

We need to establish what background knowledge (i.e. axioms) we may use in doing proofs throughout the quarter. We will assume the following. Retain this list for reference. We will add to it as needed. If in doubt about what may be assumed, please ask. When using one of these to justify a step in a proof, use its name, if it has one. Otherwise, refer to it by number and a brief description (e.g. ABK 8–mult. by a neg. number)

Assumed Background Knowledge (ABK)

1. Arithmetic (Examples:  $1 + 1 = 2$ ; “minus times minus is plus” and other rules of signs; dividing is the same as multiplying by the reciprocal; subtracting is adding the opposite; common denominators; . . .)
2. Commutative and associative properties of addition and multiplication of real numbers; distributive property of real numbers
3. Adding, subtracting, or multiplying both sides of an equality by the same real number preserves equality.
4. Dividing both sides of an equality by the same *nonzero* real number preserves equality.
5. Saying  $a < b$  is the same as  $b > a$  and  $a \leq b$  is the same as  $b \geq a$  and  $b = a$  is the same as  $a = b$ .
6. Adding or subtracting both sides of an inequality by the same real number preserves the inequality.
7. Multiplying or dividing both sides of an inequality by the same *positive* real number preserves the inequality. (Example: If  $x < 2$ , then  $3x < 6$ .)
8. Multiplying or dividing both sides of an inequality by the same *negative* real number *reverses* the inequality. (Example: If  $x < 2$ , then  $-3x > -6$ .)
9. Transitivity of equality and inequality: If  $a \geq b$  and  $b \geq c$ , then  $a \geq c$ . Same for  $=$ ,  $>$ ,  $<$ ,  $\leq$ .
10. Law of Trichotomy. If  $a$  and  $b$  are real numbers, then exactly one of the following is true:  $a = b$ ,  $a < b$ , or  $a > b$ .
11. Closure.  $\mathbb{C}$ ,  $\mathbb{R}$ ,  $\mathbb{Q}$  are closed under addition, subtraction, multiplication, and division by *nonzero numbers*.  $\mathbb{Z}$  is closed under addition, subtraction, and multiplication.  $\mathbb{N}$  is closed under addition and multiplication. (Example: If  $a, b \in \mathbb{R}$ , then  $a + b \in \mathbb{R}$ .)
12. If  $a$  is a real number and  $n$  is an odd natural number, then  $a$  has a unique  $n^{\text{th}}$  root in the real numbers.
13. For any real numbers  $a$  and  $b$ ,  $|ab| = |a||b|$ .
14. For any real number  $c$ , for any *positive* real number  $d$ ,  $|c| < d$  if and only if  $-d < c < d$ . That also holds with  $<$  replaced by  $\leq$ .
15. The quadratic formula.

16. Domains and ranges of the elementary functions from algebra and trigonometry (rational, exponential, logarithm, sine, cosine, etc.). Trig and logarithm identities.
17. Substitution. Any part of any formula may be replaced by anything equal to it.