Engaging Preservice Teachers and Elementary-Age Children in Transformational Geometry

Tessellating T-Shirts

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The NCTM (2000) Geometry Standard states that students in grades K–2 should "investigate and predict the results of putting together and taking apart two- and three-dimensional shapes" and that students in grades 3–5 should investigate "transforming shapes"—goals that surprise the preservice elementary teachers whom we teach. The Maine Learning Results (MLR) expects the state's students in prekindergarten through grade 2 to describe two-dimensional shapes as well as use positional language. Requiring translations of two-dimensional shapes supports this expectation. Students in grades 3–4 are expected to "use transformations," while students in grades 5–8 are expected to use coordinate systems and geometric tools to measure and construct two-dimensional figures. Finally, the MLR expects secondary school students to use coordinates to represent figures and their respective transformations. Taken collectively, experiences such as these set students up for success if they find themselves using the language of linear algebra and matrices to understand linear transformations. The MLR expectations, through activities such as "Tessellating T-shirts," are revealing and underscore the importance of mathematics for our preservice teachers. Lessons and curricular decisions can and should include fun activities but cannot lose sight of the mathematical underpinnings.

Many preservice teachers believe these topics should be taught in high school, although few were taught these when they themselves were in high school. Recently, a preservice teacher reported that her fourth-grade daughter was immersed in the same mathematics that she herself was investigating in our undergraduate mathematics course for teachers. Hearing this, many preservice students felt encouraged that their coursework was relevant. All our preservice teachers explore transformational geometry in their mathematics course and focus on this in their methodology course, but they have few opportunities to work with elementary-age children in this area.

In introducing transformational geometry to our preservice teachers as well as to elementary-age students, we focus on tessellations. We often rely on the work of the famed Dutch “mathematician’s artist,” M. C. Escher (1898–1972), whose prints include many tessellations, as well as artwork using other mathematical concepts, such as Möbius strips. The M. C. Escher Foundation (www.mcescher.com/) makes reproductions of Escher’s work easy to find on the Internet, in calendars, on posters, and in books. The contemporary American artist Jim McNeill (1967–) also produces tessellation art; an example is his “Escher Bowl.” Some elementary school art teachers may have access, through their art supply catalogs, to posters and a video featuring McNeill as he demonstrates various tessellation techniques. Demonstration clips are also available on the Internet at www.jimmcneill.com/. In addition, illustrations of tile floor and wall patterns from Middle Eastern art and American quilt art include tessellated planes. Pattern blocks, a staple of the elementary school classroom, may be a readily available resource to motivate a lesson on tessellations.

Lindquist and Clements (2001) remind us that “we need to ask what new concepts students will learn, what reasoning skills we will help them develop, and what connections we are making with other areas of mathematics” (p. 415). Tessellations offer rich classroom interactions that allow students to realize that the tiled floor their desk rests on is a tiled plane. The mathematics associated with transformational geometry further connects students to their mathematics education. Tessellations provide a rich opportunity for preservice teachers to connect and motivate geometric learning in their future classrooms.

We take multiple approaches to tessellations depending on our audience. Here we discuss our approaches to two different audiences.

Figure 1

To create the pattern for a shape that will tessellate, students draw a design from corner to corner on one side of a three-by-three-inch square (a). They then cut this section out, move it (translate it), and tape it to the opposite side of the square (b).

a. Cut along dashed segments | b. Place tape over the dashed line

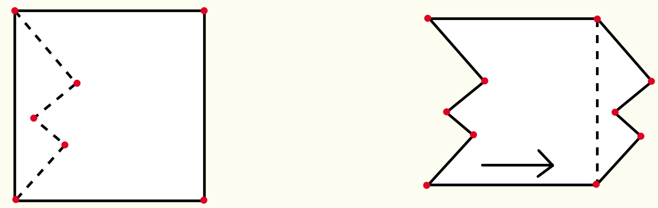

Figure 2

After repeatedly tracing the pattern template, students can create pictures by adding color and imagination.

Preservice Elementary Teachers

Our preservice teachers have consistently been surprised at the ease of creating translating tessellations. We usually begin the introduction by looking at the tessellations of the floor tiles, ceiling tiles, and possibly the cement block walls.

The preparation for this classroom activity involves cutting out three-inch squares, usually from

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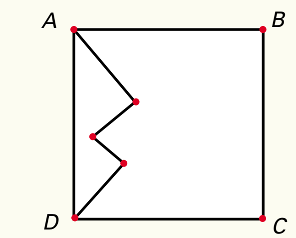
Figure 3

Students explore the mathematical relationships between the area and the perimeter of the original square (a) and the area and the perimeter of the tessellating design (b). They discover that while the area of the two shapes remains the same, the perimeter changes.

a.
Area $ABCD = 9.29\text{ cm}^2$
Perimeter $ABCD = 12.19\text{ cm}$



b.
Area $A_1B_1C_1D_1 = 9.29\text{ cm}^2$
Perimeter $A_1B_1C_1D_1 = 14.62\text{ cm}$

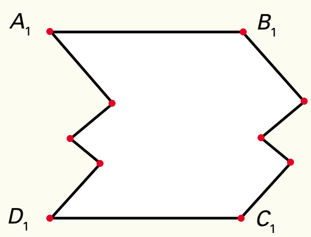


Figure 4

A sidewalk tessellation

old manila folders or three-by-five-inch index cards (the stiffness of these materials allows for numerous tracings). For ease of introducing tessellations, we have the teachers cut a design from one side of their square, beginning at a corner and ending at the other corner; translate this to the other side of the square; and tape it into place (see **fig. 1**). This strategy allows the square's top edge and its bottom edge to maintain their congruence. Certainly, if students understand that this process works from side to side they will realize that they are not restricted to changing just one edge. Creative students realize that they can translate a cutout from side to side as well as from top to bottom on the same initial square. With elementary-age children, using physical motions—having them stand and slide sideways one step for a translation tessellation, twirl for a rotation tessellation, look in a mirror for reflection, and sidestep and turn for a slide reflection tessellation—helps them internalize these concepts. Such movements also help students connect the new vocabulary terms to their meanings.

Using a piece of newsprint or other large sheet of paper, the preservice teachers can trace and translate their shapes, create pictures by placing appropriate character marks, and color their designs. (They are never too old for coloring!) With a little creativity, the shape shown in **figure 2** can become a man's face.

At this stage, it is critical that the mathematics not be lost and that vocabulary opportunities are recognized. Direction and distance are both associated with the translation, which can be both horizontal and vertical. The introduction of color creates another attribute and subsequent pattern to recognize.

We also investigate the mathematical relationships between the area and the perimeter of the original square and the tessellating design (see **fig. 3**). Preservice teachers frequently conjecture correctly that the area is constant but pause when asked about the perimeter. If confusion arises with respect to area, we can translate the cutout back to its original position to visually convince them that the area has remained constant. Helping teachers understand that the perimeter has changed may offer a teachable moment. Using a ruler, they can physically measure the new perimeter. They may also offer conjectures by measuring with their "eyes" and then checking these measurements against actual ones.

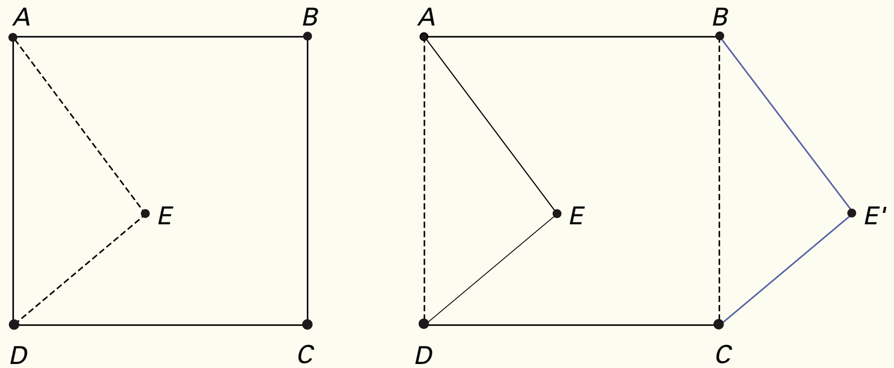
Elementary-Age Children

Tessellating T-shirts are exciting for children, particularly when the design is their own! We have successfully done this activity with students in

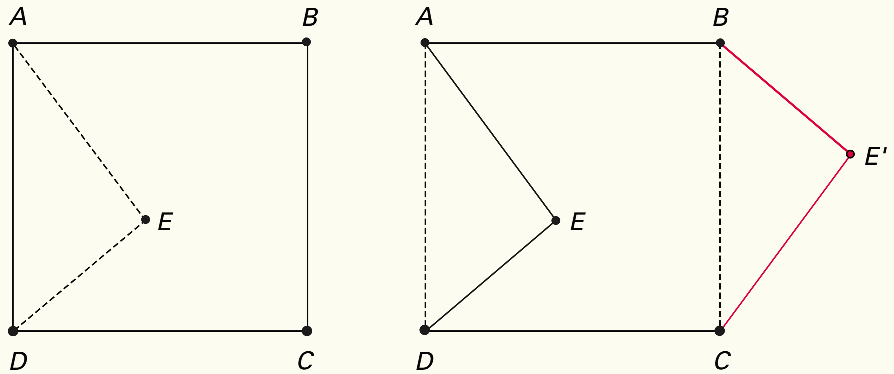
Figure 5

Student errors lead to interesting dilemmas that can present problem-solving opportunities. Figure (a) shows the intended shape of a student-created pattern. Figure (b) demonstrates what happened after the cut section was inverted and attached upside down. Figure (c) depicts a comparison between the correct figure and the incorrectly assembled pattern.

a. Correct translation



b. Incorrect flip of cutout section



c. Comparison of correct construction (blue lines) and incorrect construction (red lines)

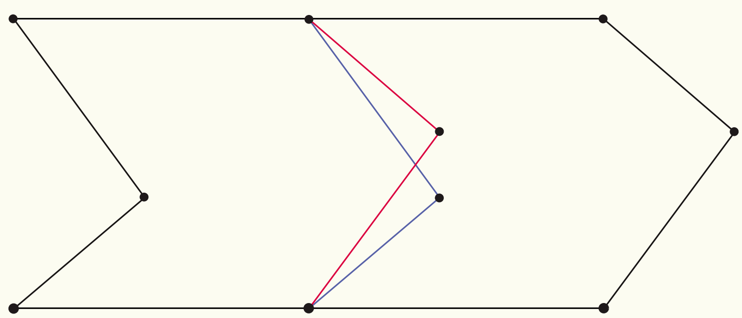


Figure 6

Another common student error is to flip the tessellating shape, rather than use a slide translation.

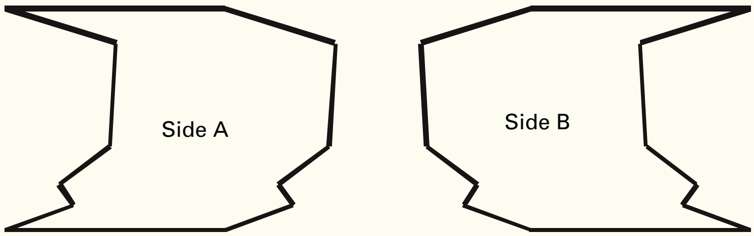


Figure 7

Coloring the tessellation

kindergarten through grade 4 and always involving preservice teachers so that they begin to understand young children's problem-solving strategies. To create the tessellating T-shirts, we begin in our preservice classroom. Then we move into a classroom of children, where we adapt the initial activity to small hands. Again using used manila folders, we cut very large squares from one side of the folders and repeat the preservice teachers' activity with the children. Our preservice teachers demonstrate the cutting and taping strategy and then assist the children with their individual tessellation designs. When the children are satisfied with their designs, we go outside with sidewalk chalk and create huge tessellations on the sidewalk (see **fig. 4**).

Having children work with these large shapes first has proven successful in preparing them to later work with three-inch squares for their T-shirt design. A three-inch square may seem very small, but elementary-age children *are* small and wear small clothes; patterns made with a larger square begin to cover the shirt in just a couple of tracings. The preservice teachers work with the children individually or in small groups to help them cut and tape their final translating piece. Some children will want to create a pattern that tessellates in both a horizontal and vertical direction, an idea that always leads to fun designs.

A few children may come up with interesting dilemmas that can lead to problem solving about other types of tessellations. Inevitably a student or two will invert their cut piece so that they attach it upside down; see **figure 5a** for the correct translation of the cutout from *A* to *B*. **Figure 5b** illustrates a flip of the cutout section, *AED,* which is translated to the opposite side, and the confusion this may create for a student trying to tessellate this figure. This situation may worry the student because the design will not work in the slide translation (see **fig. 5c**). However, with a little guidance, students can figure out what went wrong and how to fix it. Another common problem is when students flip their tessellating shape (see **fig. 6**), thinking this may satisfy a translation, which it does not. Generally, students are very quick to correct this problem.

Creating tessellating T-shirts is a straightforward activity. After the students have repeatedly traced their tessellating design on a large piece of newsprint, they color their design with fabric crayons (**fig. 7**). Such designs can be ironed onto polyester or cotton-blend T-shirts for great personal mathematical artwork. The more firmly the children color their designs, the brighter the colors appear on their shirts. The authors and preservice teachers operate the ironing stations; all we need are two or three irons and a clean, flat surface. We place a piece of newsprint inside the shirt to prevent colors from transferring to the opposite side of the child's shirt as we iron. The students place their colored tessellating shape on their shirt precisely where they want their design to transfer. The astute reader recognizes that the final outcome ironed on a T-shirt is indeed a reflection of the students' original design.

The final touch is applying the student's name to the T-shirt. We use one copy of their tessellating design and the MIRA math tool to create an iron-on design for their shirt front (see **fig. 8**). Students are not always aware that by writing their name with fabric crayon and ironing the name onto their shirt, they are creating a reflection of their name. If there is a mirror in the classroom, we ask students to use large print and spell their name out on a sheet of paper. The student then reflects this version of his or her name in the mirror to visually understand how, if ironed onto a shirt, it will appear. This process reinforces the need to do two reflections so that the name is "readable" on the shirt.

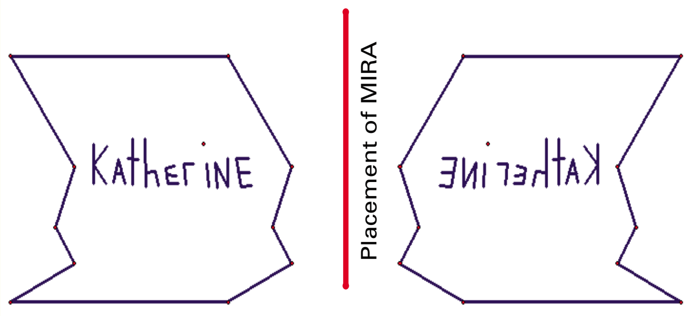
Conclusion

Is this mathematics boring? We do not think so. The mathematics for our preservice teachers—measuring, locating, reflecting, and translating—is often new because of the way we approach these topics. The children's enthusiasm reminds the preservice teachers that young students find learning exciting! We try to model our instruction for the preservice teachers in a way that, we hope, they will use to teach and engage youngsters in their classrooms.

This activity goes beyond the basic ideas of teaching tessellations. It situates geometry into the wider context of history and art and gives students a foundation that will help them understand fractals. Most important, however, it opens them to the idea that geometry can be fun!

Figure 8

Result of using a MIRA for name reflection



Reference

Lindquist, M. M., and D. H. Clements. "Geometry Must Be Vital." *Teaching Children Mathematics* 7 (March 2001): 409–15.