Lab 2

Numeration Systems
Place Value, Addition and Subtraction of Whole Numbers

Objectives:

1. The teacher will understand how to work with, model and use appropriate terminology in describing base 10 and other numeration systems.

2. The teacher will understand how to work with and model standard numerals, digits and place value in base 10 and other numeration systems using counting sticks, beans sticks or Multibase Blocks.

3. The teacher will understand a variety of addition situations and be able to distinguish between them.

4. The teacher will understand how to model addition of whole numbers using Bean Sticks & Cuisenaire Rods in a variety of addition settings..

5. The teacher will understand a variety of subtraction situations and be able to distinguish between them. The teacher will understand how to model subtraction of whole numbers using Bean Sticks & Cuisenaire Rods in a variety of subtraction settings..

Terms and Ideas to Know

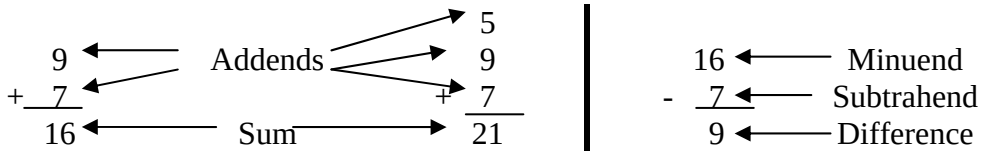
- Written symbols for numbers are called **NUMERALS.**

- A logically organized collection of numerals is called a **NUMERATION SYSTEM**.

- In the United States we use a **BASE TEN** numeration system. In base ten we use the ten **DIGITS** {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}. When we reach more than 9 objects we use numerals based on our organized, place value focused numeration system.

- Historically there are many different numeration systems. Exploring these systems can help children understand the critical concept of place value, as well as providing interesting links between their mathematics and history curricula.

- Terminology



TOPIC: NUMERATION SYSTEMS

- ***Materials: WORLD OF FIVE MONEY SET***

1. For this problem we will live in a special ***World of Five***. In the special ***World of Five*** we can only count using Pennies, Nickels, Quarters, 125-cent bills and 625-cent bills.

What is the relationship between the numbers: 1, 5, 25, 125 and 625?

These directions are given in regular base 10 numbers. You have to convert (as directed) to the special ***World of Five***.

a. Start with 12 pennies, 7 nickels and 8 quarters. Notice this is not the most efficient set of money in the ***World of Five***. Describe here how you can trade in some coins for a more efficient set of coins or bills. As a group be sure to use the coins and bills to figure this out!

The most efficient set is: _____125-cent bills _____Quarters _____Nickels _____Pennies

b. Start with 3 pennies, 5 nickels and 6 quarters and 7 125-cent bills. Notice this is not the most efficient set of money in the ***World of Five***. Describe here how you can trade in some coins for a more efficient set of coins or bills. As a group be sure to use the coins and bills to figure this out!

The most efficient set is: _____625-cent bills _____125-cent bills

_____Quarters _____Nickels _____Pennies

c. If you always utilized the most efficient set of coins and bills (and you never have "too much" money) in the ***World of Five,*** then:

You never need more than _____Pennies You never need more than _____Nickels

You never need more than _____Quarters You never need more than _____125-cent bills

You never need more than _____625-cent bills

Is there any issue with filling out that last blank?

d. What are the digits in the ***World of Five***? (see page 1)

e. How are 1, 5, 25, 125 and 625 related? How are they all related to 5?

f. Why aren't there any 50-cent coins in the ***World of Five***? (Note 50 is a base 10 number here)

g. What is the most efficient set of money in the ***World of Five*** that will make $9.63) Carefully show your conversion here. As a group be sure to use the coins and bills to figure this out!

Let's move back to base 10 for a review of familiar terms.

Example: A Base Ten <u>STANDARD NUMERAL</u>: 3215 OR $(3215)_{ten}$

We use this subscript to denote the base.
We always denote the base if it is not base ten.

The **<u>EXPANDED NUMERAL</u>**: **uses 1, 10, 10^2** and **10^3** to denote increasing place values.

$$3215 = \underbrace{[3 \times 10^3] + [2 \times 10^2] + [1 \times 10^1] + [5 \times 1]}$$

This is the EXPANDED NUMERAL

We connect the ideas of place value and digits by saying the following.

In BASE 10
For the standard numeral 3215

- ***The place value*** of the ***digit*** 3 is 10^3. We note that there are 3 thousands
- ***The place value*** of the ***digit*** 2 is 10^2. We note that there are 2 hundreds
- ***The place value*** of the ***digit*** 1 is 10. We note that there is 1 ten
- ***The place value*** of the ***digit*** 5 is 1. We note that there is 5 ones.

2. .

a. What base are we using in the ***World of Five***? Base _____

b. Look back at the efficient set you determined for your money in 1a. Use this information to fill in the following blanks:

In BASE _____ [The World of Five]

- ***The place value*** of the ***digit*** __1__ is _____ This means that there are _____ of the _____ bills
- ***The place value*** of the ***digit*** __4__ is _____ This means that there are _____ of the _____ coins
- ***The place value*** of the ***digit*** __4__ is _____ This means that there are _____ of the _____ coins
- ***The place value*** of the ***digit*** __2__ is _____ This means that there are _____ of the _____ coins

In the ***World of Five*** what is the standard numeral for this set of money? (_____)$_{\text{five}}$

Comment: Here we use the notation (_)$_{\text{five}}$ so that we don't confuse our standard numeral here in the ***World of Five*** with our usual notation for base 10.

c. Look back at the efficient set you determined for your money in 1b. Use this information to fill in the following blanks:

In BASE _____ [The World of Five]

- ***The place value*** of the ***digit*** _____ is _____ This means that there are _____ of the _____ bills
- ***The place value*** of the ***digit*** _____ is _____ This means that there are _____ of the _____ bills
- ***The place value*** of the ***digit*** _____ is _____ This means that there are _____ of the _____ coins
- ***The place value*** of the ***digit*** _____ is _____ This means that there are _____ of the _____ coins
- ***The place value*** of the ***digit*** _____ is _____ This means that there are _____ of the _____ coins

In the ***World of Five*** what is the standard numeral for this set of money? (_____)$_{\text{five}}$

Final Comment: Note that the 1, 5, 25, 125 and 625 are NOT base five numerals. They are base 10 numerals and they represent the **place values** in the ***World of Five***. Since we use base 10 in the United States we commonly use powers of the base in our expanded numerals when we talk about bases other than base 10. For example in Computer Science we use base 2 (1s and 0s) but when we refer to an expanded numeral for base two it looks, for example, like this:

$$(10010)_{two} = [1 \times 2^4] + [0 \times 2^3] + [0 \times 2^2] + [1 \times 2^1] + [0 \times 1]$$

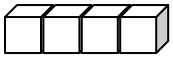
We use the digit 2 (from base 10) here in our place value powers even though there is no 2 in base 2. We do this because **we** think in base 10.

- ***Materials: BASE _____ MULTIBASE BLOCKS***

3. What base do you think these blocks are in? Explain you answer here.

Each of these blocks has a name and these names extend for all similar blocks. These are called **MULTIBASE BLOCKS** and the set you have is called specifically **BASE MULTIBASE BLOCKS** (Fill in the base number)

- Draw simple sketches of the increasing sized blocks in and fill out the rest of the table.

BLOCK	NAME	# SMALL CUBES
	**SMALL CUBE**	1 small cube
	**LONG**	_____small cubes
	FLAT	_____small cubes
	BIG CUBE	_____small cubes

- What is the relationship between the number of small cubes in these blocks (the numbers in the last column)?

h. As a group, start with 6 small cubes, 7 longs and 5 flats. Is this the most efficient set of base four Multibase Blocks? Describe here how you can trade in some blocks for a more efficient set of base four Multibase Blocks. As a group be sure to use the blocks to figure this out!

This is the most efficient set: _____Big Cubes _____Flats _____Longs _____Small Cubes

4. Use your understanding of numeration systems to fill out the following for the set of blocks in the previous problem.

In BASE FOUR

- ***The place value*** of the ***digit*** _____ is _____ This means that there are _____ of the _____
- ***The place value*** of the ***digit*** _____ is _____ This means that there are _____ of the _____
- ***The place value*** of the ***digit*** _____ is _____ This means that there are _____ of the _____
- ***The place value*** of the ***digit*** _____ is _____ This means that there are _____ of the _____

In the ***BASE FOUR*** what is the standard numeral for this set of blocks? (_____)$_{four}$

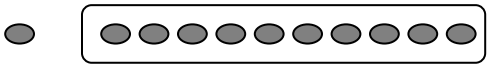
Note this standard numeral uses the subscript for base FOUR.

5. Suppose that you have 354 small cubes. What is the most efficient set of base four Multibase Blocks that has the same number of small cubes? Carefully show your conversion here. As a group be sure to use the blocks to figure this out!

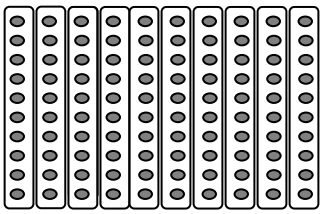
6. Describe here any similarities or differences you see between the numeration system used in the ***World of Five*** and the numeration system for using base four Multibase Blocks.

TOPIC: PLACE VALUE, ADDITION AND SUBTRACTION OF WHOLE NUMBERS

- ***Materials: BEAN STICKS***



BEAN **BEAN STICK**



BEAN RAFT, not to scale Glue sticks on back to hold together

7. Open a bag of **BEAN STICKS**. These particular bean sticks, as you can see, are useful for representing base 10 numbers. 10 bean sticks can be glued together to make a **BEAN RAFT**.

a. If each bean is ***1*** then: Each bean stick is _____ and a bean raft is _____

Addition: Case 1—You can add to join one amount to another amount

b. As a group decide how to use the bean sticks to model the answer to:
If I have 15 beans and my friend has 12 beans then how many beans do we have together?
Give a brief summary of your discussion here and draw pictures of bean sticks to show your work.

Addition: Case 2—You can add two parts to find the whole amount

8. As a group decide how to use the bean sticks to model the answer to:
If I have 17 new fruit stickers and I have 15 new star stickers then how many stickers do I have all together?
Give a brief summary of your discussion here and draw (labeled) pictures of bean sticks to show your work.

9. In the previous problem, to use the most efficient set of beans and bean sticks to show your final answer you had to trade or regroup 10 beans into one bean stick. Mathematically what is this called?

Explain how the bean stick work relates to the pencil and paper algorithm that you would use here:

$$\begin{array}{r} 17 \\ +\ 15 \\ \hline \end{array}$$

Subtraction: Case 1—Take Away

10. As a group decide how to use the bean sticks to model the answer to:
If I have 17 new fruit stickers and I give 8 of my new stickers to my sister then how many stickers do I have all together?
Give a brief summary of your discussion here and draw (labeled) pictures of bean sticks to show your work. Be sure to start by modeling 17 as 1 bean stick and 7 beans.

11. In the previous problem, in order to take away 8 beans you had to trade or regroup one bean stick into 10 beans. Mathematically what is this called? _____

Explain how the bean stick work relates to the pencil and paper algorithm that you would use here:

$$\begin{array}{r} 17 \\ -\ 8 \end{array}$$

Subtraction: Case 2—Comparison

12. As a group decide how to use the bean sticks to model the answer to:
If I have 17 new fruit stickers and my sister has 9 new fruit stickers, how many more stickers do I have?
Give a brief summary of your discussion here and draw (labeled) pictures of bean sticks to show your work. Be sure to start by modeling 17 as 1 bean stick and 7 beans.

Subtraction: Case 3—The missing addend $9 + \_\_\_\_\_ = 19$
The missing addend ↑

13. As a group decide how to use the bean sticks to model the answer to:
If I have 19 new fruit stickers, 9 are apple stickers and the rest are cherry stickers. How many cherry stickers do I have?
Give a brief summary of your discussion here and draw (labeled) pictures of bean sticks to show your work. Be sure to start by modeling 19 as 1 bean stick and 9 beans.

➢ ***Materials: CUISENAIRE RODS***

Introductory comments on Cusisenaire Rods

- ***This is the relationship between the ten different Cuisenaire Rods:***

Orange Rod = 10 white rods
Blue Rod = 9 white rods
Brown Rod = 8 white rods
Black = 7 white rods
Dark Green = 6W
Yellow = 5W
Purple= 4W
LG= 3W
Red
W

LG stands for Light Green
Red is 2 white rods
White is the smallest rod

Why are these good?

One of the really great aspects of Cuisenaire Rods is that you can use each of them to represent different numbers, depending on the circumstance and what you are trying to show. All that matters is their relative size compared to each other.

Terms

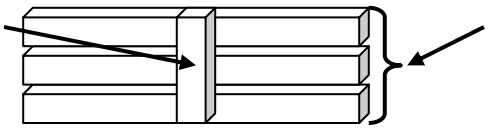
➢ If you join rods together in a horizontal row this is called a **TRAIN**. For example:

Orange Rod	Yellow Rod

This is an ORANGE-YELLOW train

- Of course, a single rod can be called a "train" too—this helps us phrase questions, explanations and answers effectively using the generic word "train".

➢ If you stack rods together in a vertical form this is called a **TOWER**. For example:

One LIGHT GREEN rod  Three BLUE rods

This stack of rods is a BLUE-LIGHT GREEN tower

- Of course, a single rod can be called a "tower" too—this helps us phrase questions, explanations and answers effectively using the generic word "tower".

➢ If you group rods together, not in a train or a tower [see example] this is called a **CLUSTER**. For example:

LG	LG	LG	LG	LG	LG	LG	LG	LG

This is a CLUSTER of 9 LG rods

- Of course, a single rod can be called a "cluster" too—this helps us phrase questions, explanations and answers effectively using the generic word "cluster".

14. Be sure to work with your group AND be sure that each person is actually working with the rods. Use the rods and the comparison chart above to answer the following questions:

a. If the white rod is 1 then the _____ rod is 7

b. If the white rod is 2 then the _____ rod is 14

c. If the _____ rod is 1 then the orange rod is 5.

Here it makes sense to physically compare two trains and each member of your group should do this!

Orange Rod = 5				
1	1	1	1	1

What color rod fits here? _____

d. If the _____ rod is 1 then the blue rod is 3. Draw and label two trains to show this.

15. Cuisenaire Rods are great for basic addition and subtraction of whole numbers. If the **white rod is 1 then the orange rod is 10**. Using trains that have as many orange rods as possible helps children add numbers by comparing "new" addition trains to familiar—emphasizing place value—trains.

BLUE = 9	BROWN = 8
ORANGE = 10	BLACK = 7

The white rod is 1 [1]

This shows the addition sentence 9 + 8 = 17

Let the white rod be 1 for all of these: [ ] The white rod is 1

a. Fill in the numbers. What addition sentence does this show? Be sure to model with the actual rods.

BROWN = _____	BLACK = _____
ORANGE = _____	YELLOW= ___

Addition sentence:

b. Draw trains to show 6 + 5. Be sure to use a comparison train to show place value. ☐ The white rod is 1

Addition sentence:

c. Draw trains to show 13 + 12. Be sure to use a comparison train to show place value. ☐ The white rod is 1

Addition sentence:

d. Label the following to show 17 – 8 as take away. ☐ The white rod is 1

ORANGE = 10	BLACK = 7

Take away ____ _____ Remains

Subtraction sentence:

e. Use take away to show 23 - 15 ☐ The white rod is 1

Subtraction sentence:

f. Use take away to show 34 - 23 ☐ The white rod is 1

Subtraction sentence:

g. Label the following to show 17 – 8 as comparison

ORANGE = 10	BLACK = 7

☐ The white rod is 1

17 is _____ more than 8

Subtraction sentence:

h. Use comparison to show 19 - 12 ☐ The white rod is 1

Subtraction sentence:

LAB TWO DISCUSSION QUESTIONS

As a group, discuss and answer the following questions. Feel free to also discuss/talk with the other groups.

1. Name another manipulative that can be uses for NUMERATION SYSTEMS.

2. Name another manipulative that can be used for PLACE VALUE, BASIC WHOLE NUMBER ADDITION AND SUBTRACTION.

3. What mathematical knowledge would you want your students to KNOW prior to introducing NUMERATION SYSTEMS.

Note: In the elementary school classroom, the primary study is the BASE 10 Numeration System. We study other bases here to emphasize the topics of a numeration system in a non-familiar setting to stimulate your learning. Some fun special projects can be done, especially with middle school students, that consider bases other than base 10. See the Inka Quipa article for an example of this.

4. What mathematical knowledge would you want to EMPHASIZE while introducing NUMERATION SYSTEMS?

5. What mathematical knowledge would you want to EMPHASIZE while introducing WHOLE NUMBER ADDITION AND SUBTRACTION?

6. Our money system has pennies, nickels, and quarters (which makes us seem like a base 5 system), as well as dimes and dollar bills and ten dollar bills (which makes us seem like a base 10 system). Why do you think we have both? (We ask this because a student may ask you the same thing…or you can ask your students this).