

# An Investigation of

# SOLAR

# NOON

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This article describes how mathematics and science were integrated in a single lesson, in the context of an extended eighth-grade science unit on astronomy. The lesson involved applying principles of geometry and using measurement and calculations to interpret observations of the sun's apparent movement across the sky during the day. An upright pointer, which is the modern equivalent of an ancient device called the *gnomon*, produced shadows that were then tracked. The observations of the shadows were used to find the direction of geographic north.

This idea is hardly new: Finding true north from the shortest shadow is among the oldest practical astronomical observations (Hogben 1937; Jackson 2004). We transformed this old idea to make it classroom accessible. Students were given a page on

which to collect data and two familiar mathematical tools, the ruler and compass, to complete the experiment.

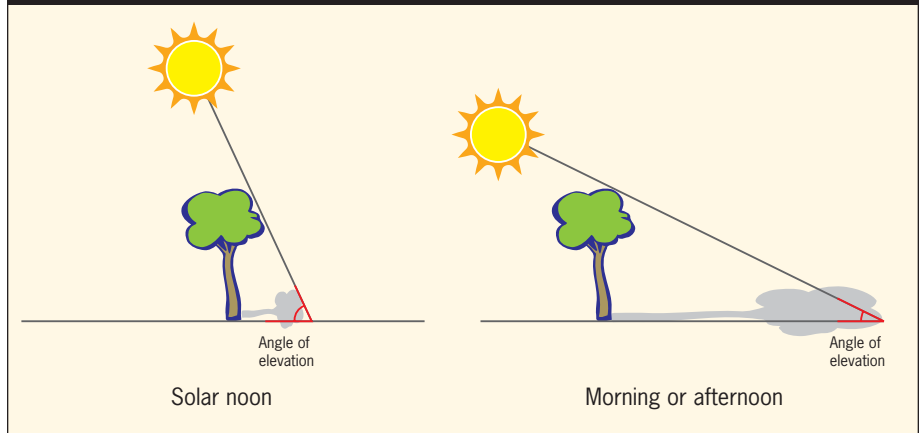
## INTEGRATING HISTORY, CULTURE, AND MATHEMATICS

History plays an important role in the study of all subjects at the Ross School, including mathematics and science. In the eighth grade, the medieval Islamic empire is a major topic of study. Islamic religious practice requires knowing how to orient oneself and how to tell time. In medieval times, the science of astronomy filled those needs. The science curriculum at the Ross School includes positional astronomy, which addresses learning from direct observations of the sky. These observations promote connections with cultural history and provide opportunities for hands-on investigations.

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**Fig. 1** At solar noon, the angle of elevation is the greatest and the shadow is the smallest that they will be all day.



### LEARNING ABOUT CLOCK NOON VERSUS SOLAR NOON

One objective of this astronomy unit connecting mathematics and science involved the conception of noon. Clock noon is an abstract concept based on the notion that the interval from midday today to midday tomorrow ought to be the same every day of the year. Before accurate mechanical clocks were perfected, there was a different concept of midday, called *solar noon*, which derived from direct observation of the sun's apparent movement across the sky. Solar noon is defined as the instant when the sun's position in the sky is highest when compared with any other time during that day. Astronomers refer to the angle from the horizon to the sun as the *angle of elevation*, so another way of defining solar noon is when the sun's angle of elevation is greatest for a particular day. At that time, the length of shadows will be the shortest. (See **fig. 1**.)

The time of solar noon is almost never the same as clock noon. Think about time zones: Clock noon and solar noon could only be the same by happenstance of location. The sun appears highest above the horizon in some location for observer A at the moment when the earth is turned to align observer A's longitude with the direction of the sun. At a different

location, observer B will see solar noon at the same moment only if A and B's longitude is the same. Observers everywhere within one time zone experience clock noon at the same moment, but there is at most one longitude in a time zone where clock noon and solar noon will coincide.

As a further complication, the time interval from solar noon one day to solar noon the next day is seldom exactly twenty-four clock hours. Solar noon changes from day to day throughout the year. If clock noon and solar noon coincide on one day, then the next day at the same location they would not. Additional information on the complexities of this idea is described in the equation of time (Green 1985). It is not our objective for students to learn the details; however, we do expect them to realize that solar noon and clock noon are different. Historically, clock noon and clock time have been in wide use only since instant communication by telegraph, telephone, and radio, which globalized our measurement of time; synchronization of distant clocks was difficult and unnecessary before these forms of communication were used, so solar noon and solar time were sufficient. In science class, the students discovered and learned three important facts about solar noon:

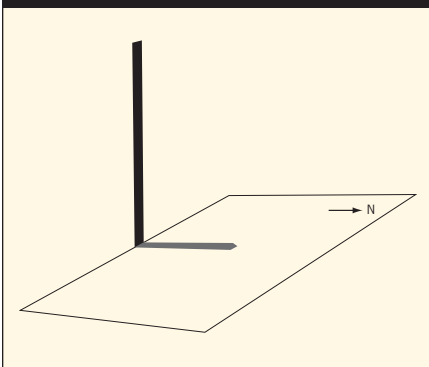
- Solar noon is the moment during the day when the sun appears the highest above the horizon and when shadows are the shortest.
- Solar noon is the moment that divides the time interval from sunrise to sunset exactly in half.
- Solar noon is the moment when shadows point directly north in the northern hemisphere or directly south in the southern hemisphere.

The students learned these facts in several ways involving multiple representations. Students used the interactive computer program *Starry Night* ([www.starrynight.com](http://www.starrynight.com)) to familiarize themselves with the daily apparent motion of the sun across the sky. They also manipulated a simple and inexpensive three-dimensional handheld model, the *Solar Motion Demonstrator* ([www.astrosociety.org](http://www.astrosociety.org)) to reinforce their understanding. To conclude the learning experience, students made outdoor sun-shadow observations and analyzed the results in an integrated mathematics and science lesson.

### SETTING UP THE EXPERIMENT

On a sunny day, with the help of their science teacher, students set up their experiment. A large piece of paper, approximately one meter square, was attached to a smooth, level surface, and a shadow stick was placed

**Fig. 2** A sheet of paper is laid flat for recording the shadow of a vertical pointer



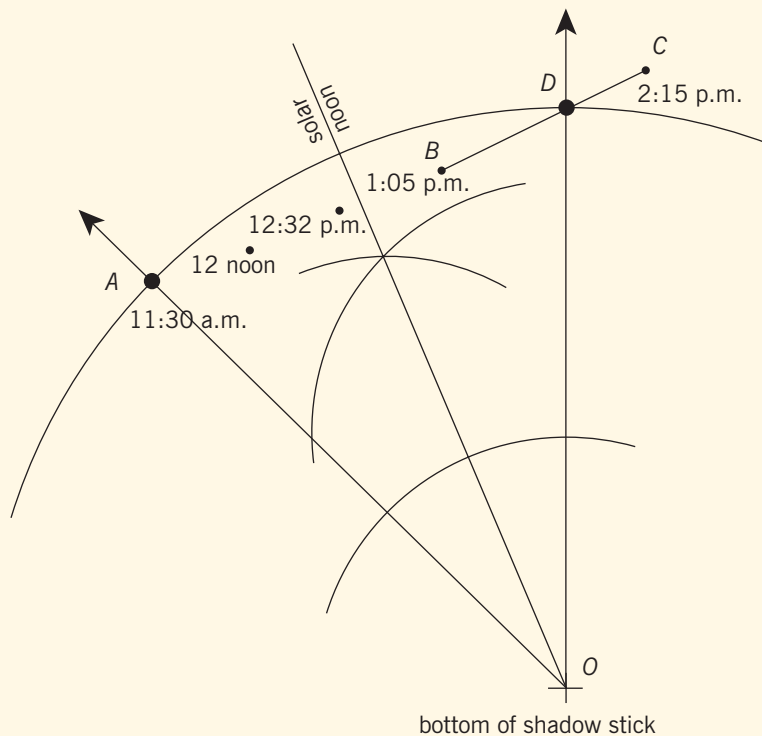
on the paper near the southern edge (see **fig. 2**). The shadow stick needs to be positioned on the southern edge of the paper so that the tip of its shadow will fit on the paper. It is necessary for students to know which directions are north and south before placing the shadow stick. To make the shadow stick, a ring stand from the science lab was used to support it. The exact location of the stick—that is, the point directly under the stick—must be marked carefully on the paper. Observations were taken at several times throughout the day, starting well before noon, and data points were recorded before and after midday. For each data point, students marked the spot where the shadow of the tip of the stick fell on the paper. The first observation was made. Students and their teacher discussed

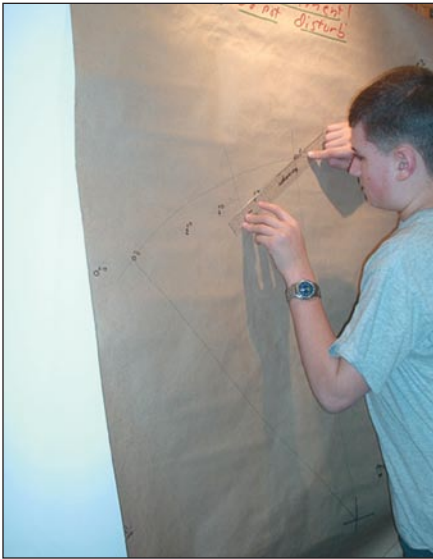
the fact that the shadow tip was fuzzy because the sun appeared as a disk, not a point of light; the mark made was roughly half a centimeter. For each observation, the time was written next to the mark. Subsequent observations were made by individual students throughout the day, between classes or during the lunch period, although five data points were sufficient.

When the experiment was finished, the science teacher took a digital photograph of the large piece of paper with the shadow points and the shadow stick clearly marked so that these elements could be transferred to an activity sheet (see **fig. 3**). The photograph was then uploaded to a computer so that each student could receive an activity sheet with the marked data points. Any other

**Fig. 3** The activity sheet shows an image of the sun shadow dots, with the student's construction of the shadow at solar noon.

Use a straightedge and a compass to construct a line pointing north.  
Measure the angle from vertical with a protractor.





means of faithfully transferring the data in geometric form to a hand-out of convenient size would serve the purpose. This completed the preparation for the mathematics and science lesson.

With guidance from their teachers, students analyzed the data from the outdoor observation and transferred that information to the activity sheet. The goal of the lesson was to calculate the time of solar noon on the day that the data were collected. The first step toward this goal was to find the location of the shadow at solar noon. To do so, one must first find two different times when the shadow lengths were exactly equal. These two times would have occurred at equal intervals before and after solar noon. We made observations in the morning and afternoon so that we could geometrically construct an afternoon shadow at the time somewhere between the two actual observations. In so doing, the shadow length in the afternoon would exactly match a morning shadow length.

The location on the original observation paper that was directly under the shadow stick must be known; it was marked by a cross at point  $O$  on the activity sheet (fig. 3). Placing the point of a compass at  $O$ ,

**Fig. 4** Once the shadow at solar noon is constructed, students can use length measurement and proportion to calculate the clock time corresponding to solar noon, which is different from 12:00.

**How to find the time of solar noon from the data sheet:**

A. With a ruler, measure the distance in millimeters between the dots at 12:32 pm and 1:05 pm and write it here: 24 mm

B. Subtract the two times to find the interval in minutes: 33 min

C. Measure the distance in millimeters from 12:32 to the bisector you drew for solar noon: 4 mm

D. Write a proportion that will give the unknown number of minutes  $x$  from 12:32 to solar noon:

$$\frac{x}{33 \text{ min}} = \frac{4 \text{ mm}}{24 \text{ mm}}$$

E. Solve for  $x$ : 5.5 minutes = 5 minutes and 30 seconds

F. Add the result to 12:32: 12:37:30

*Note:* Times of 12:32 p.m. and 1:05 p.m. are unique to the measurements made by this group of students.

students adjusted the compass opening so that the pencil point touched a morning observation, point  $A$  at 11:30 a.m., and then they drew an arc from that point toward the afternoon observations. For most, this arc did not touch any of the afternoon observations: The time after noon when the shadow length equaled the morning length will fall between two afternoon observations. For example, at point  $B$  at 1:05 p.m., the shadow length was too short to match the morning observation and at point  $C$  at 2:15 p.m., the shadow was too long. Students had to estimate where the shadow length equaled the morning length by drawing a line from  $B$  to  $C$ , then seeing where this line intersected the arc through point  $A$ , at point  $D$ . Once point  $D$  was found, students knew where the shadow length in the afternoon equaled the



morning shadow length; the shadow at solar noon was then constructed by bisecting the angle  $AOD$ . The bisector located the shadow position at solar noon and the direction of north.

Once students had this bisector, they were able to measure its distance from neighboring observations. They used proportions to estimate the time of day that solar noon occurred; a second activity sheet guided them (fig. 4).

Students were given another activity sheet that contained the original data, as well as data points taken on a different day, when the time of solar noon was not the same (fig. 5). Care must be taken to preserve the orientation of the experiment. The paper and shadow stick should be in the exact locations they were on the first day. The students repeated their steps, then performed an analysis on the new set of points. They then realized that the angle bisector still pointed north, even though the time was different. This result was both unexpected and surprising at first to the students. However, they remem-

bered the three main facts about solar noon and were able to understand their findings.

This activity took one forty-seven-minute class period, not including the data collection and preparation of the activity sheets. At the end of class, the students had a better understanding of the difference between solar noon and clock noon. They had also mastered the skill of bisecting an angle and reviewed solving proportions.

### EXTENDING THE ACTIVITY

To extend this activity, students could be asked to check the accuracy

of their calculation of solar noon by finding the times of sunrise and sunset. This information can be found in a local newspaper; in an online almanac, such as the U.S. Naval Observatory's "Data Services" at [aa.usno.navy.mil](http://aa.usno.navy.mil); or by using the software program *Starry Night*. Once the times of sunrise and sunset are known, students can calculate the time halfway between them. Any discrepancy will typically be much smaller than the time difference between clock noon and solar noon.

This project, integrating mathematics, science, and history, gave students a new slant on the workings of the sun.

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*This article is dedicated to Ralph Abraham, founding mentor of the Ross School.* ●

**Fig. 5** An activity sheet shows the image of sun shadow dots observed on two different days, October 15 and October 18, although the time of solar noon has changed, the direction of north remains the same.

Use a straightedge and a compass to construct a line pointing north. Measure the angle from vertical with a protractor.

