Nothing "Basic" about Basic Facts:

Exploring Addition Facts with Fourth Graders

To many, the word *basic* implies something that is "simple," "straightforward," or "easy." In mathematics classrooms, however, teaching and learning the "basic" facts is anything but simple. Helping students achieve mastery of basic facts—meaning that students are able to give a quick response (in 3 seconds or less) without resorting to an inefficient computational strategy, such as counting (Van de Walle 2001)—has proved to be a challenge. It is not uncommon for teachers to encounter eighth-grade students who still struggle to master the addition facts, even though those facts are typically introduced in the first grade.

The twenty-two students in one of the authors' (McGee's) fourth-grade class were no exception. Asked to solve an addition fact, such as 8 + 6, these fourth graders resorted to finger counting, tallying, head bobbing, and touch-point counting. These methods of computing, although appropriate in the earlier grades when students are first introduced to the addition facts, are not effective when, in the upper grades, they are asked to add larger numbers and work with multiplication facts. Not surprisingly, such inefficient methods of computing tend to get in the way of learning concepts and skills that depend on being able to compute accurately, efficiently, fluently, and flexibly.

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In this article we share our explorations of the challenges in, and possibilities for, helping fourth graders improve their computational proficiency of addition facts. This investigation entailed first assessing and then designing instruction that addressed the students' difficulties with the addition facts. Here we share our detailed insights into these fourth graders' difficulties, as well as the effects of the instructional strategies and activities we designed and implemented. We begin by presenting a brief overview of different strategies for solving addition facts and ways of helping students become more efficient at solving them.

Strategies for Solving Addition Facts

When students are presented with an addition fact, such as 8 + 6, aside from recalling it from memory, students can resort to several strategies—direct modeling, counting, and deriving the fact (see **table 1**). Depending on which strategy they use, solving any of the facts in **figure 1** can be more or less difficult. For students who rely on direct modeling, figuring out any of these facts can be time-

consuming because they would first have to count each number separately and count them again once the two are put together. Counting on from the larger of the two numbers, by comparison, is a more efficient strategy than direct modeling, especially when the number to be added on is less than 5, but it becomes time-consuming and less efficient with addends greater than 5.

Using the derived-fact strategy with addends that are close together or when the second addend is greater than 5 is not only a more efficient method but also a more powerful strategy. Consider, for instance, how the knowledge of 8 represented as "5 and 3 more" or 8 represented as "2 away from 10" might help you figure out the following:

5 + 3	8 + 5	8 + 6
8 – 3	8 – 4	13 – 8

Computational strategies such as those described above are the stepping-stones toward developing computational proficiency, or what the National Research Council (NRC) (2001) calls *procedural fluency.* It means being skillful in carrying out procedures flexibly, accurately, efficiently, and appropriately. Procedural fluency is intertwined with the other strands that formulate what the NRC calls *mathematical proficiency,* and as such, it is not to be learned or taught separately from the other strands.

Instruction that aims to help students learn and master the basic facts, therefore, should aim to develop students' procedural fluency as much as it should aim to develop the other strands of mathematical proficiency. These strands include *conceptual understanding*—comprehension of mathematical concepts, operations, and relations; *strategic competence*—ability to formulate, represent, and solve mathematical problems; *adaptive reasoning*—capacity for logical thought, reflection, explanation, and justification; and *productive disposition*—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Table 1

Strategies for solving addition facts

Direct Modeling	Counting On	Derived Facts
Joining All 4 + 5 = ____ Makes a set of 4 (cubes, fingers, or tallies) and another set of 5. Joins the two sets, and counts the joined sets by pointing at each object with each count.	**From First Number** 4 + 5 = ____ Starts counting on from the first addend, then counts 5, 6, 7, 8, 9. The answer is the last number said.	**Doubles** 4 + 7 = ____ Sees 4 + 7 in relation to 4 + 4. Because 7 is three more than 4, add 3 more to 4 + 4, getting 11.
Adding On 4 + 5 = ____ Models the two addends and counts only the second set. Says 4, then points at each object in the second set and counts on.	**From Larger Number** 4 + 5 = ____ Starts counting from the larger number, then counts on.	**Doubles + 1** 6 + 7 = ____ Realizes that 6 + 7 is one more than 6 + 6 , so 12 + 1 is 13. **Sums to 10** 6 + 8 = 6 + 4 + 4, or 8 + 2 + 4.

Figure 1

Which is easiest/hardest?

$$\begin{array}{r} 6 \\ +\,6 \\ \hline \end{array} \qquad \begin{array}{r} 7 \\ +\,3 \\ \hline \end{array} \qquad \begin{array}{r} 5 \\ +\,7 \\ \hline \end{array} \qquad \begin{array}{r} 7 \\ +\,9 \\ \hline \end{array}$$

To help students gain such computational proficiency with basic facts, teachers have two choices. One choice is to provide practice that aims at memorizing the facts, or what Brownell (1956/1987) called *repetitive practice*. The other choice is to provide practice that aims at developing increasingly more efficient strategies for fact retrieval, what Brownell called *varied practice*. To further explore this distinction, let us imagine two classrooms in which students are practicing their addition facts using the typical one hundred addition-facts worksheet. In the "repetitive practice" class, the timer is set and students compete either with one another or with their own "personal best" to get the most correct answers in the least amount of time—the focus is on developing quick, automatic, and correct answers to each individual addition fact. In contrast, the "varied practice" class focuses on noticing relationships between the facts and figuring out what strategies could be used to solve which facts. In this class students are asked, for example, to circle all the facts they could solve by using the make-a-ten strategy.

Repetitive practice that focuses on timed tests, flash cards, and drills can produce the desired results—students answer accurately and quickly, as many teachers can attest. The difficulty with this kind of practice, however, is that it does not draw students' attention to connections among and beyond the facts and to computing methods that are meaningful or useful later on. Varied practice, by contrast, focuses on explicit practice of strategies—how they work and when to use them (for example, how you can use 5 + 5 to help you figure out what 8 + 5 is)—and also helps students learn their basic facts but in a qualitatively different way. Students who learn their basic facts through varied practice learn not only the facts but also a way of thinking and working in mathematics that is very useful beyond the context of learning basic facts.

Exploring Students' Difficulties

As mentioned previously, students in McGee's fourth-grade class were having difficulties solving the simplest of addition facts. To learn more about the source of their difficulties, we designed an addition-facts test (see **fig. 2**). The test included facts that could be solved using derived-facts strategies, such as using doubles or near doubles (highlighted in blue), and facts or near-facts that could be solved using the make-a-ten-or-higher-decade strategy (highlighted in yellow).

Notice that we included a row of near-addition facts (having one of the addends greater than 9). These facts were meant to help us learn whether the students would use known addition facts to solve the near-facts and whether they would solve them in ways similar to, or different from, the actual addition facts. Also notice that the students were asked not to use their fingers but to circle the facts that were the hardest. Although they were not penalized for using fingers or pressured to complete the test within a set amount of time, both of these factors were noted. The purpose of the test, the teacher assured her students, was to find out which facts were most troublesome to them and to use that information to help them get better at figuring the facts out.

A close look at the students' responses revealed that McGee's class had two kinds of difficulties when solving addition facts—difficulties with accuracy and difficulties with their choice of strategy. In terms of accuracy, we found that only eight of the twenty-two students (36 percent) answered correctly fourteen to fifteen of the fifteen addition facts in the pretest. Furthermore, we found that six students (27 percent) correctly completed only eight or fewer of the facts and that one student could solve only one of the addition questions.

The most difficult number sentences (left blank, circled, or solved incorrectly) were two of the near-facts, 19 + 8 (by thirteen students) and 15 + 6 (by twelve students), which could be solved with the make-a-ten-or-higher-decade strategy. The most difficult facts were 5 + 7 (for nine students) and 8 + 6 (for eight students), both of which could be solved by using doubles. These responses and the amount of time students took to complete the pretest suggested that they were using inefficient strategies to solve addition facts. Classroom observations also revealed that the fourth graders resorted to various inefficient counting methods rather than to derived-fact strategies.

Figure 2

Addition-facts pretest and posttest

NAME: ______________________ GRADE: __________

Solve the following computations without using your fingers.
Circle the ones that were the hardest.

2 + 2	2 + 4	3 + 5	5 + 7	7 + 9
6 + 6	7 + 3	9 + 2	8 + 6	10 + 7
13 + 7	5 + 16	15 + 6	18 + 6	19 + 8

Figure 3

Practice through problem solving

Exploring Doubles

Choose a number and add it to itself. Now make the first number one more and make the second number one less. What number did you get? Try it with other numbers and see what happens. What would happen if you make the first number two more and make the second number two less?

- What is the pattern?
- Can you explain what is happening?
- How is this helpful?

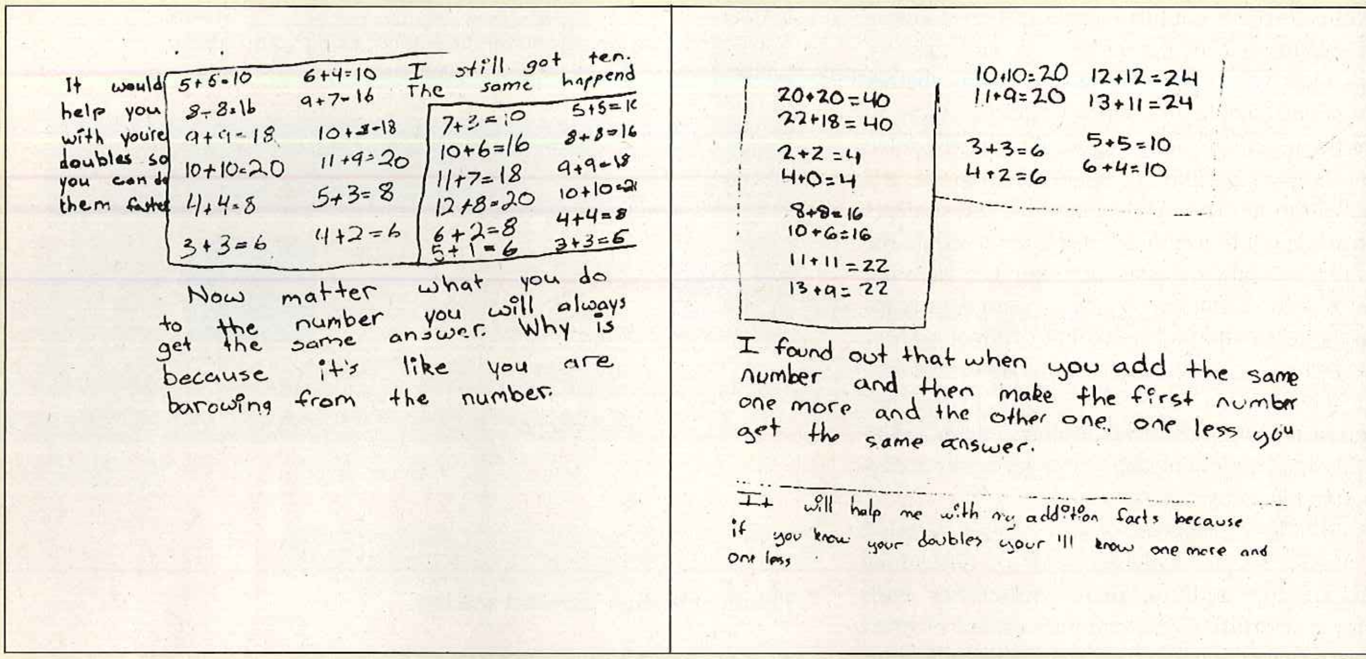
First Teaching Experiment: Practice through Problem Solving

Following Van de Walle's (2001) and NCTM's (2000) views, we considered the learning of basic facts as a problem-solving activity and therefore used the "exploring doubles" problem (see **fig. 3**) to introduce students to the using-doubles strategy. This problem aims to develop all of the NRC's (2001) mathematical proficiency strands that we discussed previously. Different from typical drill and practice, this particular problem allows students to choose the pairs of numbers they want to work with and focus their attention on patterns and relationships between different number sentences. The problem's questions also encourage students to think about how they could use what they might uncover in their explorations.

Figure 4 shows two samples of the students' work on this problem. Close examination of students' work revealed that this task helped students not only practice single-digit and multidigit addi-

Figure 4

Students' responses to "exploring doubles"



tions but also notice patterns and relationships between their number sentences. We were particularly encouraged by hearing students discuss the usefulness of this strategy (adding and subtracting the same number to each double) to figure out addition questions. One stated, "Well, now in math I will know what's the answer to what. I don't have to use my hands," and another said, "It's helping you, because what if you didn't know, say, 8 + 8, just do the plan of highering one and lowering the other and you will know more about addition."

This last statement is intriguing, and although we are not certain of what the student really meant (because this student chose not to share her thinking during class discussion), we can imagine using this particular procedure to derive unknown facts. Imagine, as the student suggested, not knowing the answer to 8 + 8. One can use the procedure "make one number higher and lower the other (by the same amount)" to change the addends until finding a number sentence that is easier to solve. That is, first change the problem mentally to 9 + 7, which might not be any easier, and then to 10 + 6, which is much easier to solve. Knowing that this adding procedure produces addition number sentences with the same answer is indeed very useful.

Second Teaching Experiment: Explicit Practice of Strategies

The second teaching experiment that we conducted was inspired by Brownell's (1956/1987) notion of varied practice and focused on the explicit practice of strategies to solve addition facts. Explicit teaching and practice of strategies is not a new idea. Mathematics educators have for a few decades advocated the idea of directly teaching strategies to children (for example, Baroody [1985]; Rathmell [1978]; Steinberg [1985]; Thornton and Toohey [1984]). Such explicit teaching of strategies, however, is not a common practice, at least not in the United States. Fuson (2003) reports that although in many countries children are taught general thinking strategies, such as "make a ten," in the United States and Canada children seldom invent these methods or learn them in textbooks. Yet the explicit teaching of strategies as is done in other parts of the world, as Fuson suggests, helps children chunk smaller numbers into larger numbers and use thinking strategies to convert an addition fact that they do not know into an addition fact that they do know.

To provide McGee's students with explicit practice of the using-doubles strategy, we designed the "working with doubles" worksheet (see **fig. 5**). In this worksheet the students were asked to (a) circle the number sentences they could solve by using a given double and (b) write two number sentences they could solve using the given double. Note that the teacher did not simply give students this worksheet without any instruction. Instead, she first introduced the exercise by working a few simpler examples with the class, and concluded the session by conducting a discussion to consolidate the students' learning.

As students worked on the worksheet, McGee noticed that some students were circling only the number sentences that equaled the sum of the given double (including those with an addend of 1 or 2). She further noticed that these students were neglecting to circle the near-double addition problems having sums of one or two more than the given double. McGee interrupted the students' individual practice and called this issue to their attention by asking, for example, whether they would use 5 + 5 to solve 9 + 1.

Our subsequent examination of the students' work on this particular worksheet suggests that a few students, like the first student in **figure 6**, in spite of the foregoing interjection, (a) continued to solve all the addition sentences even though completing the exercise was unnecessary and (b) continued to circle number sentences that had a 1 or a 2 as an addend. Most students, however, showed work that, like the work of the second student in **figure 6**, suggested that they understood the point of the exercise; that is, that using doubles is helpful when the addends are "high" and that it is most useful when addends are close together. In the second half of the worksheet, all the students wrote number sentences that were near doubles with sums that were the same as, and different from, the given doubles.

In the postexperiment discussion, we found further evidence of, and opportunity for, students' insights. McGee began the discussion by asking students which additions were easier to solve by counting up. On the chalkboard she posted 8 + 5 or 10 + 3, followed by 9 + 2 or 6 + 5. Everyone agreed that counting up would be useful to add an addend that is less than 5. Then unexpectedly a student stated that "to solve 6 + 5, what I would do is make 6 + 4 . . . equals 10, and plus 1." McGee took this student's contribution as an opportunity to name and make explicit the make-a-ten strategy. She asked the students, "Which is easier to add, 8 + 5 or 10 + 3?" and repeated the question for other examples, such as 8 + 4 (or 10 + 2) and 9 + 5 (or 10 + 4).

Figure 5

Explicit practice of strategies

Working with Doubles

Name: __________

Circle the number sentence you can solve by using the double as a guide.

Double						
(1)	5 + 5	9 + 1	6 + 5	4 + 2	7 + 3	6 + 4
(2)	8 + 8	6 + 10	8 + 4	9 + 8	10 + 7	7 + 9

Write 2 number sentences you can solve using the following doubles.

(3)	4 + 4	A.	B.
(4)	7 + 7	A.	B.

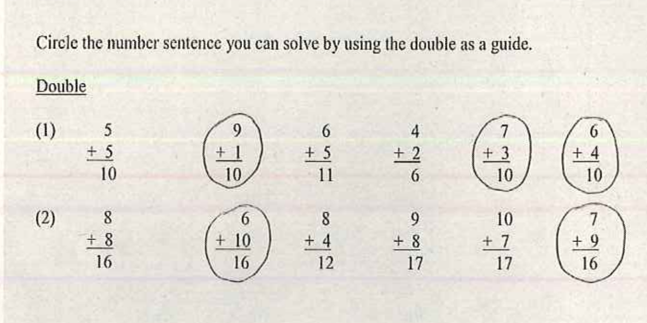
To conclude the discussion, McGee wrote on the chalkboard the facts 3 + 5, 4 + 6, 5 + 7, 6 + 8, and 7 + 9 and asked students to indicate which double they would use to solve each one. In a chorus students answered, "4 doubled, 5, 6, 7, and 8 doubled," and the teacher wrote these responses on the chalkboard. When asked if they had noticed any patterns, a fourth grader pointed out that "the answer was the number in between, doubled," meaning that the number between the two addends doubled. The teacher then identified this approach as another useful strategy. Repeating what her student had said, she explained that when the two addends differ by 2 (that is, are in a skipped-by-1 sequence), for example, 3 + 5 and 4 + 6, an easy way to find the answer quickly is to double the (skipped) number in between.

Exploring Students' Learning

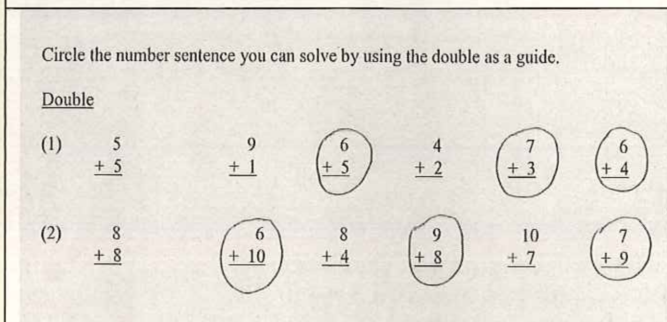
After six sessions of explicit practice of such strategies as the ones reported here (conducted once or twice a week over a period of a month), McGee's students completed the addition-facts posttest. Students' responses in the posttest and

Figure 6

Students' responses to "working with doubles"



Sample 1: Student solves all additions and circles only those with the same sum as the given double.



Sample 2: Student circles number sentences with addends that differ by one or two from the given double.

posttest interviews were very encouraging. All students improved their accuracy, that is, eighteen of the twenty-two students (82 percent) solved correctly fourteen or fifteen of the fifteen addition facts, and everyone was able to solve correctly twelve or more of the test's addition facts. Most encouraging was to find that the one student who in the pretest had been able to solve only one addition fact could correctly solve fourteen of the fifteen addition sentences in the posttest. We also noted that students completed the posttest much quicker than before and did not seem to use finger-counting strategies. These results suggest that these fourth graders had become more efficient at selecting and using strategies to solve addition facts.

When we interviewed six of the fourth graders with the lowest pretest scores, we also found more evidence that they had become more proficient and deliberate in their use of strategies. When asked how they had solved particular number sentences, students spoke about using the counting-up, using-doubles, and make-a-ten strategies. The following exchange shows that these fourth graders were naming the strategy they were using and were able to explain how they were using it.

Interviewer. Can you tell me how you solved 2 + 4?

Student. I knew 2 + 2, so I just added 2.

Interviewer. How about 9 + 7?

Student. I used 8 + 8 is 16; I used the double.

Interviewer. Why did you use 8 + 8?

Student. Seven is taking away 1 from 8, and 9 is 1 + 8.

Conclusion

Our investigation into the use of varied practice, although based on a small sample of fourth graders who were struggling with their addition facts, suggests that this approach can lead to significant improvements. Our teaching experiments show that varied practice that focuses on problem solving and on the explicit practice of strategies can help students develop the kind of mathematical proficiency advocated by the NRC (2001). Our data suggest that McGee's fourth graders improved their computational accuracy, efficiency, and flexibility (what the NRC calls *procedural fluency*) as well as other aspects of the NRC's mathematical proficiency—conceptual understanding, strategic competence, adaptive reasoning, and productive disposition.

Designing varied practice that focuses on problem solving entails searching for and constructing problems that focus students' attention on number patterns and additive relationships. Number riddles, such as the "exploring doubles" problem that we featured previously, invite students to notice and explore relationships between and among number sentences. As we noted, unlike repetitive practice with exercise drills, varied practice through such number-riddle problems entices students not only to solve individual facts but to search for possible relationships among them.

The explicit practice of strategies also requires careful selection and construction of questions that focus students' attention away from merely solving individual addition facts and, instead, promote using a known fact to figure out the unknown ones. The "working with doubles" worksheet can easily be adapted to practice other doubles, and it can be adapted to the make-a-ten strategy (for example, how does thinking of 10 as "7 and 3 more" help you when solving the following addition facts?). Although finding and designing tasks with a focus on varied practice and making time for students to work on them could be challenges, especially in the upper grades, our findings suggest that this extra time is well worth our efforts.

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