Properties

NCTM Standards

Research has shown that learning about number and operations is a complex process for children (e.g., Fuson). p. 32



Closure Property for Addition If you were to select any two whole numbers, their sum would be another whole number. This fact is expressed by saying that the whole numbers are **closed for the operation of addition.** In general, the word *closed* indicates that when an operation is performed on any two numbers from a given set, the result is also in the set, rather than outside the set. For example, the set of whole numbers is not closed for subtraction, because sometimes the difference between two whole numbers is a negative number. Consider another example. If we select any two numbers from the set of odd numbers $\{1, 3, 5, 7, \ldots\}$, the sum is not another odd number. So the set of odd numbers is not closed for addition. To test for closure, students sometimes find it helpful to draw a circle and write the numbers from a given set inside. Then if the set is *closed*, the results of the operation will be inside the circle. If the given operation produces *at least one* result that is outside the circle, the set is *not closed* for the given operation.

Closure Property For every pair of numbers in a given set, if an operation is performed, and the result is also a number in the set, the set is said to be **closed for the operation.** If one example can be found where the operation does not produce an element of the given set, then the set is **not closed for the operation.**

EXAMPLE E

Determine whether the set is closed or not closed for the given operation.

- **1.** The set of odd numbers for subtraction.
- **2.** The set of odd numbers for multiplication.
- **3.** The set of whole numbers for division.

Solution 1. The set of odd numbers is not closed for subtraction. For example, 23 - 3 is not an odd number. 2. The set of odd numbers is closed for multiplication; the product of any two odd numbers is another odd number. 3. The set of whole numbers is not closed for division. For example, $\frac{2}{3}$ is not a whole number.

Identity Property for Addition Included among the whole numbers is a very special number, zero. Zero is called the **identity for addition** because when it is added to another number, there is *no change*. That is, adding 0 to any number leaves the identity of the number unchanged. For example,

$$0 + 5 = 5 \qquad 17 + 0 = 17 \qquad 0 + 0 = 0$$

Zero is unique in that it is the only number that is an identity for addition.

Identity Property for Addition For any whole number *b*,

0 + b = b + 0 = b

and 0 is the unique identity for addition.

Associative Property for Addition In any sum of three numbers, the middle number may be added to (associated with) either of the two end numbers. This property is called the **associative property for addition**.

EXAMPLE F

$$\underbrace{147 + (20 + 6)}_{\uparrow} = \underbrace{(147 + 20) + 6}_{\uparrow}$$

Associative property for addition

Associative Property for Addition For any whole numbers *a*, *b*, and *c*,

a + (b + c) = (a + b) + c

When elementary school students compute by breaking a number into a convenient sum, as in Example G, the *associative property of addition* plays a role. Arranging numbers to produce sums of 10 is called *making 10s*.

EXAMPLE G

$$8 + 7 = \underbrace{8 + (2 + 5)}_{\uparrow} = \underbrace{(8 + 2) + 5}_{\uparrow} = 10 + 5 = 15$$

Associative property for addition

Commutative Property for Addition When two numbers are added, the numbers may be interchanged (commuted) without affecting the sum. This property is called the **commutative property for addition.**

EXAMPLE H

$$\underbrace{257 + 498}_{\uparrow} = \underbrace{498 + 257}_{\uparrow}$$

Commutative property for addition

Commutative Property for Addition For any whole numbers *a* and *b*,

a+b=b+a

As the addition table in Figure 3.5 on the next page shows, the commutative property for addition roughly cuts in half the number of basic addition facts that elementary school students must memorize. Each sum in the shaded part of the table has a corresponding equal sum in the unshaded part of the table.



onnection						♥				
		+	0	1	2	3	4	5	6	
t num-		0	0	1	2	3	4	5	6	
n the larger.		1	1	2	3	4	5	6	7	
nued, will e a palin-		2	2	3	4	5	6	7	8	
ie 3.2		3	3	4	5	6	7	8	9	
and		4	4	5	6	7	8	9	10	
naking		5	5	6	7	8	9	10	11	/
		6	6	7	8	9	10	11	12	
		7	7	8	9	10	11	12	13	
	→	8	8	9	10	(11)	12	13	14	
		9	9	10	11	12	13	14	15	

EXAMPLE I

If we know that 3 + 8 = 11, then, by the commutative property for addition, 8 + 3 = 11. What do you notice about the locations of these sums in the addition table?

Solution The sums of 3 + 8 and 8 + 3 are in opposite parts of the table. If the shaded part of the table is folded onto the unshaded part of the table, these sums will coincide. That is, the table is symmetric about the diagonal from upper left to lower right.

The commutative property also enables us to select convenient combinations of numbers when we are adding.

EXAMPLE J

The numbers 26, 37, and 4 are arranged more conveniently on the right side of the following equation than on the left, because 26 + 4 = 30 and it is easy to compute 30 + 37.

$$26 + 37 + 4 = 26 + 4 + 37 = (26 + 4) + 37 = 30 + 37$$

Commutative property

for addition

INEQUALITY OF WHOLE NUMBERS

The inequality of whole numbers can be understood intuitively in terms of the locations of numbers as they occur in the counting process. For example, 3 is less than 5 because it is named before 5 in the counting sequence. This ordering of numbers can be illustrated with a number line. A number line is formed by beginning with any line and marking off two points, one labeled 0 and the other labeled 1, as shown in Figure 3.6 on the next page. This unit segment is then used to mark off equally spaced points for consecutive whole numbers. For any two numbers, the one that occurs on the left is less than the one that occurs on the right.

HISTORICAL HIGHLIGHT

\rightarrow	1	\times	52	=	52
\rightarrow	2	\times	52	=	104
	4	\times	52	=	208
\rightarrow	8	\times	52	=	416
	_	1 ⊢4 5'	52 04 <u>16</u> 72		

One of the earliest methods of multiplication is found in the Rhind Papyrus. This ancient scroll (ca. 1650 B.C.E.), more than 5 meters in length, was written to instruct Egyptian scribes in computing with whole numbers and fractions. Beginning with the words "Complete and thorough study of all things, insights into all that exists, knowledge of all secrets . . .," it indicates the Egyptians' awe of mathematics. Although most of its 85 problems have a practical origin, there are some of a theoretical nature. The Egyptians' algorithm for multiplication was a succession of doubling operations, followed by addition as shown in the example at the left. To compute 11×52 , they would repeatedly double 52, then add *one* 52, *two* 52s, and *eight* 52s to get *eleven* 52s.

NUMBER PROPERTIES

Four properties for addition of whole numbers were stated in Section 3.2. Four corresponding properties for multiplication of whole numbers are stated below, along with one additional property that relates the operations of addition and multiplication.

Closure Property for Multiplication This property states that the product of any two whole numbers is also a whole number.

Closure Property for Multiplication For any whole numbers *a* and *b*,

 $a \times b$ is a unique whole number.

Identity Property for Multiplication The number 1 is called an **identity for multiplication** because when multiplied by another number, it leaves the identity of the number unchanged. For example,

 $1 \times 14 = 14$ $34 \times 1 = 34$ $1 \times 0 = 0$

The number 1 is unique in that it is the only number that is an identity for multiplication.

Identity Property for Multiplication For any whole number *b*,

 $1 \times b = b \times 1 = b$

and 1 is the unique identity for multiplication.

Commutative Property for Multiplication This number property says in any product of numbers, two numbers may be interchanged (commuted) without affecting the product. This property is called the **commutative property for multiplication.** For example,

$$347 \times 26 = 26 \times 347$$

Commutative property for multiplication Commutative Property for Multiplication For any whole numbers *a* and *b*,

 $a \times b = b \times a$

NCTM Standards

Using area models, properties of operations such as commutativity of multiplication become more apparent. p. 152 The commutative property is illustrated in Figure 3.17, which shows two different views of the same rectangular array. Part (a) represents 7×5 , and part (b) represents 5×7 . Since part (b) is obtained by rotating part (a), both figures have the same number of small squares, so 7×5 is equal to 5×7 .



As the multiplication table in Figure 3.18 shows, the commutative property for multiplication approximately cuts in half the number of basic multiplication facts that elementary school students must memorize. Each product in the shaded part of the table corresponds to an equal product in the unshaded part of the table.

EXAMPLE B

Since $3 \times 7 = 21$, we know by the commutative property for multiplication that $7 \times 3 = 21$. What do you notice about the location of each product in the shaded part of the table relative to the location of the corresponding equal product in the unshaded part of the table?

Solution If the shaded part of the table is folded along the diagonal onto the unshaded part, each product in the shaded part will coincide with an equal product in the unshaded part. In other words, the table is symmetric about the diagonal from upper left to lower right.

×	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21)	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

Figure 3.18

Notice that the numbers in the rows of the multiplication table in Figure 3.18 form arithmetic sequences, for example, 2, 4, 6, 8, \ldots and 3, 6, 9, 12. \ldots One reason that children learn to count by 2s, 3s, and 5s is to acquire background for learning basic multiplication facts.

Figure 3.17

Associative Property for Multiplication In any product of three numbers, the middle number may be associated with and multiplied by either of the two end numbers. This property is called the **associative property for multiplication**. For example,

$$\underbrace{6 \times (7 \times 4)}_{\uparrow} = \underbrace{(6 \times 7) \times 4}_{\uparrow}$$
Associative property
for multiplication

Associative Property for Multiplication For any whole numbers
$$a$$
, b , and c ,
 $a \times (b \times c) = (a \times b) \times c$

Figure 3.19 illustrates the associative property for multiplication. Part (a) represents 3×4 , and (b) shows 5 of the 3×4 rectangles. The number of small squares in (b) is $5 \times (3 \times 4)$. Part (c) is obtained by subdividing the rectangle (b) into 4 copies of a 3×5 rectangle. The number of small squares in (c) is $4 \times (3 \times 5)$, which, by the commutative property for multiplication, equals $(5 \times 3) \times 4$. Since the numbers of small squares in (b) and (c) are equal, $5 \times (3 \times 4) = (5 \times 3) \times 4$.



The commutative and associative properties are often used to obtain convenient combinations of numbers for mental calculations, as in Example C.

EXAMPLE C

Figure 3.19

Try computing $25 \times 46 \times 4$ in your head before reading further.

Solution The easy way to do this is by rearranging the numbers so that 25×4 is computed first and then 46×100 . The following equations show how the commutative and associative properties permit this rearrangement.

Distributive Property When multiplying a sum of two numbers by a third number, we can add the two numbers and then multiply by the third number, or we can multiply each number of the sum by the third number and then add the two products.

For example, to compute $35 \times (10 + 2)$, we can compute 35×12 , or we can add 35×10 to 35×2 . This property is called the **distributive property for multiplication** over addition.

$$35 \times 12 = \underbrace{35 \times (10 + 2)}_{\uparrow} = \underbrace{(35 \times 10) + (35 \times 2)}_{\uparrow}$$

Distributive property

Distributive Property for Multiplication over Addition For any whole numbers *a*, *b*, and *c*,

$$a \times (b + c) = a \times b + a \times c$$

One use of the distributive property is in learning the basic multiplication facts. Elementary schoolchildren are often taught the "doubles" (2 + 2 = 4, 3 + 3 = 6, 4 + 4 = 8, etc.) because these number facts together with the distributive property can be used to obtain other multiplication facts.

EXAMPLE D How can $7 \times 7 = 49$ and the distributive property be used to compute 7×8 ?

Solution

$$7 \times 8 = \underbrace{7 \times (7+1)}_{\bigcirc} = \underbrace{49+7}_{\bigcirc} = 56$$

Distributive property

The distributive property can be illustrated by using rectangular arrays, as in Figure 3.20. The dimensions of the array in (a) are 6 by (3 + 4), and the array contains 42 small squares. Part (b) shows the same squares separated into two rectangular arrays with dimensions 6 by 3 and 6 by 4. Since the number of squares in both figures is the same, $6 \times (3 + 4) = (6 \times 3) + (6 \times 4)$.





Order of Operations

Compatible Numbers for Estimation Using compatible numbers becomes a powerful tool for estimating products when it is combined with techniques for performing mental calculations. For example, to estimate $4 \times 237 \times 26$, we might replace 26 by 25 and use a different ordering of the numbers.

 $4 \times 237 \times 26 \approx 4 \times 25 \times 237 = 100 \times 237 = 23,700$

EXAMPLE J

Use compatible numbers to estimate these products.

- **1.** $2 \times 117 \times 49$
- **2.** $34 \times 46 \times 3$

Solution 1. $2 \times 117 \times 49 \approx 2 \times 117 \times 50 = 100 \times 117 = 11,700$. 2. $34 \times 46 \times 3 = (3 \times 34) \times 46 \approx 100 \times 46 = 4600$.

Front-End Estimation This technique is similar to that used for computing sums. The leading digit of each number is used to obtain an estimated product. To estimate 43×72 , the product of the leading digits of the numbers is $4 \times 7 = 28$, so the estimated product is 2800.

$$43 \times 72 \approx 40 \times 70 = 2800$$

Similarly, front-end estimation can be used for estimating the products of numbers whose leading digits have different place values.

$$61 \times 874 \approx 60 \times 800 = 48,000$$

 EXAMPLE K
 Use front-end estimation to estimate these products.

 1. 64×23 2. 68×87 3. 237×76 4. $30,328 \times 419$

 Solution 1. $64 \times 23 \approx 60 \times 20 = 1200$ 2. $68 \times 87 \approx 60 \times 80 = 4800$ 3. $237 \times 76 \approx 200 \times 70 = 14,000$

 Use front-end estimation to estimate these products.



Order of Operations

Special care must be taken on some calculators when multiplication is combined with addition or subtraction. The numbers and operations will not always produce the correct answer if they are entered into the calculator in the order in which they appear.

EXAMPLE L

Compute $3 + 4 \times 5$ by entering the numbers into your calculator as they appear from left to right.

Solution Some calculators will display 35, and others will display 23. The correct answer is 23 because multiplication should be performed before addition:

 $3 + 4 \times 5 = 3 + 20 = 23$

Mathematicians have developed the convention that when multiplication occurs with addition and/or subtraction, the multiplication should be performed first. This rule is called the **order of operations.**

Technology
ConnectionSome calculators are programmed to follow the order of operations. On this type of calculator, any combination of products with sums and differences and without parentheses can be computed by entering the numbers and operations in the order in which they occur from left to right and then pressing \blacksquare . If a calculator does not follow the order of operations, the products can be computed separately and recorded by hand or saved in the calculator's memory.**EXAMPLE M**Use your calculator to evaluate $34 \times 19 + 82 \times 43$. Then check the reasonableness of your answer by using estimation and mental calculations.

Solution The exact answer is 4172. An estimate can be obtained as follows:

 $34 \times 19 + 82 \times 43 \approx 30 \times 20 + 80 \times 40 = 600 + 3200 = 3800$

Notice that the estimation in Example M is 372 less than the actual product. However, it is useful in judging the reasonableness of the number obtained from the calculator: It indicates that the calculator answer is most likely correct. If $34 \times 19 + 82 \times 43$ is entered into a calculator as it appears from left to right and if the calculator is not programmed to follow the order of operations, then the incorrect result of 31,304 will be obtained, which is too large by approximately 27,000.

PROBLEM-SOLVING APPLICATION

There is an easy method for mentally computing the products of certain two-digit numbers. A few of these products are shown here.

$25 \times 25 = 625$	$24 \times 26 = 624$	$71 \times 79 = 5609$
$37 \times 33 = 1221$	$35 \times 35 = 1225$	$75 \times 75 = 5625$

The solution to the following problem reveals the method of mental computation and uses *rectangular grids* to show why the method works.

Problem

What is the method of mental calculation for computing the products of the two-digit numbers shown above, and why does this method work?

Understanding the Problem There are patterns in the digits in these products. One pattern is that the two numbers in each pair have the same first digit. Find another pattern. **Question 1:** What types of two-digit numbers are being used?

Devising a Plan Looking for patterns may help you find the types of numbers and the method of computing. Another approach is to represent some of these products on a grid. The following grid illustrates 24×26 ; the product is the number of small squares in the rectangle. To determine this number, we begin by counting large groups of squares. There are 6 hundreds.

entered before pressing the exponential key or after, as shown by the following keystrokes.



Numbers that are raised to powers frequently have more digits than the number of places in the calculator's view screen. If you try to compute 4^{15} on a calculator with only eight places in its view screen, there will not be room for the answer in positional numeration. Some calculators will automatically convert to scientific notation when numbers in positional numeration are too large for the view screen (see Section 6.3), and others will print an error message such as *Error* or *E*.

ORDER OF OPERATIONS

The rules for *order of operations*, discussed in Section 3.3, can now be extended to include division and raising numbers to powers. The order of operations requires that numbers raised to a power be evaluated first; then products and quotients are computed in the order in which they occur from left to right; finally, sums and differences are calculated in the order in which they occur from left to right. An exception to the rule occurs when numbers are written in parentheses. In this case, computations within parentheses are carried out first.

EXAMPLE R	Evaluate the following expressions.
	1. $4 \times 6 + 16 \div 2^3$
	2. $4 \times (6 + 16) \div 2^3$
	3. $220 - 12 \times 7 + 15 \div 3$
	4. $24 \div 4 \times 2 + 15$
	 Solution 1. 26 (First replace 2³ by 8; then compute the product and quotient; then add). 2. 11 (First replace 6 + 16 by 22; then replace 2³ by 8; then compute the product and quotient). 3. 141 (First replace 12 × 7 by 84 and 15 ÷ 3 by 5; then compute the difference and sum). 4. 27 (First compute 24 ÷ 4; then multiply by 2; then add 15).
Technology Connection	Calculators that are programmed to follow the order of operations are convenient for com- puting expressions involving several different operations. You may wish to try problem 3 in

Example R on your calculator, entering in the numbers and operations as they appear from left to right and then pressing the equality key, to see if you obtain 141.

PROBLEM-SOLVING APPLICATION

The following problem involves numbers in exponential form and is solved by using the strategies of making a table and finding a pattern.

Problem

There is a legend that chess was invented for the Indian king Shirham by the grand visier Sissa Ben Dahir. As a reward, Sissa asked to be given 1 grain of wheat for the first square of the chessboard, 2 grains for the second square, 4 grains for the third square, then 8 grains, 16 grains, etc., until each square of the board had been accounted for. The king was surprised

Scientific Notation

EXAMPLE Q

Determine approximate percents by replacing the numerators and/or denominators by compatible numbers.

1.
$$\frac{16}{62}$$
 2. $\frac{300}{2490}$ **3.** $\frac{42}{87}$
Solution **1.** $\frac{16}{62} \approx \frac{15}{69} = \frac{1}{4} = 25\%$ or $\frac{16}{62} \approx \frac{16}{64} =$

Solution 1.
$$\frac{16}{62} \approx \frac{15}{60} = \frac{1}{4} = 25\% \text{ or } \frac{16}{62} \approx \frac{16}{64} = \frac{2}{8} = \frac{1}{4} = 25\%.$$
 2. $\frac{300}{2490} \approx \frac{300}{2500} = \frac{3}{25} = \frac{12}{100} = 12\%.$ 3. $\frac{42}{87} \approx \frac{1}{2} = 50\%.$

SCIENTIFIC NOTATION

Very large and very small numbers can be written conveniently by using powers of 10. Consider the following example: Some computers can perform 400,000,000 calculations per second. Using a power of 10, we can write

$$400,000,000 = 4 \times 10^8$$

Decimals that are less than 1 can be written by using negative powers of 10. For example, the thickness of the average human hair, which is approximately .003 inch thick, can be written as

$$.003 = \frac{3}{1000} = \frac{3}{10^3} = 3 \times 10^{-3}$$

where $\frac{1}{10^3} = 10^{-3}$. In general, for any number $x \neq 0$ and any integer *n*,

$$\frac{1}{x^n} = x^{-n}$$

Any positive number can be written as the product of a number from 1 to 10 and a power of 10. For example, 2,770,000,000 can be written as 2.77×10^9 . This method of writing numbers is called **scientific notation**. The number between 1 and 10 is called the **mantissa**, and the exponent of 10 is called the **characteristic**. In the preceding example, the mantissa is 2.77 and the characteristic is 9.

The following table contains examples of numbers written in scientific notation. Fill in the two missing numbers in the table.

	Positional Numeration	Scientific Notation
Years since age of dinosaurs	150,000,000	1.5×10^{8}
Seconds of half-life of U-238	142,000,000,000,000,000	$1.42 imes 10^{17}$
Wavelength of gamma ray (meters)	.000000000003048	$3.048 imes 10^{-13}$
Size of viruses (centimeters)	.000000914	
Orbital velocity of Earth (kilometers per hour)		4.129×10^{4}

Solution .000000914 = 9.14×10^{-7} ; $4.129 \times 10^{4} = 41,290$.

Numbers written in scientific notation are especially convenient for computing. The graph in Figure 6.37 on the next page shows increases in the world's population. It wasn't until 1825 that the population reached 1 billion (1×10^9) ; by 2010 it was 6.9 billion (6.9×10^9) . Since

NCTM Standards

In the middle grades, students should . . . develop a sense of magnitude of very large numbers. For example, they should recognize and represent 2,300,000,000 as 2.3×10^9 in scientific notation and also as 2.3 billion. p. 217

EXAMPLE R

there is about 1.998×10^3 square yards of cultivated land per person, the total amount of cultivated land worldwide, in square yards, is

$$(6.9 \times 10^9) \times (1.998 \times 10^3)$$

Rearranging these numbers and using the rule for adding exponents, we can rewrite this product as

$$(6.9 \times 1.998) \times 10^{12}$$

Finally, we compute the product of the mantissas (6.9×1.998) and write the answer in scientific notation:

$$(6.9 \times 1.998) \times 10^{12} = 13.7862 \times 10^{12} = 1.37862 \times 10^{13}$$

So there is approximately 1.37862×10^{13} , or 13,786,200,000,000 square yards of cultivated land in the world. Notice in the preceding equation that 13.7862 is not between 1 and 10, so we divide by 10 to obtain the mantissa of 1.37862 and then increase the characteristic (the power of 10) from 12 to 13 to obtain an answer in scientific notation.



The preceding example illustrates the method of computing products of numbers in scientific notation: (1) Multiply the mantissas (numbers from 1 to 10); and (2) add the characteristics to obtain a new power of 10. Sometimes, to write an answer in scientific notation, it will be necessary to divide the mantissa by 10 and increase the characteristic by 1, as in the above example.

EXAMPLE S	Compute each product and write the answer in scientific notation.
	1. $(6.3 \times 10^4) \times (5.21 \times 10^3)$
	2. $(1.55 \times 10^4) \times (8.7 \times 10^{-6})$
	Solution 1. $(6.3 \times 5.21) \times (10^4 \times 10^3) = 32.823 \times 10^7$, but since 32.823 is not between 1 and 10, a requirement for scientific notation, we replace it by 3.2823×10 .
	$32.823 \times 10^7 = 3.2823 \times 10 \times 10^7 = 3.2823 \times 10^8$
	2. $(1.55 \times 8.7) \times (10^4 \times 10^{-6}) = 13.485 \times 10^{-2}$, but since 13.485 is not between 1 and 10, we replace it by 1.3485×10 .

 $13.485 \times 10^{-2} = 1.3485 \times 10 \times 10^{-2} = 1.3485 \times 10^{-1}$

Figure 6.37

Calculators that operate with scientific notation will display the mantissa and the charac-

keystrokes for entering 4.87×10^{17} and the view screen are shown here.

Chapter 6 Decimals: Rational and Irrational Numbers



teristic whenever a computation produces a number that is too large for the screen. The

To compute with numbers in scientific notation, the numbers and operations can be entered as they are written from left to right. The following keystrokes compute (4.87 \times 10¹⁷) \times (9.2 \times 10⁵) and display the answer in scientific notation. Notice that parentheses are not needed as long as the calculator is programmed to follow the order of operations.



Try the following product to see if your calculator uses scientific notation: $2,390,000 \times 1,000,000$. This product equals 2.39×10^{12} , where the mantissa is 2.39 and the characteristic is 12. Notice that the base of 10 does not appear in the calculator view screen of the first display in Figure 6.38, but it does in the second screen. Most calculators that display numbers in scientific notation show the mantissa and the characteristic but not the base 10.



If a number is too small for the view screen, it will be represented by a mantissa and a negative power of 10. Use your calculator to compute .0004 \times .000006, which is .0000000024, or 2.4×10^{-9} in scientific notation. If this is computed on a calculator whose view screen has only eight places for digits and no scientific notation, it may show a product of 0, or an error message. The calculator in Figure 6.39 on the next page shows a mantissa of 2.4 and a characteristic of $^{-9}$ with the letter E indicating an exponent of $^{-9}$.

Figure 6.38



Figure 6.39

PROBLEM-SOLVING APPLICATION

Problem

Two elementary school classes have equal numbers of students. The ratio of girls to boys is 3 to 1 in one class and 2 to 1 in the other. If the two classes are combined into one large class, what is the new ratio of girls to boys?

Understanding the Problem To obtain a better understanding of the ratios, let's select a particular number of students and compute the number of girls and boys. Suppose there are 24 students in each class. Then the class with the 3-to-1 ratio has 18 girls and 6 boys. **Question 1:** How many girls and how many boys are in a class of 24 students with the 2-to-1 ratio?

Devising a Plan One approach is to *make a drawing* representing the two classes and indicate their ratios. The following figures illustrate the girl-to-boy ratios in the two classes and show that each class is the same size. **Question 2:** Why can't we conclude from these figures that the ratio of girls to boys in the combined class is 5 to 2?



Carrying Out the Plan To obtain information from the sketches of the classes, we need to subdivide the parts so that each figure has parts of the same size. The smallest number of such parts is 12, as shown in the following figure. The combined class will have 24 equal parts. **Question 3:** What is the ratio of girls to boys in the combined class?

