Math 252 **Maple Area Lab** **Spring 2013**

Goal:
Approximate area between the graph of a function and the horizontal axis using rectangles, and when possible, improve approximations and compare to the actual area.

F1: Quick Help List **Ctrl F1: Help** **F5: Toggle between Math and Text**

Open and save a new Maple document
Use the basic "Starting with Maple" directions on the Directions and Reference Page

Maple Commands (reference only, the lab starts on the next page)

1. Starting in Maple (do this at the top of every Maple worksheet)
   >with(student): (Loads needed Maple commands
   >with(plots): : suppresses a display of what loads)

2. Sketching Left and Right Rectangles

   a. Left Rectangles
   This example plots the function $y = x^2$ in the window $0 \le x \le 3,\ 0 \le y \le 9$ and adds four left rectangles
   > leftbox(f(x), x=0..3, 4) (Maple sets the *y* range when it uses leftbox)
   with eight left rectangles
   > leftbox(f(x), x=0..3, 8) (The last digit is the number of rectangles)

   b. Right Rectangles
   > rightbox(f(x), x=0.3, 4)

3. Displaying Left and Right Sums with Sigma Notation

   a. Left Sum
   This example displays the formula for the area under $y = x^2$ from $0 \le x \le 3$ with four left rectangles in sigma notation.
   > leftsum(f(x), x=0..3, 4)

   b. Right Sum
   > rightsum(f(x), x=0..3, 4)

4. Evaluating Left and Right Sums with Maple

   a. Evaluate Left Sum
   This example evaluates the formula for the area under $y = x^2$ from $0 \le x \le 3$ with four left rectangles.
   > evalf(leftsum(f(x), x=0..3, 4)) (The f in evalf is for function)

   b. Evaluate Right Sum
   > evalf(rightsum(f(x), x=0..3, 4))

Maple Area Lab Activities
Do all work in a Maple worksheet and, as directed, have Maple carry out all computations.

I. Area Under a Line

1. **Function:** Define $f(x)$ to be $2x+3$. To indicate multiplication, use * (> $f := x->2*x+3$)

2. **Exact Area** Geometrically (not calculus), what is the exact area under $y = 2x+3$ from $0 \le x \le 4$? Use a mix of text and math input to show and explain your work.

3. **Display Left Rectangles** Use leftbox to display $f(x)$ with eight left rectangles on $0 \le x \le 4$.

4. **Subintervals** If you divide the interval $0 \le x \le 4$ into eight subintervals of equal length (as shown in #3), the right and left endpoints are $x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7$ and x_8. What is the value of each of these endpoints? Use math typing with _ to indicate a subscript.

5. **Left Approximating Area Under the Curve**

   a. Write the area of $y = 2x+3$ from $0 \le x \le 4$ using eight left rectangles as a sum using function notation for the y values/
   (example: $L_3 = (f(1)+f(2)+f(3))*.25$)

   b. Rewrite the area of $y = 2x+3$ from $0 \le x \le 4$ using eight left rectangles as a sum by substituting in the function values
   (example: $L_3 = (3+4+5)*.25$)

   c. Rewrite the area of $y = 2x+3$ from $0 \le x \le 4$ using eight left rectangles in sigma notation using the leftsum command.

   d. Use evalf to evaluate the leftsum (L_8) value.

   e. What is the percent error between the left sum with eight rectangles (L_8) computation and the actual area under the curve? ((error/actual)*100). Use evalf to display in decimal form. Is this percent error over or under the actual area? How do you know?

6. **Right Approximating Area Under the Curve**

   a. Use rightbox to display $f(x)$ with eight right rectangles on $0 \le x \le 4$.

   b. What is the area under $y = 2x+3$ from $0 \le x \le 4$ using eight right rectangles (R_8)?

   c. What is the percent error between the right sum with eight rectangles (R_8) computation and the actual area under the curve? Is this percent error over or under the actual area?

   d. What is the percent error between the average of the leftsum (L_8) and the rightsum (R_8) with eight rectangles computation and the actual area under the curve? Is this percent error over or under the actual area? How do you know?

II. Area Under a Curve.

1. **Function:** Define $g(x) = x^2 + 3$.

2. **Subintervals and Rectangles**

   a. Use leftbox to display $g(x) = x^2 + 3$ with 50 left rectangles on $1 \le x \le 6$

   b. Use rightbox to display $g(x) = x^2 + 3$ with 50 right rectangles on $1 \le x \le 6$

   c. What is the length of each subinterval in parts a) and b)?

3. **Approximating Area Under the Curve**

   a. Use leftsum with 50 rectangles (L_{50}) to approximate the area under $g(x) = x^2 + 3$ from $1 \le x \le 6$. Is this approximation over or under the actual area? How do you know?

   b. Use rightsum with 50 rectangles (R_{50}) to approximate the area under $g(x) = x^2 + 3$ from $1 \le x \le 6$. Is this approximation over or under the actual area? How do you know?

   c. What is the average of the left and right sum estimations for 50 rectangles? Is this approximation over or under the actual area? How do you know?

   d. Use increasingly large values of *n* for the number of rectangles to estimate the actual area under $g(x) = x^2 + 3$ from $1 \le x \le 6$. What do you think the actual area is? Show your calculations.

One copy / partner pair: Email your correctly named Maple worksheet to fleschb@wou.edu

Email subject line: Maple Area