**Exam 2 Solutions**

1. (14 pts) Let $f(x) = -4x^2 - 8x + 5$.

   (a) (4 pts) Find the $x$-intercepts of $f(x)$.

$$-4x^2 - 8x + 5 = 0$$
$$\Longrightarrow -4(x^2 + 8x - 5) = 0$$
$$\Longrightarrow -4(2x+5)(2x-1) = 0$$
$$\Longrightarrow x = -\frac{5}{2}, \frac{1}{2}$$

   (b) (3 pts) Give the exact intervals where $f(x)$ is decreasing.

$$x > -1 \text{ look at graph.}$$

   (c) (4 pts) Write $f(x)$ in the vertex form.

$$x - \text{coordinate of vertex is } \frac{-b}{2a} = \frac{8}{-8} = -1$$

   Thus vertex is at $(-1, 9)$. Thus the vertex form is $f(x) = -4(x+1)^2 + 9$.

   (d) (3 pts) Solve the inequality $f(x) > -7$

$$f(x) > -7 \Longrightarrow -4x^2 - 8x + 5 > -7 \Longrightarrow 4x^2 + 8x - 12 < 0$$

   Solve $4x^2 + 8x - 12 = 0$, i.e., $4(x+3)(x-1) = 0$. This means $x = -3, 1$ The parabola $4x^2 + 8x - 12$ points up, so $4x^2 + 8x - 12 < 0$ for $-3 < x < 1$.

2. (10 pts; 5 pts each) Solve each of the following quadratic equations: (a) $x(6-x) = -16$

$$6x - x^2 = -16 \Longrightarrow x^2 - 6x - 16 = 0 \Longrightarrow (x-8)(x+2) = 0 \Longrightarrow x = -2, 8$$

   (b) $2x^2 + 3x - 4 = 0$

$$x = \frac{-3 \pm \sqrt{9+32}}{4} = \frac{-3 \pm \sqrt{41}}{4}$$

3. (5 pts) Write the following in standard form: $\dfrac{-2+i}{(3-i)^2}$

$$\frac{-2+i}{(3-i)^2} = \frac{-2+i}{9-6i+i^2} = \frac{-2+i}{8-6i} \cdot \frac{8+6i}{8+6i} = \frac{-16-4i+6i^2}{84-36i^2} = \frac{-22-4i}{100} = -\frac{22}{110} - \frac{4}{100}i$$

4. (8 pts) Let $h(x) = |x|$.

   (a) (4 pts) Find the equation of the function that shifts the graph of $h(x)$ 2 units to the left, then reflects the graph about the $y$-axis, and shifts the graph 3 units downward.

   Shift to the left: $|x + 2|$
   Reflect about $y$-axis: $|-x + 2|$ (substitute $-x$ for $x$)
   Shift down 3 units: $|-x + 2| - 3$

   (b) (4 pts) Graph the equation you found in part (a). Be sure to label your graph appropriately.
   When done, graph looks like $|x|$ shifted right 2 and down 3.

5. (7 pts) A farmer wants to fence a rectangular area by using the wall of a barn as one side of the rectangle and then enclosing the other three sides with 160 feet of fence. Find the dimensions of the rectangle that give the maximum area inside.

   $2W + L = 160 \Longrightarrow L = 160 - 2W.$

   $A = LW = (160 - 2W) \cdot W = -2W^2 + 160W$ This parabola has a vertex at $W = -160/-4 =$ 40ft Thus $L = 160 - 2W = 160 - 2 \cdot 40 = 80$ft.

6. (13 pts) Consider the function $g(x) = -3x^4 + 8x^3 + 6x^2 - 24x$.

   (a) (2 pt) State the degree of $g(x)$. **4**

   (b) (2 pt) Find the leading coefficient of $g(x)$. **-3**

   (c) (3 pts) Find any local maximum(s) (both the $x$-coordinate and $y$-coordinate for each point). **According to calculator: (-1, 19) and (2, -8)**

   (d) (3 pts) Find any absolute minimum(s). **NONE...look at graph**

   (e) (3 pts) Find any absolute maximum(s). **(-1, 10)**

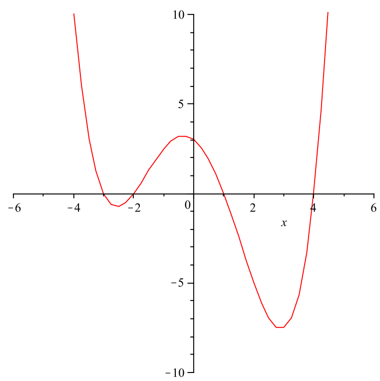
7. (10 pts) Consider the graph of a polynomial $h(x)$ below.

   (a) (2 pts) Use the graph to determine if the function graph below is even, odd, or neither. **NEITHER**

   (b) (5 pts) Use the graph to write $h(x)$ in its complete factored form.

   $h(x) = a(x+3)(x+2)(x-1)(x-4)$ based on the roots at $x = -3, -2, 1, 4$. We need the leading coefficient. We know $h(0) = 3$, so $3 = h(0) = a \cdot 3 \cdot 2 \cdot -1 \cdot -4$ Thus $3 = 24a$, i.e., $a = \frac{1}{8}$.

   (c) (4 pts) Describe the end behavior of $h(x)$.
   **h(x) rises to the left and right** or you can say, $h(x) \to \infty$ as $x \to \pm\infty$




8. (7 pts) Find the complete factored form of the polynomial $f(x) = 49x^4 + 7x^3 + 429x^2 + 63x - 108$ given that $-\dfrac{4}{7}$ and $3i$ are zeros.

   $f(x)$ has roots at $3i, -3i, -\dfrac{4}{7}$. The factored polynomial looks like $49(x-3i)(x+3i)(x+\frac{4}{7})(x-$ other root). We need to find the other root. There are various ways to do this. One way is to say $49 \cdot -3i \cdot 3i \cdot \frac{4}{7} \cdot$-(some number r) $= -108$. This means $49 \cdot -9i^2 \cdot \frac{4}{7} \cdot -r = -108$. Since $i^2 = -1$, we get $-r = \dfrac{-108}{49 \cdot 9 \cdot \frac{4}{7}}$, i.e. $r = \frac{3}{7}$. Thus $f(x) = 49(x-3i)(x+3i)(x+\dfrac{4}{7})(x-\dfrac{3}{7})$

9. (16 pts) The table lists the percentage $P$ of high school seniors who had used marijuana within the previous month in the United States for various years $y$. In this table $y = 0$ corresponds to 1975 and $y = 20$ to 1995.

| $y$ (yr) | 0 | 3 | 5 | 10 | 15 | 20 |
|---|---|---|---|---|---|---|
| $P$(%) | 27 | 37 | 33 | 25 | 12 | 21 |

   (a) (2 pts) Find the number of turning points in the data. **2**

   (b) (2 pts) Find the minimum degree of a polynomial that models the data. **3**

(c) (4 pts)Use regression to find a polynomial function $f$ that models the data.
**We should use a cubic based on our answer to (b).**
$f(x) = .0258x^3 - .77208x_4^2.7933x + 27.242$

(d) (3 pts) Use $f$ to estimate marijuana use in 1997. **Take** $f(22) = 33.9\%$.

(e) (2 pts)Did your estimate in part (d) involve interpolation or **extrapolation**.

(f) (3 pts) Since 1975, which year had the lowest percentage of students using marijuana?
**According to the calculator, the minimum occurred when** $x = 16$**, i.e. 1991.**

10. (15 pts; 3 pts each) Answer the following questions:

(a) If an even function $f$ has an absolute minimum of $-6$ at $x = -2$, then what else can be said about $f$?

**This means** $f(x)$ **also an absolute minimum of** $-6$ **at** $x = 2$ **since it is even.**

(b) Could a cubic function with real coefficients have only imaginary zeros? If so, give an example. If not, explain why not.

**NO! Imaginary roots come in conjugate pairs, so you can only have an even number of imaginary roots. Thus you can never have all 3 roots of a cubic function being imaginary.**

(c) Could a quadratic function with real coefficients have only imaginary zeros? If so, give an example. If not, explain why not.

**YES! Check out** $f(x) = x^2 + 2$.

(d) Suppose $f(x) = a(x - h)^2 + k$. How do you know whether $f$ has an absolute maximum or absolute minimum? Does $a, h$, or $k$ help determine this? What is the extreme value for $f$?

If $a > 0$, we have an absolute minimum. If $a < 0$, we have an absolute maximum. $(h, k)$ is the vertex, so the extreme value is $k$ and occurs at $x = k$.

(e) Consider the polynomial $f(x)$ of degree 5 below. Find the number of real and imaginary roots of $f(x)$.

**1 real root means 5 - 1 = 4 imaginary roots**