Name: _____

Calculus 252 Problems, Fall 2005, Set 4

Edited from Rutgers University

Please write the solution to these problems and hand them in at the Lab next week.

1. a. Consider the following improper integrals (where $k > 1$):

$$\int_k^\infty e^{-x^2}dx \qquad\qquad \int_k^\infty xe^{-x^2}dx$$

Graph the integrands and determine which integral is larger. Do the integrals converge or diverge? (*Hint:* Try a substitution to evaluate one of the integrals.)

b. Using part *a*. above, prove that

$$\int_k^\infty e^{-x^2}dx \leq \frac{1}{2}e^{-k^2}$$

c. It is known that

$$\int_0^\infty e^{-x^2}dx = \frac{\sqrt{\pi}}{2}.$$

Use this and integration by parts to calculate

$$\int_0^\infty x^2e^{-x^2}dx.$$

(*Hint:* Write the integrand as $x(xe^{-x^2})$.)