

## MTH 355 Review for Exam 2 on Monday 11/23

- Induction
- Sections from Text: §5.4, 6.1 – 6.3, 7.1 – 7.5, 8.1
- Multinomial Theorem
- Inclusion-Exclusion Principle
- Combinations with Repetition
- Derangements
- Finding and Solving Linear (Homogeneous and Nonhomogeneous) Recurrence Relations
- Fibonacci Sequence
- Generating Functions (Ordinary and Exponential): Setting up and using for recurrence relations
- Catalan Numbers
- DO THE HOMEWORK. Solutions on the web!

### MTH 355 Exam 2 from 2007

1. pg. 159 #41
2. pg. 198 #1 – 3
3. pg. 200 #21
4. pp. 260 – 262 #26, 29, 32, 38c, 45
5. pg. 315 #1
6. Find the number of ways in which the letters  $a, a, b, b, c, c, d, d$  can be arranged so that two letters of the same kind never appear consecutively. Be sure to explain your work.
7. SET UP THE APPROPRIATE GENERATING FUNCTION. DO NOT CALCULATE AN ANSWER, BUT INDICATE WHAT YOU ARE LOOKING FOR: FOR EXAMPLE, THE COEFFICIENT OF  $x^{10}$ .  

A customer wants to buy six pieces of fruit, including at most two apples, at most two oranges, at most two pears, and at least one but at most two peaches. How many ways are there to buy six pieces of fruit if any two pieces of fruit of the same type, for example, any two peaches are indistinguishable?
8. Let  $h_n$  be the number of ways to perfectly cover a  $1 \times n$  board with squares and dominoes in such a way that no two dominoes are consecutive. Find, **but do not solve**, a recurrence relation and initial conditions satisfied by  $h_n$ . (Initial conditions means find  $h_n$  for a few values of  $n$ .)

9. Use the method of characteristic roots to solve the recurrence  $b_n = -7b_{n-1} + 18b_{n-2}$  with initial conditions  $b_0 = 0$  and  $b_1 = 8$ .
10. Let  $f_0 = 0, f_1 = 1, f_2 = 1, \dots$  be the Fibonacci sequence. By evaluating  $f_1^2 + f_1^2 + f_2^2 + \dots + f_n^2$  for small values of  $n$ , conjecture a general formula and then prove it, using induction. For the proof by induction, DON'T PROVE THE BASE CASE. To help you out, consider the table below)

$n$	$f_n$	$f_1^2 + f_2^2 + f_3^2 + \dots + f_n^2$
1	1	1
2	1	2
3	2	6
4	3	15
5	5	40
6	8	104
7	13	273
8	21	