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By Kyungsoon Jeon

Reflecting on PEMDAS

Forgo rule-driven math instruction. Use these helpful tips to teach the order of operations.

When I asked my preservice elementary teachers what *order of operations* is and why it is important for children to learn, they recited “PEMDAS” or “Please Excuse My Dear Aunt Sally” but typically became silent when questioned about the importance of the order of operations. I had not realized the significance of their silence until one of my students told me the following:

> My niece is in second grade. She said that her teacher had taught 7 – 3 + 11 as 7 minus 3 is 4, and 4 plus 11 is 15. I think her teacher is wrong because it should be 3 plus 11 first, then subtract 14 from 7, which is –7. PEMDAS says that you do addition first and then do subtraction later. So, the correct answer is –7.

I presented this expression to one of my college math classes for elementary school and found that the majority of my students agreed that the teacher was teaching the wrong way. A surprisingly small number of students remembered that addition and subtraction are done in the order of their appearance from left to right, and multiplication and division are done in the order of their appearance from left to right. However, the group of students who correctly remembered the rule failed to provide an explanation to support why the rule is as it is; and more important, they did not have the confidence to justify the correctly memorized rule to their classmates.

I came to realize how this cut-and-dried rule does not furnish students with any understanding about the concept behind it, such as why a certain order of operations must be consistent for written mathematical communications. Reys and Fennell

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once suggested that the mathematics component of preservice elementary education must be examined if real change is to occur at the elementary school level (2003, p. 278). In an attempt to bring such change to our classrooms, in this article I examine what students understand about the order of operations and offer some helpful hints to correct their misunderstanding. I share three stories about PEMDAS that I have collected while instructing classes of preservice elementary school teachers of mathematics since that eye-opening moment with my student's story about her niece.

PEMDAS story 1

I gave the following problems (Billstein, Libeskind, and Lott 2004, p. 208) to students in the first content course for preservice teachers. Initially, my goal was to assess their knowledge of and skills with the operations on integers before starting the chapter on the topic. I suggest that the reader solve the following problems before reading the results from my students.

1. $2 - 5 \bullet 4 + 1$
2. $2 - 3 \bullet 4 + 5 \bullet 2 - 1 + 5$
3. $2 + 16 \div 4 \bullet 2 + 8$

After all students had worked on the problems, I asked what answers they came up with as candidates for the correct solutions. We created a table on the board as the students shared their answers as well as their solution processes (see **table 1**). The answers are listed in the order of their frequency. For example, more students answered –19 than –17 for the first problem.

The majority of students arrived at answers such as –19, –26, and 12 for questions 1, 2, and 3, respectively. For example, in the first problem, most students did $5 \bullet 4 = 20$ first; then they moved to the addition of 20 + 1, and then they subtracted 21 from 2. This way of applying PEMDAS is consistent with the way the students answered the second and third problems as well. Van de Walle, Karp, and Bay-Williams (2010, p. 474) mention that PEMDAS can lead students to think that addition is done before subtraction and multiplication comes before division, although this phrase is a commonly used mnemonic. Students in my classes for preservice elementary school teachers typically think of PEMDAS as the strict order of parentheses-exponents-multiplication-division-addition-subtraction.

To help students correct their misunderstanding, I suggested that we do the multiplication first; that is, $2 - 5 \bullet 4 + 1$ becomes $2 - 20 + 1$. Then I asked if they recalled that subtracting a number is the same as adding the additive inverse of the number; for example, $7 - 3 = 7 + (-3)$. Once they agreed, we re-represented $2 - 20 + 1$ as $2 + (-20) + 1$. Then, for quite a long period of time, students discussed the right order of operation for this new expression. This time they found that the expression uses only addition, meaning that whenever both addition and subtraction are used, one can do whichever is first, from left to right. An expression such as $2 + 16 \div 4 \bullet 2 + 8$ can also be re-represented as $2 + 16 \bullet (1/4) \bullet 2 + 8$ by re-representing the division using multiplication. Because $16 \bullet (1/4) \bullet 2$ is all multiplication, it too can be performed from left to right. Eventually my students modified their PEMDAS rules for both multiplication and division and for addition and subtraction. When they focused on the inverse relationship between multiplication and division and between addition and subtraction, they were able to explain why the operations are done in the order of their appearance from left to right.

TABLE 1

In the first content course for preservice teachers, evaluation problems were compiled into a table on the board along with answers the students provided.

	Problem	Student answers	Correct answer
1.	$2 - 5 \bullet 4 + 1$	–19, –17	–17
2.	$2 - 3 \bullet 4 + 5 \bullet 2 - 1 + 5$	–26, 4	4
3.	$2 + 16 \div 4 \bullet 2 + 8$	12, 18, 16	18

PEMDAS story 2

I gave the following problem to my elementary methods students for their mathematics journals:

> Describe a real-life situation that could be represented by the expression $5 + 8 \times 6$.

When I collected the journals, approximately 30 percent of the class had presented situations like the following four problems:

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1. During the weekly art lesson, Ms. Jones, the art teacher, set up craft tables for the next incoming class. On the left side of a table, she put 5 feathers, and on the right side, she put 8 buttons. Students are going to use these objects to decorate picture frames for their moms. Ms. Jones repeated this step for the next 6 tables. How many craft objects are available for her students to use?

2. You were trying to figure out how much you were going to get paid for babysitting for two nights. If you watched the children for 5 hours one day and 8 hours the next day, you would simply add those two numbers together and come up with 13 total hours of babysitting. Let's say that you were paid \$6 an hour for your time. You would multiply 13 times 6, and that would give you 78. You would have figured out that you were paid \$78 for that much work on the weekend.

3. In each row of our garden, my mom wants to plant 5 tomato plants and 8 onion plants. The garden can hold 6 rows. How many plants will be needed to fill the garden with tomatoes and onions? (This problem definitely communicates the expression $5 + 8 \times 6$.)

4. Ms. Harper's classroom has five boys and eight girls, and they each get a total of six cookies for participating in the fundraiser for Hurricane Katrina victims. How many cookies would be given out?

The word problems that the students created with $5 + 8 \times 6$ were, in fact, for the mathematical expression $(5 + 8) \times 6$. The scenarios from other students were similar and represent the same misunderstanding that the students had in the process of taking the mathematical expression into a daily application. It is more interesting to note that these same students could calculate the expression correctly by using PEMDAS. However, when the question was not about getting the numerical answer but finding a real-life situation that is represented by such an expression, they failed to demonstrate their knowledge.

Story-writing activities helped students

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learn and apply the order of operations in a context-based setting. Considering that the predominant school experiences that students have with the order of operations is to simplify expressions by applying the correct order of operations, the story-writing activity was a helpful intervention. I learned that other questions, such as whether parentheses are necessary, are important points to clarify. Moreover, class discussion must include how two operations that produce the same numerical answer can be different in the story-writing activity. For example, How is your story for 6 + 9 – 4 different from a story for 6 + (9 – 4)?

PEMDAS story 3

A function machine was used when students learned about algebraic concepts, especially in generalizing number patterns. At the same time, a table was drawn to process the information:

> Two goes in the function machine. It comes out as three. Four goes in. It comes out as five. Seven goes in. It comes out as eight. Ten goes in. It comes out as eleven.

Students raised a hand when they thought they could correctly guess the rule. Then we recorded the rule (see **table 2**) as 1 greater than the original number. We used the symbol ☐ to represent an input value for the function machine. We recorded the output value using the symbol plus 1: ☐ + 1. (See **table 2** for three particular examples that I chose for our class discussion.) I asked my students how they would find an original number when they knew an output value in the first pattern had the rule ☐ + 1.

Student 1: Subtract 1 from the output value.

How about finding your input value if you know your outputs are 2, 4, 6, 8, and so on, 2 × ☐?
Student 2: Divide the numbers by 2.

How about finding the original number if you know your outputs are 1, 3, 5, 7,…, 2 × ☐ – 3?
Student 3: Divide by 2 and add 3.

[*Pausing for a moment*] **Can you explain why?**
[*Almost every student chants*] PEMDAS!

[*Pausing for a longer moment*] **Let's work with 7, which is the output in this number pattern: 2 × ☐ – 3. I divide it by 2, as you said, which is 3.5, and add 3. Now I have 6.5. It's not the same as 5, which we used as the input value for 7, is it? Work with another output value, and see if dividing by 2 and adding 3 works for it.**
Student 1: No. They don't give the input values back.

Can you think of why it didn't work? In other words, what should we do to find the original number when the output is 2 × ☐ – 3?

The discussion in this classroom vignette shows again the misconception that my students held with PEMDAS. They used a rule for evaluating expressions to solve an equation. Once we confirmed that dividing by 2 and then adding 3 is not the correct way to get our input values, we made an analogy between recording and rewinding a videotape and doing and undoing a series of mathematical operations. For example, when a person is being videotaped as a walker and then as a runner, the rewinding process would be a runner and then a walker in the reverse direction. The movements themselves also get reversed. So when the order of operations is to multiply by 2 and subtract 3, the reversing process should reverse the order so that adding 3 is the first operation to perform, and dividing by 2 is the second step to follow. We made it clear that we must use the inverse of each operation as well as reverse the order of operation. As an extension, I asked students whether the following two expressions are the same:

$2n - 3$
$(n - 3) \times 2$

TABLE 2

These values were used in the function machine activity.

Input	Output	Input	Output	Input	Output
2	3	1	2	2	1
4	5	2	4	3	3
8	9	3	6	4	5
10	11	4	8	5	7
…	…	…	…	…	…
☐	☐ + 1	☐	2 × ☐	☐	2 × ☐ –3

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I did not give students the written expressions as they are printed here. Instead, I provided a verbal description: "You will multiply a number by 2 and subtract 3 from the product. Is it the same if you subtract 3 from a number and then multiply that difference by 2?" The initial response was that the two verbal expressions are the same. When I wrote the formal expressions on the board, many of the students said they were changing their mind: The two expressions were not the same. PEMDAS did help them see the difference between the two expressions in the written format, $2n - 3$ and $(n - 3) \times 2$. PEMDAS could also help them to learn how the difference in the order of the operations in the two expressions results in two different algebraic relationships.

Helpful tips for teachers

Bassarear does an excellent job of intentionally bringing up the potential problem of the use of PEMDAS when computations use more than one operation: "Do this problem with pencil and paper and then on your calculator: $3 \times 4 - 8 \div 2$." If a student does each operation in the order in which it appears in the problem, she will get the answer of 2. However, a calculator will yield the answer of 8. The teacher can then emphasize the fact that getting two different answers from the same problem has led to rules called the *order of operations*. These rules help one decide the order so that each expression can have only a single value. Of course, most calculators are programmed to obey the order of operations (Bassarear 2005, p. 199).

Another helpful activity for children would be to have them experience that different answers come up depending on different placements of parentheses:

$$(3 \times 4) - (8 \div 2)$$
$$3 \times (4 - 8) \div 2$$

Explore how many different answers exist. Also explore why an expression that has parentheses

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yields the same answer as another expression that does not have parentheses:

$(3 \times 4) - (8 \div 2)$
$3 \times 4 - 8 \div 2$

Help them understand the efficiency, that is, how the order of operations eliminates the need to use parentheses. Emphasize that the focus of discussion is not on the correct answer but on reasoning about what is happening when an expression is written in one way or another.

You can also have children insert parentheses to make an expression true. For example, have them insert parentheses so that these two expressions are true:

$2 + 3 \times 6 + 9 = 47$
$2 + 3 \times 6 + 9 = 39$

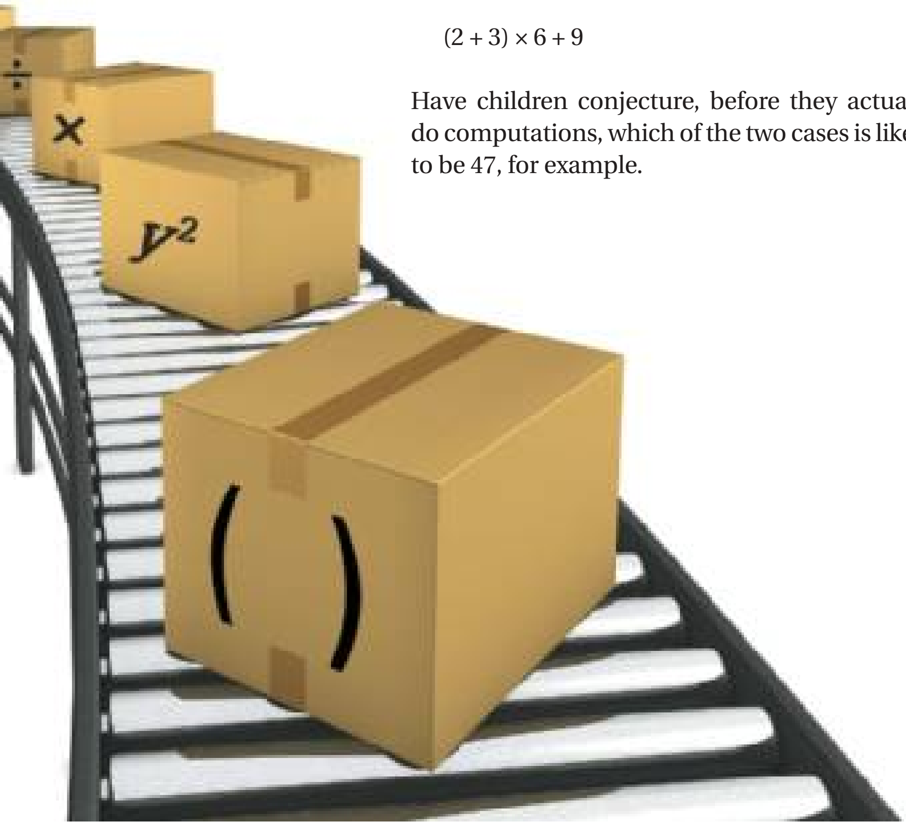
I recommend using a calculator because it may support young children's development of number sense and operation sense. Placement of parentheses around multiplication—

$2 + 3 \times (6 + 9)$

—will yield a large value in comparison to placement of parentheses around addition, which will yield a smaller value:

$(2 + 3) \times 6 + 9$

Have children conjecture, before they actually do computations, which of the two cases is likely to be 47, for example.



Van de Walle, Karp, and Bay-Williams (2010) comment that PEMDAS is a useful mnemonic. However, they point out that it can lead students to think that addition is done before subtraction and that multiplication comes before division (p. 475). Golembo (2000) suggests that an improvement might be writing the mnemonic in rows to indicate order:

P	Parenthesis
E	Exponents
MD	Multiplication or Division (whichever is first from left to right)
AS	Addition or Subtraction (which is first from left to right)

The universally agreed-on system helps students see that the order of multiplication and division is interchangeable, as is that for addition and subtraction, as long as they perform the operations from left to right (p. 575).

Be aware of the fact that many times students' only experience is to simplify expressions or evaluate an expression by applying the order of operations (Van de Walle, Karp, and Bay-Williams 2010). So having children write expressions that indicate the proper order of operations using story-writing activities would be helpful for both a teacher's own assessment process and for children's understanding of the rule. Encourage children to use a context of their choice to fit the expression they have created. The following is my scenario, for example, for my student who shared her second-grade niece's story with 7 – 3 + 11 in the beginning of this article:

> So, you think addition should be done first, and then subtraction is done later. Let me give you a story, and you write an expression while I give you the story.
>
> "Hannah went to pick apples at an orchard. She picked 7 apples and gave 3 apples to her younger sister, Erin. Then their mom gave Hannah 11 apples. How many apples does Hannah have now?"

My student wrote 7 – 3 + 11, and then she thought for a while and said that the answer should be 15. More important, she realized that the subtraction had to be done first—not by

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using PEMDAS but by using the context and the understanding about her own expression.

At the beginning of the article, I cited Reys and Fennell's suggestion to bring real change at the elementary school level. They also suggested that this change should be directed at emphasizing a rigorous curriculum focused on *understanding* important concepts and developing proficiency in skills required to solve problems (2003, p. 278). In my opinion, *teaching for understanding* should be the key element of teaching mathematics. PEMDAS, in this sense, surprised me many different times in my own classroom. However, its use definitely taught me a lot about the way I should examine problems that my students bring to the classroom.

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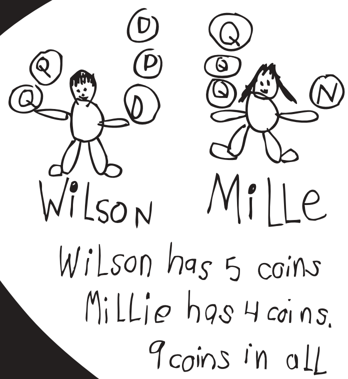

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