Using Research to Develop Computational Fluency in Young Mathematicians

How often have you been inspired to implement strategies learned at a terrific professional development session only to be disappointed by the results in your own classroom? I've confronted this situation many times, especially with respect to mathematics teaching. I am a certified elementary education teacher and reading specialist who teaches second grade in a self-contained classroom. During my undergraduate training, I had only one mathematics content course, and, as a result, for twelve years I have struggled to teach mathematics proficiently. In an attempt to fill in the gaps in my own conceptual knowledge of mathematics and my implementation of traditional mathematics curricula, I have attended many professional development sessions. However, I have not always fully understood the concept being taught or the research behind the content. I gained knowledge, but overall attempts to improve my instruction were not effective. Fortunately, the story does not end here. During the past four years, I have worked with colleagues to

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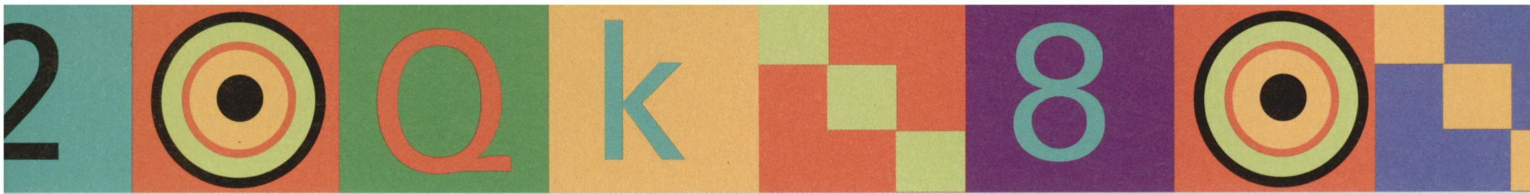


improve my teaching of mathematics by conducting classroom-based inquiry to meet my students' specific mathematical needs. This article describes my journey as an elementary teacher learning to use research-based methods to develop and improve my second-grade students' computational fluency.

Research-Based Methods

While taking a graduate course, I read *Young Mathematicians at Work: Constructing Number Sense, Addition, Subtraction* (Fosnot and Dolk 2001), which describes an approach to teaching computation by developing strategies based on number sense. I was particularly interested in how the teachers in the Mathematics in the City project, described in this book, developed short minilessons to focus on specific strategies to help children develop efficient mental mathematics computation. Through the minilessons, the teachers guided children through the mental process of addition and subtraction, for example, and then through a discussion of addition and subtraction problems, a process that made the strategies they used more explicit. In the minilessons, the teachers created "strings," a structured series of related problems intended to develop and highlight number relationships and operations. A typical minilesson would involve the teacher posing a problem for students to solve mentally. The teacher would give the students about a minute to let them solve the problem and then would call on three or four students to describe how each solved it. The teacher would listen to each student's explanation, represent his or her computation strategies on the board, and then move on to a related problem in the string.

I began to wonder how I could foster mental mathematics strategies in my classroom. I observed that most of my students, when computing two-digit numbers, were using only the splitting strategy. This solution strategy is a combination of the use of expanded notation (step 1) and the application of the commutative property of addition (step 2). The splitting strategy had been introduced in the fall, and many students found it a helpful way to compute. For example, the students would solve 34 + 25 by splitting (expanding) 34 into 30 + 4 and 25 into 20 + 5, then add 30 + 20 = 50 and 4 + 5 = 9, and then combine 50 + 9 = 59 (see **fig. 1**). The students stuck with this strategy, even when it was not efficient. I wanted to challenge my students to become accurate, fluent, and efficient when computing (Russell 2000). Fosnot and Dolk's work became the foundation of my own inquiry into more efficient mental computation strategies for addition.

A School for Professional Development

My pre-K–grade 8 neighborhood public school is part of a unique partnership involving the school district, teachers' union, and local university and was developed with the participation of parents, teachers, university faculty, community members, and neighborhood groups. Our school adopted a textbook series based on NCTM's Standards (1989, 2000). (For more information on Standards-based curricula, visit www2.edc.org/mcc.) In this program, children are not directly taught algorithms but rather are taught to develop their own strategies and ways of making sense of mathematical situations. A central part of the school's mission is to be a center for best practices and professional development, and over the past four years a professor from the university has conducted graduate-level courses in mathematics education at the school site. During the 2005–2006 school year, our course work centered on foundations of computational fluency. Given the needs of the learners in my classroom and our course work, I decided to implement Fosnot and Dolk's (2001) ideas—in particular, their idea of problem strings—to further develop my students' computational fluency. To collect data during each problem string minilesson, I represented my students' mental computation on a white board

Figure 1

Example of splitting strategy

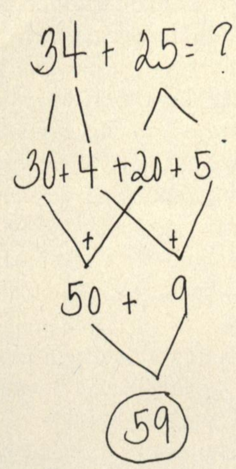
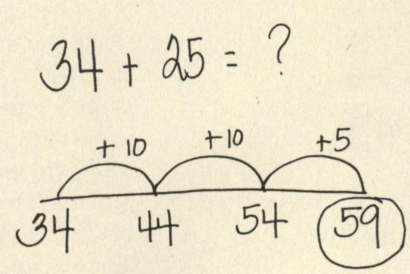


Figure 2

A student's mathematical thinking modeled on an open number line



and later transferred the work onto chart paper for future reference.

Implementation and Refinement

While taking the course and observing my students, I discovered that they were beginning to develop a sense of place value and were noticing and using patterns on the hundreds chart, but many of them were not making leaps of ten mentally. As Fosnot and Dolk (2001) note, making leaps of ten is an important landmark strategy, and one tool to help children develop this skill is the open number line. As Hiebert et al. (1997) have found, using tools effectively helps children develop deeper meanings of mathematics. The hundreds chart, which up to this point had been our main tool for developing computational strategies, does not represent numbers linearly, as an open number line does. Some children need a model like the open number line to keep a record of their counting and help them think while experimenting with patterns and relationships and thus developing number sense. **Figure 2** illustrates how the open number line might be used to represent a strategy for adding 25 to 34 by making jumps of ten and five.

As my students began to develop ideas about place value, they became fluent in splitting numbers into manageable, or "friendlier," numbers, such as tens and ones. Then I led the children in a discussion about whether splitting both numbers is the most efficient way to solve that problem. I asked the children, "Can you solve the problem faster?" Kevin replied, "I can use the up-and-down strategy, but some of us get mixed up when we use that strategy." The "up-and-down" strategy is the term my students use to describe standard addition algorithms in which the addends are written vertically. As stated earlier, the university partnership curriculum does not introduce algorithms but allows students to compute in ways that make sense to them. Of course, many students have already been exposed to standard algorithms at home. To address this juxtaposition, we recognize that using the standard algorithm is one strategy for computation but accept other strategies that are developmentally appropriate for each student.

I designed my instructional intervention so that the core of my class's mathematics workshop would continue to be conceptual investigations, but I intended to teach short minilessons using problem strings at least twice a week for three months. The initial purpose of the problem strings would be to help children make jumps of ten, a strategy that involves keeping one number whole and adding tens to it. For example, to solve 34 + 25, the children would initially compute 34 + 10 + 10 + 5. Over time, the goal is for students to begin to add all the tens at one time. For example, they would solve 34 + 25 by computing 34 + 20 + 5. As each child explained his or her mental process, I would record his or her thinking on an open number line.

Initially, I used the problem strings straight from Fosnot and Dolk (2001), but eventually I developed strings collaboratively with my professor and other teachers who were also taking the course and implementing strings in their own classrooms. Further, we collected data by recording the strategies the students were using and reflected on the minilessons. See **figure 3** for some examples of the problem strings I used to help my students learn to make jumps of ten.

Figure 3

Four examples of problem strings

no. 1	no. 2	no. 3	no. 4
37 + 4	42 + 9	46 + 7	27 + 6
37 + 14	42 + 19	46 + 17	27 + 16
37 + 34	42 + 39	46 + 47	27 + 36

When I first introduced the "jumps of ten" strategy, my students were in the middle of a geometry unit that they had been working on for a month during the core of our mathematics workshop. It was important to me that the students work on mental mathematics computation during this unit so that they would not regress during the remaining two months of my teaching intervention. During this unit, the students worked with addition and subtraction strategies to solve problems using numbers up to 100. Students worked with partners to explore structures and patterns within the hundreds chart, add multiples of five and ten, use the hundreds chart to keep track of a total amount, calculate the distance between two numbers on the hundreds chart, solve addition and subtraction problems involving numbers up to 100, reduce an equation with multiple addends to an equation having only two addends, use multiples of tens and ones to find the difference between two numbers, and check a solution by using a different strategy. I began to notice that more and more students were using the open number line as a tool to make jumps of ten as they solved these problems during mathematics workshop.

During the minilessons, I recorded the students' mental computation strategies on a white board, a process that allowed the students to see one another's mental thinking step by step. I used the open number line to show the students' thinking while linking it back to the splitting strategy they had previously used, a step that enabled them to see how inefficient splitting could be at times. Having another strategy to use helped my students become flexible, fluent, and accurate while solving mental mathematics computations. Following is a vignette of a problem-string minilesson that uses these problems: 62 + 10, 62 + 30, 62 + 39, and 56 + 38.

Teacher. Mathematicians, we are going to solve a short string to start off math workshop. Are you ready?
Students. You won't be able to trick us today!
Teacher. I want you to think about how we have been making jumps of ten in our head to solve problems. The first one is easy. *[Teacher writes 62 + 10 on the white board. Immediately thumbs go up.]*
Manuel. I started at 62 and added 8 to equal 70 and then added 2 to equal 72.
Teacher. Did anyone else do it differently?
Emma. I did it quicker. I didn't break 10 up. I just added 62 + 10 = 72. Look, you can just jump down 10 on the hundreds chart. [See **fig. 4.**]
Teacher. Any other ways? *[No one volunteers.]* OK, I want you think about our first problem to solve our second problem. *[Teacher writes 62 + 30 on the white board. After ten seconds, most thumbs are up.]*
Nim. I know 62 + 10 = 72, so I just added 72 + 10 = 82, 82 + 10 = 92.
Zayton. I did it like Naomi, kind of. I knew 62 + 10 = 72, but I added the 20 left, 72 + 20 = 92.
Cheng. I just added 62 + 30 together and knew it equaled 92. [See **fig. 5.**]
Teacher. Fantastic, mathematicians. I love how you are using what you already know about numbers to solve these problems. Continue thinking about the problems that have come before to solve the next one. This next problem is hard, but I want you to stretch your thinking.
[Teacher writes 62 + 39 on the white board.]
Helen. I just added 62 + 10 = 72, 72 + 10 = 82, 82 + 10 = 92, 92 + 9 = 101.
Teacher. Did anyone use the previous problem to help solve this one?

Figure 4

Representation of student's thinking when solving 62 + 10

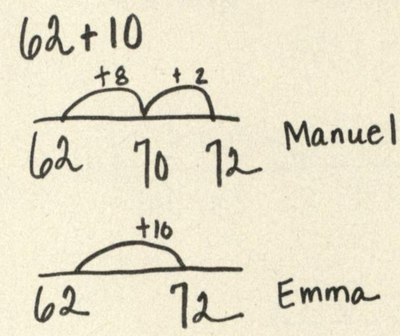
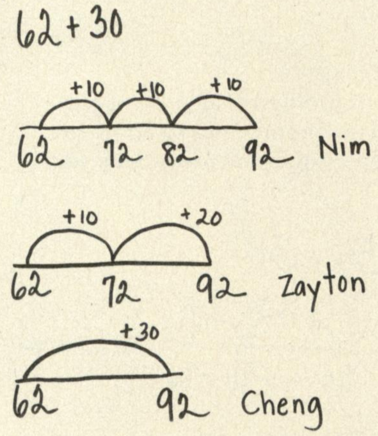


Figure 5

Varied student approaches for solving 62 + 30



Yu. I did. I know that 62 + 30 = 92, so I just added 92 + 9 =101. I stopped for a second when I got to 100, but I knew the next place was 1, so I counted 101.
Doug. I did something a little different. I knew that 62 + 40 = 102, and then I just subtracted 1 from 102 to get 101.
Teacher. Doug, can you tell us more?
Doug. I know that 39 is one less than 40, so I just subtracted one from the total to get the answer.
Ibra. I like how you did that, Doug. I might try that way sometime. [See **fig. 6.**]
Teacher. Okay, here is the last problem in the string. It is different from the other problems, but I want you to think about what you did to solve the other

Figure 6

Another problem in the string: 62 + 39. Students used different strategies, displayed on open number lines.

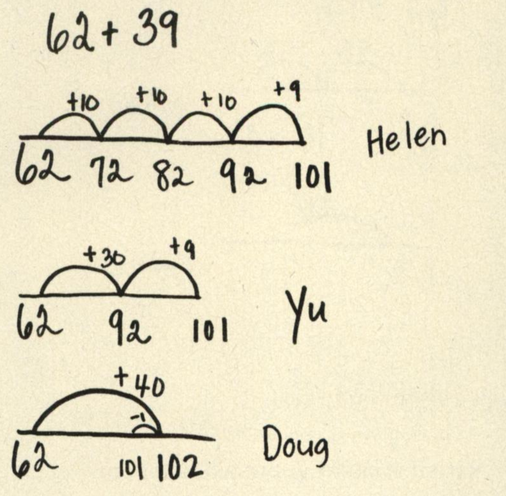
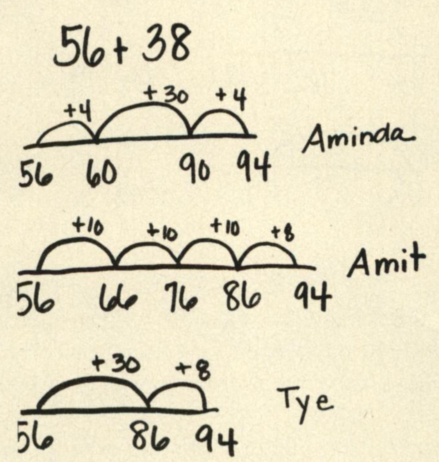


Figure 7

A different problem—56 + 38—gives students an opportunity to apply the strategies used in the previous problem string.



problems. *[Teacher writes 56 + 38 on the white board.]*

Aminda. I started at 56 and added 4. That equals 60. Then, I added 60 + 30, and that equaled 90, and then I counted up 4, and that equaled 94.

Teacher. Aminda, you made jumps of ten in your head, do you know what other strategy you used?

Aminda. Emma showed me how to go to the next friendly number, so I have been trying to use it to solve problems.

Teacher. Who else would like to share?

Amit. I started at 56 and made jumps of ten in my mind: 56 + 10 = 66, 66 + 10 = 76, 76 + 10 + 86, and 86 + 8 = 94.

Teacher. Great job making the jumps of ten in your head.

Tye. I just broke 38 into 30 and 8. I added 56 + 30 = 86 and then added 86 + 8 = 94.

Teacher. It must have been hard to keep track of those numbers in your head. Fantabulous!

Tye. I have been practicing! [See **fig. 7**.]

Teacher. Well done, mathematicians. I can really see how you have been using what you already know about numbers to solve other problems in your mind. I am so proud of your hard work today.

Impact

In analyzing my students' daily work, I found that many students began to use the open number line as a tool to solve computation problems as well as word problems. They began to talk with one another about jumps of ten and in their spare time make up computation problems for their partner to solve using an open number line.

Not all my students were developmentally ready to make jumps of ten mentally. Three were still counting on the hundreds chart by ones. However, a handful of students who originally were counting by ones were now making jumps of ten on the hundreds chart. The majority of my students, who originally were making jumps of ten on the hundreds chart and splitting numbers, were now making jumps of ten mentally and using the open number line in their daily work. **Figure 8** shows two examples from my students' mathematics journals. As part of a review for our annual standardized testing, the students were asked to solve two-digit addition problems, including 83 + 16. One student used an open number line to solve this problem (see **fig. 8a**): He started at 83, made a jump of ten (83 + 10 = 93), and then added on the ones (93 + 6 = 99). In the second example, the students, as part of a word problem unit, were asked to solve the following word problem:

> 19 children are playing tag on the blacktop. 12 children joined in. How many children are playing tag now?

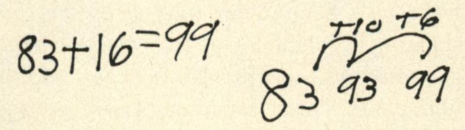
One student used the open number line to solve this problem (see **fig. 8b**). The student started on the open number line, made a jump of 10 (19 + 10 = 29), and then jumped by ones on the open number line (29 + 2 = 31).

I interviewed my students about the open number line as a tool for solving computation problems. Nim stated, "Using the open number line to make jumps of ten is easier and quicker that using number trees [splitting]. It is not as much work, and it is not as confusing." Ibra pointed out, "I can make jumps of 10 or 20, or even more, in different ways. The open number line is just another way to help me keep track of my jumps." Yu summed up, "Using the open number line is easy. I don't have to use as many muscles." Cheng believed the open number line was "quick and easy." Mohammad declared, "The open number line helps you figure out problems faster, but I still like using the splitting strategy. I can use the open number line to check my work." When making jumps of ten, Emma discovered that she could also use the open number line to record getting to a "friendly" number. This strategy is similar to the jumps of ten strategy but involves adding ones until a friendly number is reached. For example, one student solved 35 + 47 in three steps: adding 47 and 30 to get 77, then adding 77 and 3 to get 80, and then adding 80 and 2 to get 82 (see **fig. 9**).

Planning, collaborating, and reflecting with my professor and colleagues on writing and implementing problem strings furthered my mathematical thinking as well as my teaching. On my own, I could have attempted to replicate Fosnot and Dolk's (2001) work in my classroom, but, without the support and collaboration of my colleagues, I would not have fully understood the research behind the mathematical ideas. And it would have been difficult to find the time required to inquire about unfamiliar concepts.

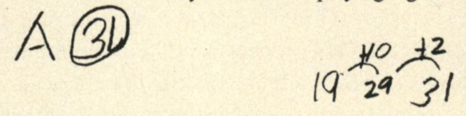
Figure 8

Examples of students' thinking as recorded in their mathematics journals



(a)

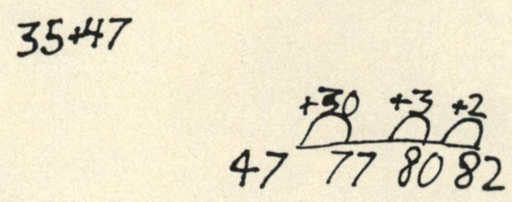
19 children are playing tag on the blacktop. 12 children joined in.
How many children are playing tag now?



(b)

Figure 9

Example of the "next friendly number" strategy



Future Considerations

Conducting classroom-based inquiry is an ongoing process. It is not just a matter of learning a new method or technique to implement in the classroom; rather, it is a constant cycle of collecting data then refining and developing new questions to foster students' mathematical thinking. I continue to use problem strings in our mental mathematics minilessons—for example, I have begun to encourage my students to make jumps of ten backward. The children wanted a successful strategy for subtraction computation, but this task was difficult for them at first. After the first three problem strings, however, the majority of my students are jumping back ten. I will continue to use this strategy in relation with the open number line and will relate it to the splitting numbers strategy.

As we continue our journey, I will also highlight the "next friendly number" strategy. Some of my students have begun to use this strategy in small-group work and have also shared it in our whole-group minilessons. Sharing this strategy has stretched the thinking of other students and readied them to take on this strategy. The students seek one another out during partnerships to continue to develop their repertoire of mental mathematics strategies.

Conclusion

Using problem strings has truly helped my students develop additional mental mathematics strategies. Further, use of the open number line to show their thinking has served as a crucial bridge as my students moved from concrete to abstract thinking. Working with problem strings and the open number line has given my students a chance to understand one another's thinking and compute in a way that makes sense to them. They have deepened their understanding of relationships among numbers and among operations, specifically between addition and subtraction. These experiences will help my students in the future as they consider more complex mathematical relationships and continue becoming competent mathematicians. The experiences have also shaped my beliefs about teacher inquiry and the benefits of using research-based methods to improve my teaching instruction and my students' performance. This process is not a quick fix; it will not help you teach a fantastic lesson on Monday, as many professional development sessions advertise. Rather, it is an ongoing succession of questioning, planning, teaching, collecting and analyzing data, collaborating, reflecting, and refining that ultimately improves your teaching and your students' mathematical thinking and reasoning.

Bibliography

Fosnot, Catherine, and Maarten Dolk. *Young Mathematicians at Work: Constructing Number Sense, Addition, and Subtraction*. Portsmouth, NH: Heinemann, 2001.

Hiebert, Joseph, Thomas P. Carpenter, Elizabeth Fennema, Karen C. Fuson, Diana Wearne, Hanlie Murray, Alwyn Oliver, and Piet Human. *Making Sense: Teaching and Learning Mathematics with Understanding*. Portsmouth, NH: Heinemann, 1997.

Mathematics in the City (MitC). Collaboration of City College and the Freudenthal Institute. http://www.mitcccny.org.

MetroMath. Collaboration of the University of Pennsylvania, Rutgers University, and City University of New York and associated school districts. http://www.metromath.com.

National Council of Teachers of Mathematics (NCTM). *Curriculum and Evaluation Standards for School Mathematics*. Reston, VA: NCTM, 1989.

———. *Principles and Standards for School Mathematics*. Reston, VA: NCTM, 2000.

Russell, Susan Jo. "Developing Computational Fluency with Whole Numbers." *Teaching Children Mathematics* 7 (November 2000): 154–59. ▲