for usage errors, such as right/write, then/than, its/it's.

Mentor Instructions

Congruent Rectangles

http://mathforum.org/pows/

In ***Congruent Rectangles*** students apply the concepts of area of a rectangle, perimeter of a rectangle, the fact that opposite sides of a rectangle are equal, and congruence. They might also use some algebraic techniques like setting up and solving a system of simultaneous equations, but the problem can be solved without the use of any formal variables.

Answer Check

After students submit their solution, we encourage them to check their answer by looking at the answer that we provide. Below is what they will see. You might use the accompanying questions as prompts to help students who are struggling, or to encourage those who have found a correct solution to improve their explanation.

The perimeter of the rectangle is 114 units.

If your answer **does not** match our answer,

- don't forget to use the small rectangles in your solution – they do matter!
- can you figure out the area of each of the small rectangles?
- notice that the horizontal line across the large rectangle is made of four widths of the small rectangle *or* three lengths of the small rectangle – four widths must equal three lengths.
- you can assign variables to dimensions of the small rectangle and use algebra to solve the problem, but algebra isn't required.

If any of those ideas help you, you might *revise* your answer, and then leave a *comment* that tells us what you did. If you're still stuck, leave a *comment* that tells us where you think you need help.

If your answer **does** match ours,

- have you said *why* you can do each step you did?
- did you make any mistakes along the way? If so, how did you find them?
- what hints would you give another student?

Revise your work if you have any ideas to add. Otherwise leave us a *comment* that tells us how you think you did—you might answer one or more of the questions above.

If you see that a student hasn't checked our answer, you might say something like, "After you submit it's a good idea to click the link to see our answer. There are hints there to help you, whether our answers agree or not." Likewise, when you see that a student has looked at our answer, but didn't appear to have benefited from the suggestions, you might say something like, "When you checked our answer, did you read the hints there? They can help you, whether our answers agree or not."

Scoring Rubric

The problem-specific scoring rubric we use to assess student solutions is a separate stand-alone document available from a link on the problem page or from Lesson 5 of the Online Mentoring Guide. Be sure to download it and refer to it when scoring.

Some important ideas about separating the categories:

- A student can choose a good strategy without interpreting the problem correctly. If the student interprets the problem incorrectly, yet picks a good strategy based on that interpretation, she/he is a Practitioner in Strategy (despite the fact that the strategy will never result a correct solution).
- Similarly, a student can be accurate without getting the problem right. Evaluate Accuracy based on whether the chosen strategy was executed correctly – even if the strategy is faulty.

It's important to separate these categories; they are not entirely dependent on one other.

Mentoring Suggestions

To score Practitioner in Strategy using a guess-and-test method, the solver needs to test numbers systematically – achieving success through skill and understanding, not pure luck. Random guessing, or guesses that are not reasonable in the context of the problem, indicate an Apprentice.

It is sometimes the case that a student's strategy has to be assumed to be unsound because they didn't say enough about what they did. In this problem, if they find the area of the small rectangles to be 108 square units and then say that the dimensions are 9 and 12 without saying anything about why, they simply have to be an Apprentice in Strategy. They may even say that they used trial and error to determine that the dimensions were 9 and 12, but without more information, we can't know that it's a well-reasoned choice. You should ask them to say more about either how they determined that 9 and 12 were the best choice, or, if they say they used guess and check, to tell you more about what they guessed and how they checked it. You can also ask them whether the dimensions could also be 6 and 18 – sometimes it's easier to explain why something doesn't work than to explain why something does.

Mentors may want to suggest or model effective representation and notation that can help improve the Clarity of an explanation:

- Lists, tables and number sentences (equations) often help the reader follow more easily than does pure text. A student might benefit from an example of how to type equations.
- Starting in about fifth grade it's preferable to use the asterisk (*) as the multiplication symbol, so as to avoid confusion with x used as a variable. Asterisks appear on many calculators and are used in spreadsheets and in many popular published curricula.
- The slash (/) is an effective way to indicate division.
- Be on the lookout for students who string equations together, e.g., 756/7 = 108/9 = 12. This creates an incorrect statement. Mentors should model more conventional ways of representing multiple calculations.

Solutions may vary in the degree of formality with which they solve the problem. When judging the Clarity of students' writing, consider their age/grade, but also consider that some students don't have a lot of practice writing mathematics. When scoring use your best judgment to determine whether the math language is "level-appropriate," but don't hesitate to provide suggestions or models that the students can use to improve their communication. And don't hesitate to be honest when saying something like, "I'm having a hard time understanding what you did when you [did something specific]." Helping you understand their thinking is good incentive for them to include more information or try to make things clearer.

Be sure to look down through the thread to see if the student has changed an answer after looking at ours. If they have, in order to score Practitioner in Strategy and Completeness, the solver's strategy must be compatible with the corrected answer and their explanation must include enough detail for you to be confident they understand the answer and have not simply changed it to match ours. A specific reference to how or why they changed an answer can be counted as reflection.

Be on the lookout for (and encourage!) exceptional insights and observations regarding number patterns and relationships, which might lead to Expert status in Interpretation and/or Completeness. Solving or checking the answer by another means can boost a score in Strategy and/or Reflection.

Our Solutions

The key concepts in this problem are area and perimeter of a rectangle, properties of rectangles (specifically that the opposite sides are equal), congruence, and the idea that the small rectangles play an important part.

There are a number of ways that students might solve the problem. Here are a few examples.

Method 1: Guess and Check, Area of Small Rectangles

Since all the small rectangles are congruent, they all have the same area. 756/7 = 108 units^2. This means that the dimensions of the small rectangles need to multiply to 108. I made a chart of possible factor pairs (I'm assuming the dimensions are integers, and will see if it works).

1, 108
2, 54
3, 36
4, 27
6, 18
9, 12

I took the first pair, 1 and 108, and wrote 108 next to each long side of the small rectangle in the picture across the bottom, and 1 next to each short side of the small rectangle across the top. Since it's a rectangle, those two sides have to be equal. It was obvious that the four 1s across the top weren't nearly long enough to match the three 108s across the bottom.

I decided to try 6 and 18. This made the top 24 and the bottom 48. Not equal.

I tried 9 and 12. That makes the top 36 and the bottom 36. They're equal! So the dimensions of the small rectangles are 9 and 12. If I label the rest of the large rectangle and find the distance all the way around, I find that the perimeter is 114 units.

(*Note:* Students might more directly see that the dimensions of the small rectangle have to be in a ratio of 3:4, because the top and bottom of the large rectangle must be equal. Then they might test the factor pairs of 108 until they find one that has a ratio of 3:4 – it's a more abstract way of doing just what was done in this example.)

Method 2: Guess and Check, Ratio of the Dimensions of the Small Rectangle

I noticed that there are four short sides of the small rectangle across the top of the large rectangle and three long sides of the small rectangle across the bottom. This means that four of the short ones must equal three of the long ones, since the opposite sides of a rectangle are equal.

I also noticed that the area of the small rectangle is 108 units^2, since there are seven small rectangles and they are all congruent (meaning they are equal). 756 divided by 7 is 108.

I decided to pick pairs of numbers with a ratio of 3:4 and see if they multiply to 108, since the short and long sides of the small rectangles must multiply to 108, since length times width gives the area of a rectangle. I made a table of the numbers and their products:

3:4	12
6:8	48
9:12	108

The last choice worked, so 9 and 12 must be the dimensions of the small rectangles. Adding these up all the way around the large rectangle gives a perimeter of 114 units.

Method 3: System of Two Equations with Two Unknowns

Assign x as the width of the small rectangle and y as the length. The top and bottom of the big rectangle are equal, and the top is 4x while the bottom is 3y. That gives us

$$4x = 3y$$

Also, we know that the area is 756 u^2, and the dimensions of the large rectangle are 4x and x + y (since the edge is a length and a width of the small rectangle). That gives us

$$4x(x + y) = 756$$

In the first equation, we solve for y to get

$$\frac{4x}{3} = y$$

Substituting this into the second equation, we solve for x.

$$4x(x + \frac{4x}{3}) = 756$$

$$4x(\frac{3x + 4x}{3}) = 756$$

$$4x(\frac{7x}{3}) = 756$$

$$28x^2 = 2268$$

$$x^2 = 81$$

$$x = 9, -9$$

We know that x can't be -9 because we are talking about lengths of segments, and lengths cannot be negative. So x = 9. We go back to the third equation and substitute 9 in for x:

$$\frac{4 * 9}{3} = y$$

$$12 = y$$

The dimensions of the small rectangle are 9 and 12. Now we find the perimeter of the large rectangle, the formula for which is 2(L + W). Remember that the top of the large rectangle is 4x and length of the sides is x + y.

$$\begin{aligned} \textit{perimeter} &= 2(4x + (x + y)) \\ &= 2(36 + 21) \\ &= 2(57) \\ \textit{perimeter} &= 114 \end{aligned}$$

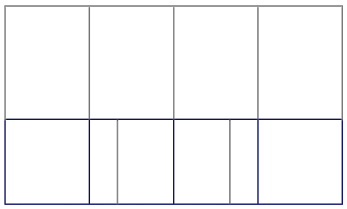
So the perimeter of the large rectangle is 114 units.

(*Note:* Students might do the same thing using the area of the small rectangle instead of the large one, which leads to slightly simpler algebra.)

Method 4: Direct Calculations of Dimensions of Small Rectangle

I noticed that each small rectangle must have an area of 108 units^2, since there are seven congruent rectangles, and 756/7 = 108. This means that the three rectangles across the bottom of the large rectangle have a total area of 324 units^2.

I extended the lines forming the three small rectangles at the top to split the bottom portion of the large rectangle into four squares:



The gray lines are the original dividers. These four squares have a total area of 324 units^2, so each has an area of 81 units^2. Since they're squares, that means they must have an edgelength of 9 units. So the short side of the small rectangles is 9 units long. Since the area of small rectangle is 108 units^2, the other side must be 12 units long. This results in a perimeter of 114 units for the large rectangle.

Please let me know if you have any questions about this, or if you'd like to discuss any part of it.

~ Annie
annie@mathforum.org

Problem:

Student:						School/Teacher:	
Date:	Age/Grade:	N	A	P	E	Commend	**Improve**
Problem Solving:	Interpretation						
	Strategy						
	Accuracy						
Communication:	Completeness						
	Clarity						
	Reflection						
						Extra:	
Student:						School/Teacher:	
Date:	Age/Grade:	N	A	P	E	Commend	**Improve**
Problem Solving:	Interpretation						
	Strategy						
	Accuracy						
Communication:	Completeness						
	Clarity						
	Reflection						
						Extra:	
Student:						School/Teacher:	
Date:	Age/Grade:	N	A	P	E	Commend	**Improve**
Problem Solving:	Interpretation						
	Strategy						
	Accuracy						
Communication:	Completeness						
	Clarity						
	Reflection						
						Extra:	
Student:						School/Teacher:	
Date:	Age/Grade:	N	A	P	E	Commend	**Improve**
Problem Solving:	Interpretation						
	Strategy						
	Accuracy						
Communication:	Completeness						
	Clarity						
	Reflection						
						Extra:	
Student:						School/Teacher:	
Date:	Age/Grade:	N	A	P	E	Commend	**Improve**
Problem Solving:	Interpretation						
	Strategy						
	Accuracy						
Communication:	Completeness						
	Clarity						
	Reflection						
						Extra:	

Problem:

Student:						School/Teacher:	
Date:	Age/Grade:	N	A	P	E	Commend	**Improve**
Problem Solving:	Interpretation						
	Strategy						
	Accuracy						
Communication:	Completeness						
	Clarity						
	Reflection						
						Extra:	
Student:						School/Teacher:	
Date:	Age/Grade:	N	A	P	E	Commend	**Improve**
Problem Solving:	Interpretation						
	Strategy						
	Accuracy						
Communication:	Completeness						
	Clarity						
	Reflection						
						Extra:	
Student:						School/Teacher:	
Date:	Age/Grade:	N	A	P	E	Commend	**Improve**
Problem Solving:	Interpretation						
	Strategy						
	Accuracy						
Communication:	Completeness						
	Clarity						
	Reflection						
						Extra:	
Student:						School/Teacher:	
Date:	Age/Grade:	N	A	P	E	Commend	**Improve**
Problem Solving:	Interpretation						
	Strategy						
	Accuracy						
Communication:	Completeness						
	Clarity						
	Reflection						
						Extra:	
Student:						School/Teacher:	
Date:	Age/Grade:	N	A	P	E	Commend	**Improve**
Problem Solving:	Interpretation						
	Strategy						
	Accuracy						
Communication:	Completeness						
	Clarity						
	Reflection						
						Extra:	

MATH 396
ONLINE MENTORING ASSESSMENT

- Your online mentoring grade is based on the quantity and quality of your replies.
- You are expected to complete ≈ 6 threads (the exact number will be determined as we go).
- New threads are only assigned to people who keep up. If you are late responding, or otherwise fail to do your work, you will not get new assignments and your grade will suffer.
- Every reply should follow the guidelines set out in the Math Forum mentoring guide.
- The more you are asked to revise your reply, the lower your grade will be for that reply.

Online Mentoring Assessment Details (each thread is ~3% of your class grade)

Part I: Score for each reply and for each reply revision per a *Change Request* **Average score calculated per thread (out of 30 points per thread)**	
Scoring—each item: Yes (1), No (0)	
Score matches the grading rubric for the problem:	**6 points**
i) Interpretation iii) Accuracy v) Clarity ii) Strategy iv) Completeness vi) Reflection	
Written Reply—each item: Yes/all (1), partial (½), missing (0)	
Content—Your reply should:	**8 points**
i) Demonstrate you understand the student's work (if student's work makes sense). ii) Give praise corresponding to the high scores you have given the student. iii) Focus on the most important misconceptions in their work or encourage them to improve upon some other aspect of the solution if work is correct (choose 2 – 3 things to focus on). iv) Help student along if they use an effective strategy give specific suggestion if it will not v) Give specific hints to help the student move forward without telling them the answer or giving them a formula. vi) Give specific hints corresponding to areas (from scoring) where student needs to improve. vii) Be concise – don't add in lots of other questions or details that are not needed. viii) Check and address the student's *Thread History*. Did they leave a note that you can respond to? Did they check their answer? If not, have you encouraged them to do so?	
Mathematics —Your reply should:	**5 points**
i) Indicate you understand the mathematics the child is using. ii) Model correct mathematical language iii) Model correct mathematical units and formulas iv) Model correct number sentences v) Be mathematically correct in every way	
Tone—Your reply should:	**5 points**
i) Start with an appropriate greeting, praising the student for at least one aspect of their reply. ii) Be polite, friendly and use upbeat, kid friendly and age appropriate language and ideas. iii) Be focused on the student and use the word "I" sparingly. iv) End with an encouragement to revise their solution, work on the Extra or try future Problems of the Week (whichever is appropriate). v) Include your name at the bottom with a little information about you or a fun fact about Oregon (you might save your "signatures" in a file, then cut and paste it into each reply)	

Spelling / Grammar / Punctuation / Formatting Each item—No issues (2), one error/issue (1), two or more errors/issues (0)	
SGPF—Your reply should:	**6 points**

i) Use correct spelling and capitalization.
ii) Use correct grammar and punctuation.
iii) Be in an easy to read format with:
- ideas separated into multiple paragraphs or bullets
- blank lines between paragraphs
- no excess spacing or other visual problems

EXCESSIVE REVISIONS and INAPPROPRIATE COMMENTS DEDUCTION

You are allowed one "free" revision of your reply, as I don't expect anyone to get it right on the first try. After your first revision request, 2 points will be deducted for each successive revision requested by me.

You will also lose a point for every comment that is not helpful, makes no sense, or is inaccurate.

Part II: Timeliness (per thread)

Deductions per response

- You are expected to check to see if you have work in your "box" approximately every day.
- If works sits in your box undone for more than a day you will start to lose points
- Your average score from Part I will be reduced 2 points for every 12 hours over one day you are late with any response. If T is the number of hours since a *New Thread* is assigned to you, a *Student Revision* is posted for you, or a *Change Request* is entered, then:

T hours	$0 < T \leq 24$	$24 < T \leq 36$	$36 < T \leq 48$	$48 < T \leq 60$	$60 < T \leq 72$	$72 < T$
Deduct	0	-2	-4	-6	-8	-10

Unassigned threads

- Since new threads are only assigned to people who keep up with their work, if you are late with your responses, you run the risk of not being assigned all of your threads.
- For unassigned threads due to lateness responding to previous threads, you will earn a score of 0 for each unassigned thread (as compared to the thread totals of the rest of the class).

Unanswered threads

- A thread that is left in your box for over 3 days (72 hours) will be completed by me.
- You will be given a timeless deduction of -10 for this thread.
- You will be given two, Part I, reply scores of 0 for this thread which will factor into the thread average score.
- Please note, there will be an actual child waiting for your response, it is unprofessional and reflects poorly on both you and WOU to leave threads unanswered.
- If for some reason you know you will not be completing a thread, please notify your professor as soon as you can.

Mathematics 396 Peer Grading Sheet

Attach this completed sheet to the front of your homework.

Name of Student: _____ **Due Date:** _____

Name of Peer Reviewer: _____ **Homework #** **1 2 3 4 5 6 7**

I. Special Problem Assessment **Chapter / Set** _____ **Problem #** _____

Complete

How complete is the solution?

0 Problem is missing 1 Partially solved 2 Solution is complete

Communication

How well is the solution communicated?

0 Poorly 1 Adequately 2 Very well

Accuracy

How accurate is the solution?

0 Incorrect 1 Partially correct 2 Correct

Additional Comments

Are there any other comments about this work you would like to share?

Score Section I (out of 6) _____

II. Completeness Check Off

For each of the remaining SEVEN assigned problems

- 2 points for a completed problem
- 1 point for an attempted problem / no work shown problem
- 0 points for a missing problem / problem with just the problem written down

Question # (fill in)							
Score (0, 1 or 2)							

Score Section II (out of 14) _____

Total Score (out of 20) _____

Mathematics 396 Peer Grading Sheet

Attach this completed sheet to the front of your homework.

Name of Student: _____ **Due Date:** _____

Name of Peer Reviewer: _____ **Homework #** **1 2 3 4 5 6 7**

I. Special Problem Assessment **Chapter / Set** _____ **Problem #** _____

Complete

How complete is the solution?

0 Problem is missing 1 Partially solved 2 Solution is complete

Communication

How well is the solution communicated?

0 Poorly 1 Adequately 2 Very well

Accuracy

How accurate is the solution?

0 Incorrect 1 Partially correct 2 Correct

Additional Comments

Are there any other comments about this work you would like to share?

Score Section I (out of 6) _____

II. Completeness Check Off

For each of the remaining SEVEN assigned problems

- 2 points for a completed problem
- 1 point for an attempted problem / no work shown problem
- 0 points for a missing problem / problem with just the problem written down

Question # (fill in)							
Score (0, 1 or 2)							

Score Section II (out of 14) _____

Total Score (out of 20) _____

Mathematics 396 Peer Grading Sheet

Attach this completed sheet to the front of your homework.

Name of Student: _____ **Due Date:** _____

Name of Peer Reviewer: _____ **Homework #** **1 2 3 4 5 6 7**

I. Special Problem Assessment **Chapter / Set** _____ **Problem #** _____

Complete

How complete is the solution?

0 Problem is missing 1 Partially solved 2 Solution is complete

Communication

How well is the solution communicated?

0 Poorly 1 Adequately 2 Very well

Accuracy

How accurate is the solution?

0 Incorrect 1 Partially correct 2 Correct

Additional Comments

Are there any other comments about this work you would like to share?

Score Section I (out of 6) _____

II. Completeness Check Off

For each of the remaining SEVEN assigned problems

- 2 points for a completed problem
- 1 point for an attempted problem / no work shown problem
- 0 points for a missing problem / problem with just the problem written down

Question # (fill in)							
Score (0, 1 or 2)							

Score Section II (out of 14) _____

Total Score (out of 20) _____

Mathematics 396 Peer Grading Sheet

Attach this completed sheet to the front of your homework.

Name of Student: _____ **Due Date:** _____

Name of Peer Reviewer: _____ **Homework #** **1 2 3 4 5 6 7**

I. Special Problem Assessment **Chapter / Set** _____ **Problem #** _____

Complete

How complete is the solution?

0 Problem is missing 1 Partially solved 2 Solution is complete

Communication

How well is the solution communicated?

0 Poorly 1 Adequately 2 Very well

Accuracy

How accurate is the solution?

0 Incorrect 1 Partially correct 2 Correct

Additional Comments

Are there any other comments about this work you would like to share?

Score Section I (out of 6) _____

II. Completeness Check Off

For each of the remaining SEVEN assigned problems

- 2 points for a completed problem
- 1 point for an attempted problem / no work shown problem
- 0 points for a missing problem / problem with just the problem written down

Question # (fill in)							
Score (0, 1 or 2)							

Score Section II (out of 14) _____

Total Score (out of 20) _____

Mathematics 396 Peer Grading Sheet

Attach this completed sheet to the front of your homework.

Name of Student: _____ **Due Date:** _____

Name of Peer Reviewer: _____ **Homework #** **1 2 3 4 5 6 7**

I. Special Problem Assessment **Chapter / Set** _____ **Problem #** _____

Complete

How complete is the solution?

0 Problem is missing 1 Partially solved 2 Solution is complete

Communication

How well is the solution communicated?

0 Poorly 1 Adequately 2 Very well

Accuracy

How accurate is the solution?

0 Incorrect 1 Partially correct 2 Correct

Additional Comments

Are there any other comments about this work you would like to share?

Score Section I (out of 6) _____

II. Completeness Check Off

For each of the remaining SEVEN assigned problems

- 2 points for a completed problem
- 1 point for an attempted problem / no work shown problem
- 0 points for a missing problem / problem with just the problem written down

Question # (fill in)							
Score (0, 1 or 2)							

Score Section II (out of 14) _____

Total Score (out of 20) _____

Mathematics 396 Peer Grading Sheet

Attach this completed sheet to the front of your homework.

Name of Student: _____ **Due Date:** _____

Name of Peer Reviewer: _____ **Homework #** **1 2 3 4 5 6 7**

I. Special Problem Assessment **Chapter / Set** _____ **Problem #** _____

Complete

How complete is the solution?

0 Problem is missing 1 Partially solved 2 Solution is complete

Communication

How well is the solution communicated?

0 Poorly 1 Adequately 2 Very well

Accuracy

How accurate is the solution?

0 Incorrect 1 Partially correct 2 Correct

Additional Comments

Are there any other comments about this work you would like to share?

Score Section I (out of 6) _____

II. Completeness Check Off

For each of the remaining SEVEN assigned problems

- 2 points for a completed problem
- 1 point for an attempted problem / no work shown problem
- 0 points for a missing problem / problem with just the problem written down

Question # (fill in)							
Score (0, 1 or 2)							

Score Section II (out of 14) _____

Total Score (out of 20) _____

Mathematics 396 Peer Grading Sheet

Attach this completed sheet to the front of your homework.

Name of Student: _____ **Due Date:** _____

Name of Peer Reviewer: _____ **Homework #** **1 2 3 4 5 6 7**

I. Special Problem Assessment **Chapter / Set** _____ **Problem #** _____

Complete

How complete is the solution?

0 Problem is missing 1 Partially solved 2 Solution is complete

Communication

How well is the solution communicated?

0 Poorly 1 Adequately 2 Very well

Accuracy

How accurate is the solution?

0 Incorrect 1 Partially correct 2 Correct

Additional Comments

Are there any other comments about this work you would like to share?

Score Section I (out of 6) _____

II. Completeness Check Off

For each of the remaining SEVEN assigned problems

- 2 points for a completed problem
- 1 point for an attempted problem / no work shown problem
- 0 points for a missing problem / problem with just the problem written down

Question # (fill in)							
Score (0, 1 or 2)							

Score Section II (out of 14) _____

Total Score (out of 20) _____

Portfolio Assignments

There will be **three** portfolio assignments during the term. For each portfolio problem, you will be given a problem to modify for one of the following groups:
Grades K – 1, 2 – 3, 4 – 5, Middle School, or English Language Learners.
You are allowed to select **each group only once** during the quarter.

Please use the ideas discussed in class about modifying a problem to make it more appropriate for your classroom

These problems should have at least three different mathematical steps to do them.

Each problem must be solved using one of the strategies assigned to it.

Try to make the problems interesting and relevant to children's lives, and thus useful for you upon entry into the teaching profession. Have fun and use your imagination!

Portfolio problem assessment (on a scale of 40 points).

Grade Level: Two (2) points are given for specifying which group your problem is for (Grades K – 1, 2 – 3, 4 – 5, Middle School, or English Language Learners).

Prescribed Strategy: Two (2) points are given for specifying which problem solving strategy your problem is illustrating.

Problem: Thirteen (13) points are given for this section of the assignment which contains the wording of the actual problem:

- Three (3) points are given for a problem that can be best solved using the prescribed strategy in the assignment.
- Three (3) points are given for writing a problem that clearly is a modification of the original problem, i.e., is related to the old problem somehow, but not certainly different as well..
- Three (3) points are given for an interesting story.
- Four (4) points are given for clarity and good use of language.

Rubric: Four (4) points for including a rubric created specifically for the problem.

Solution: Eight (8) points for a complete and correct solution that uses the prescribed strategy and would receive a perfect score according to your rubric.

Verification: Three (3) points are given for a verification of your solution using a different method than the prescribed strategy.

Include a section giving *Comments for Teachers.*

- Two (2) points are given for giving two ideas for extending or generalizing the problem for a different grade level.
- Two (2) points are given for describing the skills this problem will help your students master.
- Two (2) points are given for explaining how the mathematics is appropriate for the group your problem is written.
- Two (2) points are given for explaining of how you modified the problem.

IMPORTANT: Please **type** and **staple** one of the scoring to each of your portfolio assignments:

Portfolio 1 DUE 4/13 Modify the following problem, where the solution must be done by either drawing a picture or using algebra.

A worm climbs three feet up a wall each day but slips two feet at night. If the height of the wall is 12 feet, how long will it take for the worm to get to the top of the wall?

Notes:

- Please be as creative as possible on this one.
- Don't use worms
- For this problem only, you cannot do it for Grades 2 – 3.

Portfolio 2 DUE 4/27 Modify the following problem, where the solution must be done by looking for a pattern.

Kazuko has a beach ball. It is colored with six vertical sections, in order: white, orange, yellow, blue, red, and green. She spins the ball, and she notices that the colors whir by very fast. If the first color to go by is white and the ball spins around so that 500 colors (including that initial white) go by, what is the 500th color?

Notes:

- Don't use beach balls on this one.
- For this problem only, you cannot do it for Grades K – 1. Ideally, there will be some division and/or multiplication involved in the solution as well.

Portfolio 3 DUE 5/11 Modify the following problem, where the solution must be done by working backwards.

Three sailors came upon a pile of coconuts. The first sailor takes one third of the coconuts. The second sailor comes up and takes half of what is left. The third sailor then takes one-fourth of what remains. Left over are exactly six coconuts which they toss to the monkeys. How many coconuts were there in the original pile?

Notes:

- Don't use coconuts on this one.

Name: _____

Portfolio Grading Sheet Spring 2011

1. Indicate Grade Level			2		1		0
2. Indicate Prescribed Strategy			2		1		0
3. Problem							
Is problem best solved by prescribed strategy?	3		2		1		0
Is problem clear modification of original problem?	3		2		1		0
Does problem portray an interesting story?	3		2		1		0
Clarity and good use of language	4	3		2		1	0
4. Rubric: Are the scores and descriptions appropriate for this problem?							
	4	3.5	3	2.5	2	1	0
5. Solution: Is it a complete and correct solution that uses the prescribed strategy and would receive a perfect score according to your rubric?							
	8	7	6	5	4	2	0
6. Verification: Do you include a verification and does it use a different strategy than the prescribed strategy?							
	3		2		1		0
7. Comments for Teachers							
Extending idea	2			1			0
Describe skills	2			1			0
Describe why age appropriate	2			1			0
Explanation of how you modified the problem	2			1			0

Total: [ ] **/40**

Name: _____

Portfolio Grading Sheet Spring 2011

1. Indicate Grade Level			2		1		0
2. Indicate Prescribed Strategy			2		1		0
3. Problem							
Is problem best solved by prescribed strategy?	3		2		1		0
Is problem clear modification of original problem?	3		2		1		0
Does problem portray an interesting story?	3		2		1		0
Clarity and good use of language	4	3		2		1	0
4. Rubric: Are the scores and descriptions appropriate for this problem?							
	4	3.5	3	2.5	2	1	0
5. Solution: Is it a complete and correct solution that uses the prescribed strategy and would receive a perfect score according to your rubric?							
	8	7	6	5	4	2	0
6. Verification: Do you include a verification and does it use a different strategy than the prescribed strategy?							
	3		2		1		0
7. Comments for Teachers							
Extending idea	2			1			0
Describe skills	2			1			0
Describe why age appropriate	2			1			0
Explanation of how you modified the problem	2			1			0

Total: [ ] **/40**

Name: _____

Portfolio Grading Sheet Spring 2011

1. Indicate Grade Level			2		1		0
2. Indicate Prescribed Strategy			2		1		0
3. Problem							
Is problem best solved by prescribed strategy?	3		2		1		0
Is problem clear modification of original problem?	3		2		1		0
Does problem portray an interesting story?	3		2		1		0
Clarity and good use of language	4	3		2		1	0
4. Rubric: Are the scores and descriptions appropriate for this problem?							
	4	3.5	3	2.5	2	1	0
5. Solution: Is it a complete and correct solution that uses the prescribed strategy and would receive a perfect score according to your rubric?							
	8	7	6	5	4	2	0
6. Verification: Do you include a verification and does it use a different strategy than the prescribed strategy?							
	3		2		1		0
7. Comments for Teachers							
Extending idea	2			1			0
Describe skills	2			1			0
Describe why age appropriate	2			1			0
Explanation of how you modified the problem	2			1			0

Total: [ ] **/40**

Problems of the Week (POWs) Assignments

Your solution will be graded according to the Math 396 POW Scoring Rubric, and thus your write up should include all your reasoning, and not just a final answer.
*The **emphasis** of these assignments is on **clear, complete and precise explanations.** Please write your solutions in a way that a "typical" student in the 5-8th grade range will be able to follow it.*

These problems were taken from a variety of sources, including *www.eduplace.com*, *http://www.cmc.uwaterloo.ca*, *Crossing the River with Dogs Instructor Resources* by Johnson, Herr and Kysh.

POW 1 DUE 4/6 Choose one of the following two problems:
Divide the number 3000...007 (there are 99 zeros in this number) by 37. What is the remainder? What are the 1st 9 and the last 9 digits of the quotient?

It takes 852 digits to number the pages of a book consecutively. How many pages are there? How many times is the digit "7" printed?

POW 2 DUE 4/20 Choose one of the following two problems:
A positive integer is called a *perfect power* if it can be written in the form a^b, where a and b are positive integers with $b \geq 2$. For example, 32 and 125 are perfect powers because $32 = 2^5$ and $125 = 5^3$.
The increasing sequence 2, 3, 5, 6, 7, 10, . . . consists of all positive integers which are not perfect powers. What is the 200th number in this sequence?

What is the smallest number, greater than zero, which the first five prime numbers as well as the first six composite numbers evenly divide?

POW 3 DUE 5/4 Choose one of the following two problems
Mr. and Mrs. Brown gave a party for their friends they have not seen for a long time. Three couples came. During the party, some of the people were so happy to see each other again, that they even shook hands. (None of the men shook hands with their own wives.) Later on, Mr. Brown asked everybody how many people he/she shook hands with. He received seven different answers. How many guests did Mrs. Brown shake hands with?

How can you cook an egg for exactly 15 minutes, if all you have is a 7 minute hourglass and an 11 minute hourglass?

POW 4 DUE 5/25 Choose one of the following two problems
Cindy leaves school at the same time every day. If she cycles at 20 km/h, she arrives home at 4:30P.M. If she cycles at 10 km/h, she arrives home at 5:15 P.M. At what speed, in km/h, must she cycle to arrive home at 5:00 P.M.?

Al and Bert must arrive at a town 22.5 km away. They have one bicycle between them and must arrive at the same time. Bert sets out riding at 8 km/h, leaves the bicycle and then walks at 5 km/h. Al walks at 4 km/h, reaches the bicycle and rides at 10 km/h. For how many minutes was the bicycle not in motion?

MATH 396 PROBLEM OF THE WEEK COVER				
Student Name		**Total Score**		/20
Instructor		× 2 =		/40
POW #		**Due Date**		

	1 point	2 points	3 points
Problem Solving			
Interpretation	Does not show much understanding of the problem.	Shows some understanding of the problem. Completes part of the problem.	Answers the main part of the problem correctly.
Strategy	Does not know how to set up the problem. OR No evidence of strategy. OR Strategy didn't work.	Strategy makes sense, but isn't enough to solve the whole problem, OR Strategy is not applied systematically. OR Strategy might verify a correct answer, but fails to explain how they found it.	Picks a sound strategy. Approaches the problem systematically, achieving success through skill, not luck. Guess-and-check approach must involve good reasoning and informed guessing.
Accuracy	Has made many errors or shows no math.	Some work is accurate. May have one or two errors. OR Shows very little arithmetic.	Work on main problem is accurate and contains no arithmetic mistakes.
Communication			
Completeness	Writes very little to explain how the answer was achieved.	Provides explanation, but no calculations OR Shows calculations but without explanation about why they were done. OR The path through the work is only partly shown.	Explains most of steps taken to solve the problem and rationale for them, with enough detail for another student to understand. Shows all key calculations.
Clarity	Explanation is very difficult to follow.	Explanation isn't totally unclear, but another student wouldn't follow it easily. OR Spelling errors / typos make it hard to understand. OR Uses incorrect or has missing units.	Attempts to make explanation readable by a peer. Shows effort to use good formatting, spelling, grammar, typing. Uses correct units where appropriate and uses correct mathematical language.
Reflection	Does nothing reflective (0 points)	Includes one reflective thing (1 point)	Includes two reflective things (2 points)
	• Checks answer with different method. • Explains where he/she got stuck. • Summarizes the process used.	• Reflects on reasonableness of answer • Comments and explains the ease or difficulty of problem. • Describes any errors made and how he/she found and corrected them.	
	1 point	**2 points**	**3 points**
Display			
Presentation	Writing is messy and hard to read, solution is not clearly organized on the page AND diagrams/charts/tables are not neatly drawn and clearly labeled.	Writing is messy and hard to read OR solution is not clearly organized on the page OR diagrams / charts / tables are not neatly drawn and clearly labeled.	Writing is clear and neat, solution is clearly organized on the page and diagrams/charts/tables are neatly drawn and clearly labeled.

MATH 396 PROBLEM OF THE WEEK COVER				
Student Name		**Total Score**		/20
Instructor		× 2 =		/40
POW #		**Due Date**		

	1 point	2 points	3 points
Problem Solving			
Interpretation	Does not show much understanding of the problem.	Shows some understanding of the problem. Completes part of the problem.	Answers the main part of the problem correctly.
Strategy	Does not know how to set up the problem. OR No evidence of strategy. OR Strategy didn't work.	Strategy makes sense, but isn't enough to solve the whole problem, OR Strategy is not applied systematically. OR Strategy might verify a correct answer, but fails to explain how they found it.	Picks a sound strategy. Approaches the problem systematically, achieving success through skill, not luck. Guess-and-check approach must involve good reasoning and informed guessing.
Accuracy	Has made many errors or shows no math.	Some work is accurate. May have one or two errors. OR Shows very little arithmetic.	Work on main problem is accurate and contains no arithmetic mistakes.
Communication			
Completeness	Writes very little to explain how the answer was achieved.	Provides explanation, but no calculations OR Shows calculations but without explanation about why they were done. OR The path through the work is only partly shown.	Explains most of steps taken to solve the problem and rationale for them, with enough detail for another student to understand. Shows all key calculations.
Clarity	Explanation is very difficult to follow.	Explanation isn't totally unclear, but another student wouldn't follow it easily. OR Spelling errors / typos make it hard to understand. OR Uses incorrect or has missing units.	Attempts to make explanation readable by a peer. Shows effort to use good formatting, spelling, grammar, typing. Uses correct units where appropriate and uses correct mathematical language.
Reflection	Does nothing reflective (0 points)	Includes one reflective thing (1 point)	Includes two reflective things (2 points)
	• Checks answer with different method. • Explains where he/she got stuck. • Summarizes the process used.	• Reflects on reasonableness of answer • Comments and explains the ease or difficulty of problem. • Describes any errors made and how he/she found and corrected them.	
	1 point	**2 points**	**3 points**
Display			
Presentation	Writing is messy and hard to read, solution is not clearly organized on the page AND diagrams/charts/tables are not neatly drawn and clearly labeled.	Writing is messy and hard to read OR solution is not clearly organized on the page OR diagrams / charts / tables are not neatly drawn and clearly labeled.	Writing is clear and neat, solution is clearly organized on the page and diagrams/charts/tables are neatly drawn and clearly labeled.

MATH 396 PROBLEM OF THE WEEK COVER				
Student Name		**Total Score**		/20
Instructor		× 2 =		/40
POW #		**Due Date**		

	1 point	2 points	3 points
Problem Solving			
Interpretation	Does not show much understanding of the problem.	Shows some understanding of the problem. Completes part of the problem.	Answers the main part of the problem correctly.
Strategy	Does not know how to set up the problem. OR No evidence of strategy. OR Strategy didn't work.	Strategy makes sense, but isn't enough to solve the whole problem, OR Strategy is not applied systematically. OR Strategy might verify a correct answer, but fails to explain how they found it.	Picks a sound strategy. Approaches the problem systematically, achieving success through skill, not luck. Guess-and-check approach must involve good reasoning and informed guessing.
Accuracy	Has made many errors or shows no math.	Some work is accurate. May have one or two errors. OR Shows very little arithmetic.	Work on main problem is accurate and contains no arithmetic mistakes.
Communication			
Completeness	Writes very little to explain how the answer was achieved.	Provides explanation, but no calculations OR Shows calculations but without explanation about why they were done. OR The path through the work is only partly shown.	Explains most of steps taken to solve the problem and rationale for them, with enough detail for another student to understand. Shows all key calculations.
Clarity	Explanation is very difficult to follow.	Explanation isn't totally unclear, but another student wouldn't follow it easily. OR Spelling errors / typos make it hard to understand. OR Uses incorrect or has missing units.	Attempts to make explanation readable by a peer. Shows effort to use good formatting, spelling, grammar, typing. Uses correct units where appropriate and uses correct mathematical language.
Reflection	Does nothing reflective (0 points)	Includes one reflective thing (1 point)	Includes two reflective things (2 points)
	• Checks answer with different method. • Explains where he/she got stuck. • Summarizes the process used.	• Reflects on reasonableness of answer • Comments and explains the ease or difficulty of problem. • Describes any errors made and how he/she found and corrected them.	

	1 point	2 points	3 points
Display			
Presentation	Writing is messy and hard to read, solution is not clearly organized on the page AND diagrams/charts/tables are not neatly drawn and clearly labeled.	Writing is messy and hard to read OR solution is not clearly organized on the page OR diagrams / charts / tables are not neatly drawn and clearly labeled.	Writing is clear and neat, solution is clearly organized on the page and diagrams/charts/tables are neatly drawn and clearly labeled.

MATH 396 PROBLEM OF THE WEEK COVER				
Student Name		**Total Score**		/20
Instructor		× 2 =		/40
POW #		**Due Date**		

	1 point	2 points	3 points
Problem Solving			
Interpretation	Does not show much understanding of the problem.	Shows some understanding of the problem. Completes part of the problem.	Answers the main part of the problem correctly.
Strategy	Does not know how to set up the problem. OR No evidence of strategy. OR Strategy didn't work.	Strategy makes sense, but isn't enough to solve the whole problem, OR Strategy is not applied systematically. OR Strategy might verify a correct answer, but fails to explain how they found it.	Picks a sound strategy. Approaches the problem systematically, achieving success through skill, not luck. Guess-and-check approach must involve good reasoning and informed guessing.
Accuracy	Has made many errors or shows no math.	Some work is accurate. May have one or two errors. OR Shows very little arithmetic.	Work on main problem is accurate and contains no arithmetic mistakes.
Communication			
Completeness	Writes very little to explain how the answer was achieved.	Provides explanation, but no calculations OR Shows calculations but without explanation about why they were done. OR The path through the work is only partly shown.	Explains most of steps taken to solve the problem and rationale for them, with enough detail for another student to understand. Shows all key calculations.
Clarity	Explanation is very difficult to follow.	Explanation isn't totally unclear, but another student wouldn't follow it easily. OR Spelling errors / typos make it hard to understand. OR Uses incorrect or has missing units.	Attempts to make explanation readable by a peer. Shows effort to use good formatting, spelling, grammar, typing. Uses correct units where appropriate and uses correct mathematical language.
Reflection	Does nothing reflective (0 points)	Includes one reflective thing (1 point)	Includes two reflective things (2 points)
	• Checks answer with different method. • Explains where he/she got stuck. • Summarizes the process used.	• Reflects on reasonableness of answer • Comments and explains the ease or difficulty of problem. • Describes any errors made and how he/she found and corrected them.	
	1 point	**2 points**	**3 points**
Display			
Presentation	Writing is messy and hard to read, solution is not clearly organized on the page AND diagrams/charts/tables are not neatly drawn and clearly labeled.	Writing is messy and hard to read OR solution is not clearly organized on the page OR diagrams / charts / tables are not neatly drawn and clearly labeled.	Writing is clear and neat, solution is clearly organized on the page and diagrams/charts/tables are neatly drawn and clearly labeled.

Math 396 Journal Assignments and Quiz Directions

There are three journal reading assignments and follow up quizzes.

The four journal articles are on **e-reserve in the library**, under Math 396.

For each journal article there is a no note, 15 point quiz on the content of the article. Each quiz will consist of three to five questions that should easily be able to answer if you have carefully read the journal article.

To study for each journal quiz, it is recommended that you carefully read the article and write yourself a simple outline of the key points and features of the article.

See your course homework page for quiz dates.

Caution: Use the library log in page to access the journal articles, text only online versions will not include critical diagrams, charts and tables in the articles. The library log in page is linked to your course homework page.

Journal 1
"Things I learned about Teaching (and Assessing) Mathematics from my Sensei," Daneile J. Brahier, Mathematics Teaching in the Middle School, August, 2005.

Journal 2
"Promote problem-solving discourse," Jonathan Bostic and Tim Jacobbe, Teaching Children Mathematics, August, 2010.

Journal 3
"Supporting children's problem solving," Victoria R. Jacobs and Randolph A. Philipp, Teaching Children Mathematics, September, 2010.