Homework Number 8 Solutions

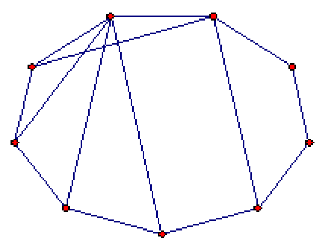
HW pp. 449 – 450:

#3 No because the graph has an odd number of odd-degree vertices (namely one).

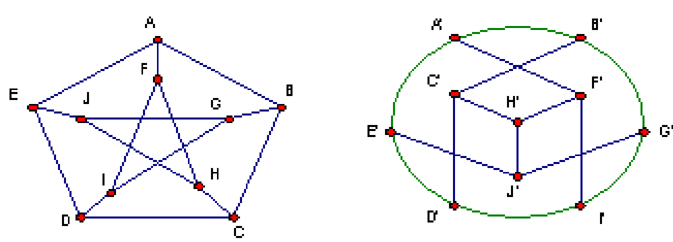
#4 No. Since 3 of the 5 vertices have maximum degree (4), this means these three vertices are adjacent to each vertex. Therefore, the degree of the other two vertices is at least 3.

#5 For each vertex v, we have $0 \leq \deg(v) \leq n-1$. This means we have n possibilities for the degree of each vertex and n vertices. If no two vertices have the same degree, then for each k such that $0 \leq k \leq n-1$, there exists a vertex v with $\deg(v) = k$. This means there is a vertex of degree 0 and a vertex of degree $n-1$. This can't be since the vertex of degree 0 is adjacent to no other vertex and the vertex of degree $n-1$ is adjacent to all the vertices. Therefore, there are only $n-1$ possibilities for the degree of each vertex(there are n vertices). By the pigeonhole principle, there exists two vertices with the same degree.

#9



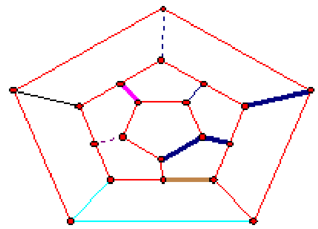
#12 Note that the 1st graph is not isomorphic to the 3rd graph because the 3rd has a 4-cycle, but the 1st graph does not. Similarly for the 2nd and 3rd graphs. Below, find an isomorphism for the 1st and 2nd graphs.



#30 K_n has an Eulerian Circuit (closed Eulerian trails) if the degree of each vertex is even. This means n has to be odd, since the degree of each vertex in K_n is $n-1$.

K_n has an Eulerian trail (or an open Eulerian trail) if there exists exactly two vertices of odd degree. Since each of the n vertices has degree $n-1$, we need $n = 2$.

#35 According to Theorem 11.2.4, you can partition the edges of our graph into $20/2 = 10$ open trails, since each of the 20 vertices is of odd degree. See the colored graph below to see the 10 different trails used.



#44 A Hamilton cycle goes through each vertex exactly once and returns back to the original vertex. A Hamilton cycle in a bipartite graph goes back and forth from the left hand side to the right hand side, and then ends on the left hand side (back at the original vertex). This means $K_{m,n}$ has a Hamilton cycle if and only if $m = n$. If $m > n$ (or vice versa), then you'll run out of right hand side vertices before you hit all the left hand side vertices (or vice versa).

A Hamilton path goes through each vertex exactly once and does not return back to the original vertex. Since $K_{n,n}$ has a Hamilton cycle, it also has a Hamilton path. If $m = n+1$ (or $n = m+1$), then K_m, n has a Hamilton path just by going $a_1, b_1, a_2, b_2, \ldots, a_n, b_n, a_{n+1}$ (where the vertices on the left hand side are $\{a_1, a_2, \ldots, a_m\}$ and the vertices on the right hand side are $\{b_1, b_2, \ldots, b_n\}$. If $m \geq n+2$, then we'd run out of right hand side vertices before we get done with the left hand side vertices.