

Calculus Lab 7—Modeling Internet Growth

Objective: Use mathematical modeling to predict the growth of Internet access.

Maple Commands:

dsolve(expr,y(x)); Invokes Maple's differential equation **solver** to solve expr for $y(x)$. To use this, first define expr to be some ordinary differential equation.

plot(expr,x=a..b); Ordinary plot() command. You can also specify range $y=c..d$.

plot({expr1,expr2},x=a..b); Plots both expr1 and expr2 on one set of axes.

In this lab, we will use Maple's differential equation solver to solve an equation for us, so we can concentrate on the results. But first we must learn to use the solver, called dsolve. Let's try it out on the differential equation

$$\frac{1}{x^2} \frac{dy}{dx} = e^y$$

1. Type **ODE:= (1/x^2)*diff(y(x),x)=x*exp(y(x));** (Here we create the equation, store it in memory, and call it ODE.)
2. Type **dsolve(ODE,y(x));** Here we apply the differential equation solver. Notice you get a constant of integration. Maple usually employs an underscore character to denote such constants, so the constant may appear as **_C1**, but can take other forms. Notice also that the may be in *implicit* form (y isn't isolated; Maple often isolates the constant of integration instead).
3. We could avoid this last problem by using a variation of the dsolve() command: Type **dsolve(ODE,y(x),'explicit');** This solves ODE and explicitly isolates $y(x)$.

Maple cannot always isolate y . If we use this last form of the dsolve command, then Maple will not return any solution when it is unable to isolate y . You should be able to solve the above equation without Maple's help because we easily recognize that it is separable.

4. Solve $\frac{1}{x^2} \frac{dy}{dx} = e^y$ by hand.

- Maple saw this too. To confirm that it did, type **infolevel[dsolve]:=2:** and repeat the above steps. Notice when you apply dsolve() that Maple now tells us *how* it solved the equation: specifically, it says that it used the fact that the equation is separable.
- To turn the infolevel feature off, type **infolevel[dsolve]:=0:**

Exercise: How fast do people catch up to technology? Let $p(t)$ be the percentage of adults in North America who have Internet access. In July 1996, the Boston Globe newspaper reported that 7.7% of adults had access via home, work, or school, but this was increasing dramatically as a function of the time t . Reasonable estimates suggested that it would double every two years, though some expected it to double yearly or even more rapidly. The volume of traffic carried by the Internet was doubling every 3 months.

It is believed that dp/dt is proportional to the product of (i) the present percentage $p(t)$ of people already using the Internet (since they tell their friends) and (ii) the number $(100\%-p(t))$ of people who do not use it (since new Internet users, known as newbies, have to come from the pool of present non-users), so

$$\frac{dp}{dt} = kp(100 - p)$$

where k is a constant of proportionality. This is called the *logistic equation*.

- Find the general solution of this equation (you may use Maple's dsolve here or hand calculations). Record your answer below.

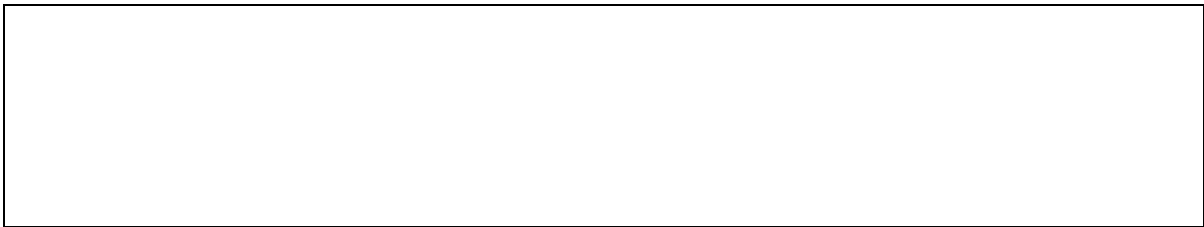
- Let $t=0$ be July 1996. Use the fact that 7.7% of adults were connected to the Internet that month to fix the constant of integration in your solution (round off your constant to the nearest whole number). (You will need a calculator to figure out $_C1$.) Record $_C1$: _____

- If the percentage of adults who have Internet access doubles yearly, then the constant k takes the approximate value $k=0.0076$ (if t is measured in years), while a doubling time of two years corresponds to $k=0.0038$. For each of these two possibilities, write down the equation for $p(t)$ *below*. There should be no undetermined constants in your equations.

10. Graph these two equations on the same set of axes, using a 40 year period. What do you notice about these graphs?



11. In what calendar year do you expect 25% of adults in North America to have Internet access in each of these two scenarios? When will 90% of adults to have Internet access?



12. A more realistic model might suggest that the maximum percentage of North American adults to have Internet access, at least for the foreseeable future, is perhaps 50% of the total population, since economics will prevent some people from ever owning a home computer and since certain occupations do not require computer access at work. How would you revise the logistic equation in light of this model?



13. Solve your revised equation (and record below) and graph the solution, assuming $k=0.0038$ (a two-year doubling time). In this model, when will 25% of adults have access? (record below)

