5

Primary Experiences in Learning What (As Well as How) to Count

Jane F. Schielack

A THREE-year-old girl, looking for someone to work with her on the computer, made her first foray into discrete mathematics. She began to list all the possible partners in her family. "Daddy could be my partner; or Mommy could be my partner; or my brother, Trey, could be my partner," she listed quickly. Then, after a moment of thought, without any outside prompting, she continued, "Or Daddy and Mommy could be partners, or Mommy and Trey could be partners, or Daddy and Trey could be partners." Satisfied that she had exhausted the possibilities, she proceeded to recruit her partner.

The *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989) calls for a K–4 mathematics curriculum that is broad in range of content, conceptually oriented, and developmentally appropriate with emphasis on students' active involvement in constructing mathematical ideas, applying mathematics, and developing the interrelationships of mathematical knowledge. Discrete mathematics, although not traditionally identified as a topic for the primary grades, can be investigated by young children in the context of familiar tasks and surroundings. Although they may practice counting given sets of objects, young children should also address, in particular situations, what *needs* to be counted.

Two of the major content strands suggested for kindergarten through grade 4 in the *Curriculum and Evaluation Standards for School Mathematics* are "Statistics and Probability" and "Patterns and Relationships." In grades 5–8, the *Standards* continues this content recommendation with a strand for patterns and functions and with separate strands for statistics and probability. For intermediate and upper elementary school students to be able to describe easily patterns of combinations and arrangements and to identify accurate sample spaces for determining probabilities, it would be helpful for them to have had many experiences in kindergarten through grade 2 in generating and recording arrangements within varying contexts. These

counting experiences can be based on actual situations familiar to children, as in the scenario above, and can be integrated into many of the activities that commonly occur in the primary-grade classroom.

COUNTING, WITH ORDER

Generating the Information

For an art project, ask kindergarten students to choose two pieces of construction paper, one for the picture and one for the frame, out of four possible colors. After the pictures are completed, display them on the floor or on the bulletin board. In an informal discussion, point out that Sue made a blue picture with a yellow frame and that John made a red picture with a white frame. Ask if the students think that these are all the ways that pictures and frames could be put together. Then help the students group the pictures by matching color arrangements. Some discussion may occur here about whether a blue frame with a yellow picture is different from a yellow frame with a blue picture. After the students decide that these *do* represent *different* arrangements, ask them if they think there are any other ways to put together a picture and frame.

At this point, if there is not already an example of a picture and frame of the same color, it may be suggested by a student or by the teacher. If the original directions were carefully worded so that phrases like "Choose two

different colors for a picture and a frame" were not included, students will be open to this possibility. Students can then make examples of any picture and frame arrangements that they think are missing.

Summarizing the Activity

It is important for students to review their thinking processes after solving a problem and to share their procedures with one another. Appropriate process questions posed by the teacher in a large-group discussion can help students organize the information they have generated:

- *How did you decide that you had found all the color arrangements?* Answers to this question might represent a wide range of sophistication from "We just kept trying and couldn't make any new ones" to "We made as many pictures as we could with blue frames, then as many as we could with yellow frames, and so on." This last response leads directly to the next question.
- *How can we arrange our information to help us know that we have found all the possible color arrangements?* By grouping the pictures according to the color of the *frame*, the teacher can draw the students' attention to the fact that for each frame color, there should be either three or four picture colors, depending on whether the frame and picture are allowed to match. By organizing these results into four groups of three (if pictures and frames must be different colors) or four groups of four (if pictures and frames of the same color are allowed), the students can see the twelve or sixteen arrangements that can be generated with the four colors. The pictures could also be grouped according to the color of the *picture*, with each one having three, or four, possible frame colors. The teacher can then arrange the pictures in a 3×4, or 4×4, array, showing both ways of grouping simultaneously.

Although it is not appropriate with most primary students to discuss using multiplication for counting, this visual representation will provide information that they can recall in later activities asking them to relate multiplication to counting arrangements. They also will have experienced the importance of independent choices versus dependent choices. They must decide whether a piece of information, in this case a color, can be combined with itself or not.

Other Suggested Activities

Ask students to address similar questions about counting with order in a wide variety of settings. A group of three students putting on a puppet show or a play with three characters can make a chart showing how many different

ways they could be assigned to the roles. Here it is important for students to realize that they are making dependent choices; they are restricted to using each person only once in each arrangement. They can then perform the plays using the six different arrangements.

Students can determine how many ways four sets of books or other materials can be arranged on the classroom shelves, placing one set on each shelf. For example, the dictionaries can be placed on the top shelf, the red readers on the second shelf, the paperback books on the third shelf, and the *Weekly Reader*s on the bottom shelf. Or the dictionaries can be placed on the bottom shelf, and so on. Students should make some judgments about which of the twenty-four arrangements would be most desirable or practical.

Then the results from the first count can be revised in light of different restrictions. For example, the dictionaries, being the heaviest books, might need to be on the bottom shelf. How will that change the number of possible arrangements? With a visual representation of the possible ways to place the books on the shelves, students can see that only six of the twenty-four arrangements will satisfy the restriction and that the results will be the same as placing three sets of books on three shelves.

Other situations that call for counting with order include the number of ways a small group of students can be assigned to the class helper chart, the number of ways five students can be lined up to go to the library, and the number of ways the types of food—such as milk, main course, vegetables, and dessert—can be arranged in the cafeteria along the tray line.

COUNTING, WITHOUT ORDER

Generating the Information

In this activity, first-grade students, working in cooperative groups of three or four, are to determine how many different colors of paint they can make with pairs of four given colors: yellow, blue, red, and white. Before actually mixing the paint, each group predicts how many colors they think they will be able to generate. Using small plastic spoons, the students mix two spoonfuls of paint of each of two colors. It is important to specify clearly that equal amounts of paint be mixed in order to eliminate a wide variety of intensities of the same color. The colors may be mixed on a sheet of waxed paper, then transferred with fingers or brush to the record sheet. Depending on how much experience the students have had with organizing information, they might develop their own method of recording results or use the chart in figure 5.1.

Most students will begin this activity by choosing colors to mix at random. They become so interested in mixing colors that it may be helpful for the

COLORS WE CAN MAKE			
First Color	mixed with	Second Color	Resulting Color

Fig. 5.1

group members to have assigned roles: one person chooses the two colors to mix, one person mixes the colors, one person records the starting colors on the record sheet, and one person records the resulting color.

When each group is sure that all possible colors have been mixed and recorded, the results are posted with the results of the other groups. With the charts on display, the groups are then asked to share their strategies and conclusions.

Summarizing the Activity

- *Did you predict that there were more or fewer colors than you found?* The predictions in early counting experiences are more or less random,

since most students do not ha
tions. If they have done the
they may make their predictio

- *Did every group generate th*
been very careful in the wor
"How many *new* colors" in or
it is possible that a group will
their results, for example, tw
blue. This type of exploration
number of results is an early e
ing with independent choices
dents can then engage in a
question meant and if they th
counted. The results might
colors, and the answer to the q
four original colors.

- *How did your group decide*
to this question, as in the act
from "We just kept trying unt
to "We mixed everything we
then everything with yellow, t
sponse, the teacher might hel
this procedure and compare it

- *Some groups have recorded*
have recorded blue mixed with
record sheet? It is highly likely
sented on the record sheets.
looking at the displays that in t
equivalent to *paint B mixed wi*

- *How is what we are counting*
the ways to put the pictures ana
sion is to generate student res
whether different orders of the
lent—whether the choices blue
arately, as with the pictures an
the paint mixtures. This questi
item, since the students' respon
each counting situation as an in
able to find some relationships
are more easily seen by the st
used in the two activities.

Resulting Color

chooses the two colors
cords the starting colors
sulting color.
rs have been mixed and
the other groups. With
hare their strategies and

r colors than you found?
re more or less random,

since most students do not have a strategy on which to base their predictions. If they have done the picture-frame activity described previously, they may make their predictions based on those results.

- *Did every group generate the same number of colors?* If the teacher has been very careful in the wording of the activity, avoiding phrases like "How many *new* colors" in order not to limit the students' explorations, it is possible that a group will include the four original colors as part of their results, for example, two spoonfuls of blue and two spoonfuls of blue. This type of exploration should be encouraged, since the increased number of results is an early experience in the difference between counting with independent choices and counting with dependent choices. Students can then engage in a profitable discussion of what the original question meant and if they think using the same color twice should be counted. The results might be grouped into new colors and original colors, and the answer to the question be presented as six new colors plus four original colors.

- *How did your group decide that you had found all the colors?* Answers to this question, as in the activity for counting with order, could range from "We just kept trying until we felt like that was all we could make" to "We mixed everything we could with red, then everything with blue, then everything with yellow, then everything with white." Using this response, the teacher might help the students list the results of following this procedure and compare it with how they recorded their final results.

- *Some groups have recorded red mixed with blue, whereas other groups have recorded blue mixed with red; why aren't both of these listed on each record sheet?* It is highly likely that these different orders will be represented on the record sheets. It should be very obvious to the students looking at the displays that in this instance *paint A mixed with paint B* is equivalent to *paint B mixed with paint A*.

- *How is what we are counting here the same as or different from counting the ways to put the pictures and frames together?* The goal in this discussion is to generate student responses that indicate an understanding of whether different orders of the same two colors are distinct or equivalent—whether the choices blue/yellow and yellow/blue are counted separately, as with the pictures and frames, or considered the same, as with the paint mixtures. This question works well as an informal assessment item, since the students' responses indicate whether they are dealing with each counting situation as an independent experience or whether they are able to find some relationships between the situations. The relationships are more easily seen by the students if the same four colors have been used in the two activities.

Other Suggested Activities

Given four ingredients such as chocolate chips, raisins, peanuts, and shredded coconut, how many different trail mixes can be made with exactly two ingredients? With exactly three ingredients? Ask the students to measure and mix the ingredients, organize the results, display the mixes in clear plastic bags on the bulletin board, and later sample them. Other situations that call for counting without order include selecting vegetables to make vegetable soup, selecting a small group of students from a larger group to sing a song, and choosing some flowers from a collection of different kinds of flowers to make a May basket.

CONCLUSION

Each of the counting activities should begin with predictions of possible results and be followed by questioning and student discussion that focus on the procedures used to find the combinations, the procedures used to decide when all the combinations have been found, and the similarities or differences between each activity and other counting situations that have been experienced.

The benefit of engaging primary students in discussions promoted by these activities is the development of their awareness of *what,* as well as *how,* to count. Certain attributes of counting situations—such as whether different orders are counted separately or not and whether choices are dependent or independent—that historically have been stumbling blocks for older students can be experienced by younger students in settings that will give them visual, auditory, and tactile references. Early experience in the need to identify such characteristics when counting provides a meaningful basis on which to build the symbolic representations of counting procedures.

REFERENCE

National Council of Teachers of Mathematics. *Curriculum and Evaluation Standards for School Mathematics.* Reston, Va.: The Council, 1989.