Early Childhood Corner

Achievable Numerical Understandings for All Young Children

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Young children aged 3 to 7 can learn a great deal about numbers. In a home or day-care environment, this learning can occur as children experience daily routines. Young children will learn to count, match, see, and compare numbers if caregivers or older children count, show objects, and point out small numbers of things. Such informal teaching can be done while children play, eat, get dressed, go up and down stairs, jump, and otherwise move through the day. These activities are engaging and fun but need to be encouraged and modeled by adults or more advanced children in the group. In larger day-care or school settings, numerical understanding results from similar informal learning opportunities combined with more structured experiences that enable all children to engage in supported learning activities with adult and peer modeling and help.

Activities for Achieving Mastery Goals

Figure 1 shows core mastery goals for numerical understanding that children aged 3 to 7 can achieve. The goals reflect a great deal of research by many people (see Fuson [1992a, 1992b] for summaries of this research) and our own experience in building a kindergarten program as part of the Children's Math Worlds project (see De La Cruz [1999]; Fuson, Smith, and Lo Cicero [1997]; Fuson, Sugiyama, and Grandau [2000]; and Lo Cicero, De La Cruz, and Fuson [1999] for descriptions of this project and Fuson et al. [2000] for a description of the equity pedagogy used in this project). All goals emphasize connections that children can and should build among oral number words, written numbers, and numerical quantities, that is, the meanings of the words and written numbers.

The grade levels in **figure 2** indicate the grades at which all children can acquire the listed understandings deeply and with fluency if they have opportunities to learn at home and at school or in day care. Three-year-olds can learn oral counting words to 8 and can be taught to see small quantities up to 5 and to recognize these quantities in varying arrangements. They can also learn the cardinal meaning of counting as telling how many things have been counted, as opposed to merely counting and attaching no significance to the final count word. Four-year-olds can work with sets of objects up to 10 and count sets that are even larger. They can also begin to recognize and visualize sets of quantities from 5 to 10 as special patterns that they see when "looking inside" numbers. For example, they can see the numbers 6 to 10 as 5 plus some more, for example, 6 is 5 and 1, 7 is 5 and 2, and so on. Children can also begin to see the doubles embedded in 4 (2 and 2) and in 6 (3 and 3).

Kindergarten children can learn "break-apart partners" for small numbers and for the number 10. Break-apart partners are pairs of numbers hiding inside a number. For example, 5 partners are 4 and 1 and 2 and 3. By modeling the breaking-apart concept through storytelling and dramatic play or drawing, children can see that one group of 5 apples

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Edited by **Douglas Clements,** *clements@buffalo.edu, and* **Julie Sarama,** *jsarama@buffalo.edu, State University of New York at Buffalo, Buffalo, NY 14260. This department addresses the early childhood teacher's need to support young children's emerging mathematics understandings and skills in a context that conforms with current knowledge about the way that children in prekindergarten and kindergarten learn mathematics. Readers are encouraged to send manuscripts for this section to "Early Childhood Corner," NCTM, 1906 Association Drive, Reston, VA 20191-9988.*

FIGURE 1

Mastery goals* for numerical understanding for all children

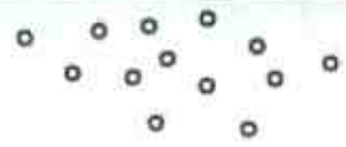
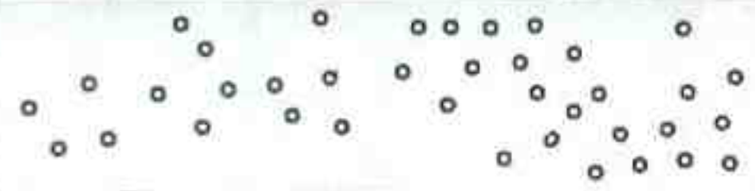
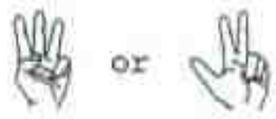
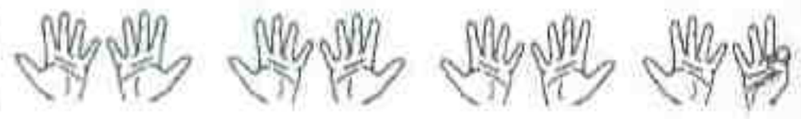
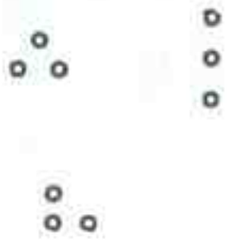
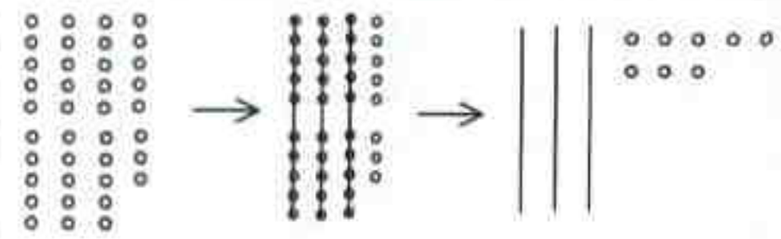
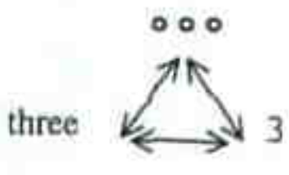
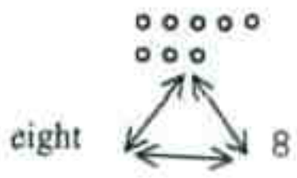
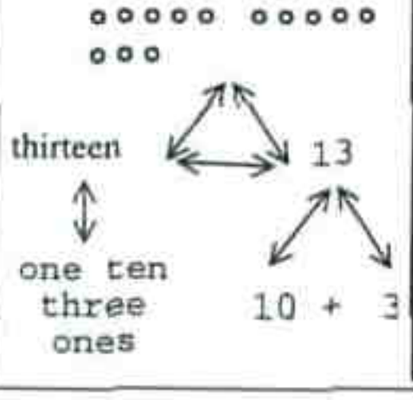
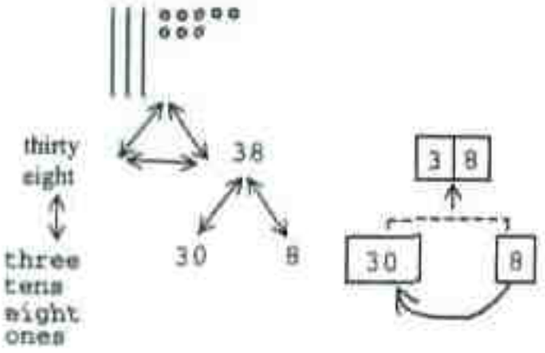
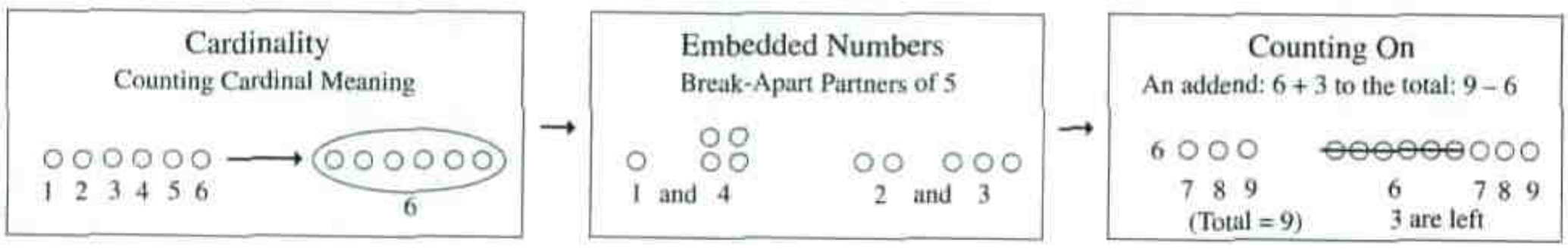
Goals	Three-Year-Olds	Four-Year-Olds	Kindergartners	First Graders
Using disorganized counting				
Counting on fingers				
Recognizing patterns				
Relating words, numerals, and physical referents				

FIGURE 2

Developmental Levels of Conceptual Structures (Fuson 1992b; pp. 252–53)



(total) can be broken into two groups (partners), for example, one group of 3 apples for Lisa and the other group of 2 apples for Aaron. Put another way, children can see the total = partner + partner relationship. Then, by putting the two groups of apples back together, children can prove to themselves that 2 and 3 really do make up 5 and see the partner + partner = total relationship. Through dramatic play, demonstration, and drawings or recordings, children begin to understand and enjoy seeing "numbers inside of numbers." In their drawings and with objects, children can show partners by leaving a space or drawing a separating mark.

We have found that kindergarten children can learn to write numerals to 10. This ability gives children a chance to become fluent in such writing and closes the gap when entering first grade between those who can write numbers readily and those who cannot.

Kindergarten children can also learn that teen numbers are composed of one group of 10 and some loose 1's. English teen and two-digit number words are complex and difficult to learn. The word structure does not correlate with the numerical parts. In contrast, in some East Asian languages, the word for *18*, for example, translates into "ten eight," and *38* translates into "three ten, eight." In the English system, teen and two-digit numbers look like two single-digit numbers written beside each other; nothing shows the 10 value for the number on the left. Young children need verbal and visual supports for understanding these number words and written numbers.

In our project, we provide this scaffolding by using *tens* and *ones* words, as well as standard number words, when working with teen and two-digit numbers. We say *13* as "thirteen" and as "1 ten, 3 ones" and say *38* as "thirty-eight" and as "3 tens,

FIGURE 3

Visual supports for learning teen and two-digit numbers

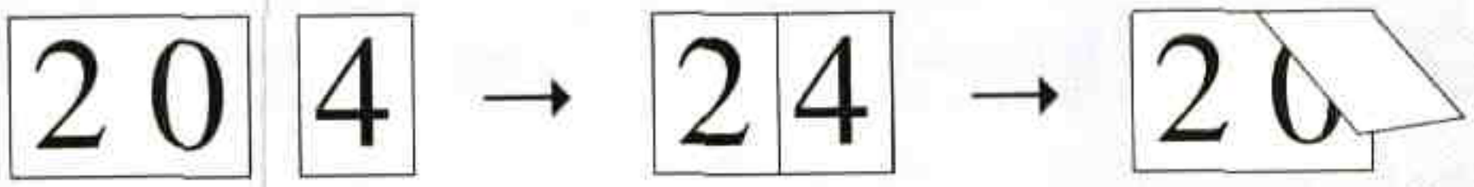


FIGURE 4

Addition and subtraction situations

All problem solving requires counting out objects, then counting or matching to solve the problem. Situations can be acted out or solved by modeling the situation with fingers or objects.

Break-Apart or Put-Together Situations

Dad had three apples. He put one in my lunchbox and the rest in a bowl. How many apples went in the bowl?

My grandmother has two cats and one dog. How many animals does my grandmother have?

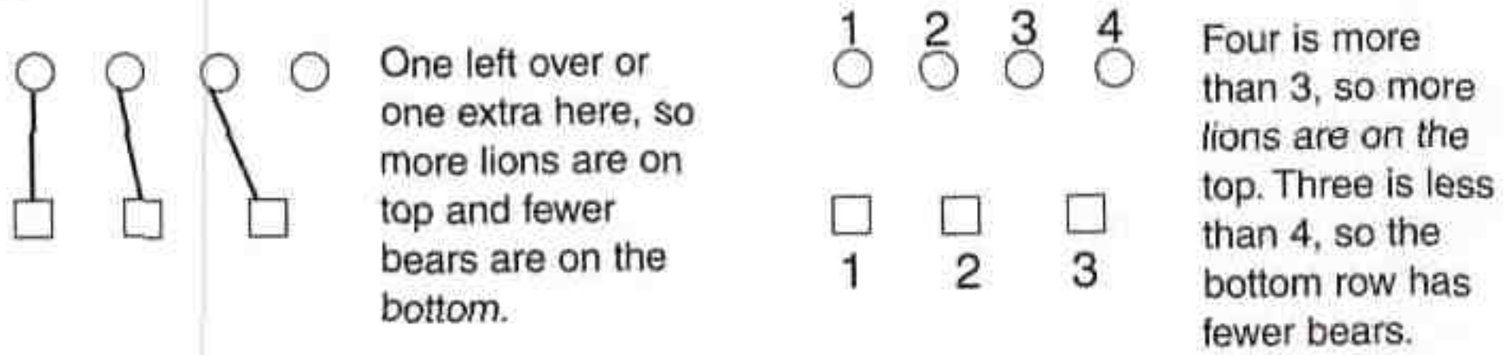
Add-to or Take-away Situations

Two birds were at the bird feeder. One more bird flew in. How many birds now?

Four puppies were playing on the porch. Two ran down the steps off the porch. How many were still on the porch?

Comparing Situations

Here are four lions and three bears. Do we have more lions or more bears? Do we have fewer lions or fewer bears?



8 ones." These words are used interchangeably and help reinforce the embedded ten-based thinking and place-value understanding. Our visual supports for these written numbers include cards that show the decade numbers as 10, 20, 30, and so on, and cards that are half as wide that show the single digits 1 through 9. The single-digit cards fit on top of the 0 on the decade-number cards to enable children to build numbers in such a way that they can still see both parts of the number they are making by flipping up the single-digit card (see **fig. 3**).

By using these cards and talking about the "invisible zero" hiding in the number, we can clarify the meaning of written numbers as "ten and three" or "twenty and four." Flashing ten fingers and smaller groups of fingers to show the value of a number is fun and builds numerical understanding. For example, 24 is two flashes of all ten fingers, then a showing of four fingers. Our first graders also learn to draw tens and ones to help them with calculations. The children first draw ten *single circles in a vertical column. Then they con*nect all the circles with a vertical line so that the line comes to mean ten. They later use a quick ten-stick (just the line) to mean ten and draw circles next to it to mean ones.

Another vital aspect of early numerical competence is understanding how to solve the types of

FIGURE 5

Numerical behavior in the home

Age (Year and Month)	Numerical Behavior
1-11	"I got two shobels [shovels]." When you count pictures in books or things, you say one number each time you lift up your finger and point it down again, but you count some things several times and others not at all. Typical counting series: "One, four, five, eight, four, five, two, six." Tonight we counted steps going up to bed (we usually do). We were on nine and you said, "One two fee four five six." First time so many correct.
2-1	You counting on your muffin (a toy): "One two three eight seven three." Several times today: "I two years old." "Make a *B*. Do it again. Two *B*s."
2-6	Two tomatoes were on the table. You said and acted out the following: "One tomato from two tomatoes leaves one. Two tomatoes from two tomatoes leaves no." *Sesame Street* does similar things.
2-7	Putting prunes back into a box, you correctly counted them up to nine. When asked how many prunes: "Three" (your standard "how many" answer at this point: three eyes, etc.).
2-10	You just asked for four olives (you love olives!). Your father gave them to you, and you said, "Two and two make four."
3-2	"Eight nine ten eleventeen twelveteen thirteen." Later, concentrating very hard: "twenty-one, twenty-two, twenty-three, twenty-four, twenty-five." Each *twen* was very long, and you raised your fingers 1, 2, 3, etc., as you counted. You count everything. You love to count.
4-9	When we were preparing bags of popcorn and peanuts for your (early) school birthday, you counted kernels of colored popcorn. In great excitement and wonder: "Ooh! I counted up to one hundred and two!" You refused to make piles of ten. You finally quit at 150.
5-7	Conversation with you in the bathtub: You: "How much is seven and seven?" I held up seven fingers and had you hold up seven fingers. You counted. You still usually will not count on from the first number; you need to count the fingers for the first number. You asked and we did nine and nine, four and four, six and six, five and five, and then ten and ten.

problems shown in **figure 4,** which lists the three different kinds of addition or subtraction situations. Four-year-olds can begin to solve word problems like these if the problems represent familiar situations and involve very small numbers. Children can solve the problems by using their fingers or objects or by acting out the story situation in all its parts; for example, children could be the puppies on the porch to model one of the problems in **figure 3.** Such activities build oral language skills and good listening and thinking skills—especially if children are asked to retell the stories in their own words, make up stories, and solve the stories of others. Such counting and story activities can occur throughout the day or at special mathematics story times.

Variations in Achieving Mastery Goals

Much variation is currently evident in the numerical understanding of young children. Most of this difference results from variations in children's experiences. The goals in **figure 1** are mastery goals that specify core achievable competencies. If children are supported in learning situations with their caregivers and teachers, they can reach these goals. Until all children experience this support, however, caregivers and teachers must continue to assist children in mastering skills from earlier levels because the children will not have had opportunities to develop them. We do not yet know how

much children can learn when they are in optimal learning situations for several years.

Many activities at specified grade levels can and should go beyond those outlined in **figure 1.** Children can always count more objects, learn more number words, and see patterns in larger numbers. Such experiences form their learning zones, in which they develop understandings for the next level of learning. Pairing children who are weak in numerical understanding with those who are stronger can build a peer-helping structure that helps all children in constructing social and conceptual knowledge and skill.

Conclusion

Figure 5 shows how young children can build numerical understandings if their natural capacities for learning are supported by adults. The table contains entries from the first author's diary about her daughter written to the daughter. These examples illustrate how numbers and counting activities can arise naturally in many different activities if they are supported by the caregiver and by other learning opportunities. Note, for example, that some scenarios influenced by *Sesame Street* appear in the diary.

With assistance in working on the achievable numerical understandings identified here, all children can reach their conceptual and social potentials. By providing a rich, numerically supportive environment, all caregivers and teachers can watch their children's natural competences and confidence flourish.

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