

## Partial HW Solutions to §4.3, §4.5, and §4.6

§4.3 solutions:

#24 :

$$B(x) = 3x^{2/3} - x$$
$$B'(x) = 2x^{-1/3} - 1 = \frac{2 - x^{1/3}}{x^{1/3}}$$

Critical numbers are  $x = 0, 8$ .

$B(x)$  increasing on the interval  $(0, 8)$  and decreasing on  $(-\infty, 0) \cup (8, \infty)$ .

$$B''(x) = -\frac{2}{3}x^{-4/3} = -\frac{2}{3(\sqrt[3]{x})^4}$$

$B(x)$  always concave down (except for at  $x = 0$ .)

#29 :

$$f(x) = \frac{x^2}{x^2 - 1}, f'(x) = \frac{-2x}{(x^2 - 1)^2}, f''(x) = \frac{6x^2 + 2}{(x^2 - 1)^3}$$

$f(x)$  increasing for  $(-\infty, -1) \cup (-1, 0)$  and decreasing for  $(0, 1) \cup (1, \infty)$

$f(x)$  concave up for  $(-\infty, -1) \cup (1, \infty)$  and concave down for  $(-1, 1)$

#37 :  $f'(x) = (x + 1)^2(x - 3)^5(x - 6)^4$

$f(x)$  increasing for  $(3, 6) \cup (6, \infty)$ .

§4.5 solutions:

#10 :

$$\lim_{x \rightarrow 0} \frac{e^{3t} - 1}{t} = \lim_{x \rightarrow 0} \frac{3e^{3t}}{1} = 3$$

#23 :

$$\lim_{x \rightarrow 1} \frac{x^a - ax + a - 1}{(x - 1)^2} = \lim_{x \rightarrow 1} \frac{a \cdot (a - 1)x^{a-2}}{2x} = \frac{a \cdot (a - 1)}{2}$$

#24 :

$$\lim_{x \rightarrow 0} \frac{1 - e^{-2x}}{\sec x} = 0$$

#28 :

$$\lim_{x \rightarrow 0^+} \sin x \ln x = \lim_{x \rightarrow 0^+} \frac{\ln x}{\frac{1}{\sin x}} = \lim_{x \rightarrow 0^+} \frac{\ln x}{\frac{1}{\csc x}} = \lim_{x \rightarrow 0^+} \frac{1/x}{-\csc x \cot x} =$$
$$\lim_{x \rightarrow 0^+} \frac{1/x}{(-1/\sin x) \cdot (\cos x / \sin x)} = \lim_{x \rightarrow 0^+} -\frac{\sin^2 x}{x \cos x} = \lim_{x \rightarrow 0^+} -\frac{2 \sin x \cos x}{\cos x - x \sin x} = 0$$

§4.6 solutions:

#6 : Since the rectangle has area 1000. This means  $lw = 1000$ , so  $l = 1000/w$ . Perimeter is  $P = 2l + 2w = 2000/w + 2w$ . Set  $P' = 0$ . This means

$$0 = -\frac{2000}{w^2} + 2 \Rightarrow w = \sqrt{1000} = l.$$

#9 :

$$x^2 + 4xy = 1200 \Rightarrow y = \frac{1200 - x^2}{4x} = \frac{300}{x} - \frac{1}{4}x.$$

$$V = x^2y = x^2 \left( \frac{300}{x} - \frac{1}{4}x \right) = 300x - \frac{1}{4}x^3.$$

$$V'' = 0 \Rightarrow 0 = 300 - \frac{3}{4}x^2 \Rightarrow x = 20.$$

Dimensions are  $20 \times 20 \times 10$ , so volume is  $4000\text{cm}^3$ .

#12 :  $lwh = 10$  and  $l = 2w$ . This means  $h = \frac{10}{2w^2} = \frac{5}{w^2}$ .

This means cost

$$C = 10 \cdot l \cdot w + 2 \cdot 6 \cdot l \cdot h + 2 \cdot 6 \cdot w \cdot h = 20w^2 + \frac{60}{w} + \frac{120}{w} = 20w^2 + \frac{180}{w}$$

Set  $C'(w) = 0$ . This gives

$$C'(w) = 40w - \frac{180}{w^2} \Rightarrow w = \sqrt[3]{4.5}$$

Volume is therefore approximately \$163.54.