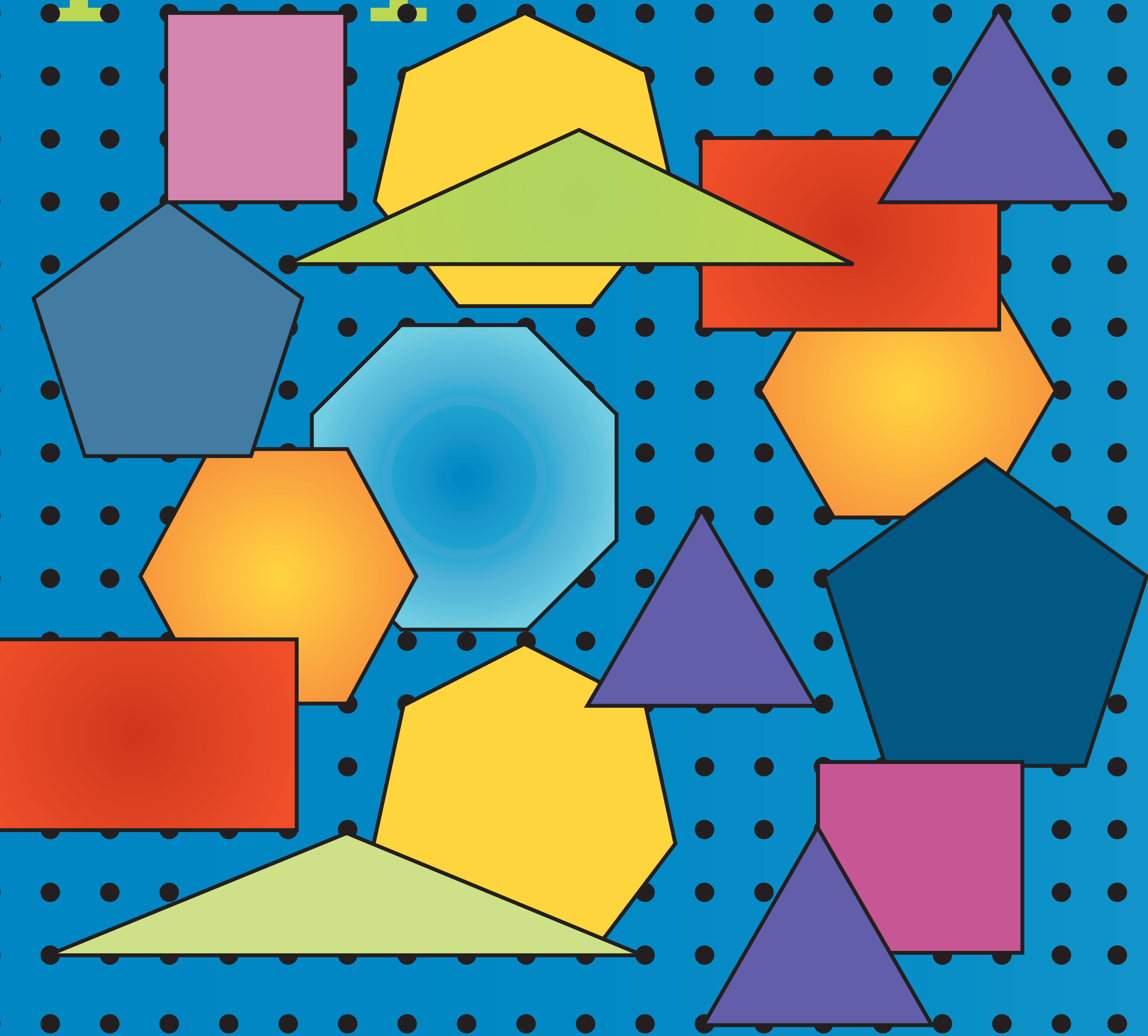
Polygon properties:



Copyright © 2010 The National Council of Teachers of Mathematics, Inc. www.nctm.org. All rights reserved.
This material may not be copied or distributed electronically or in any other format without written permission from NCTM.

What is possible?

Sorting shapes and solving riddles develop and advance children's geometric thinking and understanding while promoting mathematical communication, cooperative learning, and numerous representations.

By Rebecca R. Robichaux and Paulette R. Rodrigue

Opening a package of crackers, an excited third grader exclaimed, "These are quadrilaterals!"

We were delighted to see her applying knowledge that she had recently gained about polygons to a nonmathematical context. As instructors of elementary mathematics methods courses, we frequently monitor teacher candidates as they instruct youngsters in grades K–8. In this integrated social studies and science lesson, a preservice teacher was conducting a Thanksgiving lesson that engaged third-grade students in a butter-making activity. Preparing to spread the butter on crackers and sample it, the child had spontaneously shared her discovery. The preservice teacher then led the class in a short review of polygons and their properties.

After the lesson, the preservice teacher concluded that as a result of how students were taught the properties of polygons—by using shape-sort and riddle activities—this student could easily identify the quadrilateral in a context outside the usual "math time." Although our students were third graders, the activities are appropriate for students in grades 3–5 and possibly for those in other grade levels if some modifications are made.

What follows is a brief summary of how children develop an understanding of the properties of geometric shapes and a description of the shape-sort and riddle activities that the preservice teacher found to be beneficial for deepening students' geometric understanding. To successfully complete the various tasks that the shape-sort and riddle activities comprise, students work in cooperative learning groups and use higher-order thinking skills. Both activities meet the Geometry Standard, which states that students should engage in activities that allow them to "analyze characteristics and properties of two- and three-dimensional geometric shapes…; develop mathematical arguments about geometric relationships; [and] use visualization, spatial reasoning, and geometric modeling to solve problems" (NCTM 2000, p. 41).

Geometric thinking and polygon properties

All students in grades 3–5 ought to be able to "identify, compare, and analyze attributes of two- and three-dimensional shapes and develop vocabulary to describe the attributes; classify two- and three-dimensional shapes

according to their properties and develop definitions of classes of shapes such as triangles and pyramids; and make and test conjectures about geometric properties and relationships and develop logical arguments to justify conclusions" (NCTM 2000, p. 164). For students to successfully meet these expectations, research indicates that they must maintain a certain level of geometric thinking (Clements and Sarama 2000; van Hiele 1999).

The van Hiele theory of geometric thinking identifies the first three levels of geometric thought as visual, descriptive, and informal deduction. At level 0, the visual level, nonverbal thinking occurs; students judge shapes by the way that they look. Students will identify a two-dimensional shape on the basis of its appearance and their mental images of that shape rather than on the shape's mathematical properties.

Students at level 1, the descriptive level, identify shapes according to their properties. A triangle is no longer a triangle because "it looks like one" but because it is a closed figure that has three sides and three angles. Students at this level are able to view specific shapes as belonging to a whole class of shapes that have certain properties. Language becomes crucial at this level because it enables students to give precise descriptions to shapes.

At level 2, the informal deduction level, students are able to logically deduce one property from another and logically order them. They can formulate definitions using properties that they already understand. For example, students functioning at the informal deduction level can deduce that all squares are rectangles because all squares possess all the properties of rectangles.

Teachers must prompt students beyond their initial tendency to sort shapes by "number of sides."

PAULETTE R. RODRIGUE

Elementary school students possess various levels of geometric thinking. To be prepared for successful completion of a typical high school geometry course, students should have passed through levels 0 and 1 of the van Hiele model and should be functioning at level 2. To acquire the thinking described for each level, "development is more dependent on instruction than on age or biological maturation, and types of instructional experiences can foster, or impede, development" (van Hiele 1999, p. 311). The two activities described in this article support students in progressing to higher levels of geometric thinking.

Instructional activities that develop geometric thinking should begin with exploration, then move toward more focused activities through which specific concepts are built and the related language is introduced, and end with consolidating activities that allow students to integrate what has been learned with what was already known (van Hiele 1999). The instructional activities in this article follow this sequence.

The "Shape sort" activity engages students in the exploration of the similarities and differences found in a large collection of polygons. This collection includes various shapes—symmetrical and nonsymmetrical, convex and concave, regular and irregular—and an assortment of angle measures. Previous research has found that providing students with a large variety of shapes, rather than only a few examples of specific shapes, enables them to eventually identify hierarchical relationships that exist between and among certain polygons, such as the hierarchical-based classes of triangles and quadrilaterals (Kay 1987).

Students then investigate polygon properties through student presentations and student-created posters. Following the shape-sorting tasks, more exploration occurs using geoboards to create shapes with various characteristics based on "clues" in the form of riddles. This riddle task eventually focuses on the notion of "impossible" polygons and why certain characteristics cannot exist within a single shape.

Finally, students consolidate their understanding during a wrap-up discussion as well

PAULETTE R. RODRIGUE

PAULETTE R. RODRIGUE

LEFT: Before they learned the terms *concave* and *convex*, students sorted within a sort according to whether their shapes had "dips."

RIGHT: Using rulers, students drew shapes onto a recording sheet for each category of each sort, noting a real-world example.

as with written reflections. Through such tasks, students deepen their geometric understanding and are more likely to avoid typical misconceptions and misunderstandings, such as thinking that two triangles can always be put together to form a square (Clements and Sarama 2000).

Shape sorting

To introduce polygon properties, each small group of students receives a large assortment of polygons that contains both regular and irregular triangles, quadrilaterals, pentagons, hexagons, heptagons, and octagons. Students' first task is to sort these polygons in as many ways as they can (Van de Walle 2007). They receive recording sheets to help them organize their thoughts with regard to the various ways in which shapes can be sorted (a recording sheet **template** accompanies the online version of this article).

The teacher observes students working on this task, encouraging them to think beyond the typical initial tendency to use "number of sides" as the only sorting criteria. While counting the sides of an irregular octagon, one student explained to her group members, "We can group together shapes with three sides, [those with] four sides, [those with] five sides, [and those with] six sides, like this: This one goes with eight sides."

With some guidance as needed, students also sort the shapes according to angle measure; whether they are regular or irregular, convex or concave, symmetrical or not; the number of lines of symmetry; whether they have parallel sides and perpendicular sides; and other less mathematical conditions, such as whether they resemble something in the real world.

Some students eventually sort within a sort. For example, after sorting by "number of sides," one group discussed how some of the four-sided shapes have dips and some do not: "Look at this one; it goes in, but these do not. See, this one makes a 'V' in the inside."

If such sorting within a sort does not occur spontaneously in due course, encourage your students to do so.

Using recording sheets, students must not only sort the shapes but also draw—using rulers—a shape of their own for each category of each sort and find an example in the real world of a shape in each category. They must explain the sorting rule or condition in writing and defend how they are certain they have sorted the shapes correctly for each condition. For example, if they have sorted on the basis of "has symmetry or does not have symmetry," then they are encouraged to explain how they are certain that each shape in the "has symmetry" stack does, in fact, have at least one line of symmetry.

While explaining how he knew the shapes in his "no symmetry" group were not symmetrical, a student explained, "There's no way to fold it so that it matches."

In explaining concave shapes, another student remarked, "All the shapes in this group have a piece missing. Like we could fill it in."

In many cases, students traced some of the shapes onto their recording sheets within their explanations to show the sorting condition they were trying to defend, such as line of symmetry or equal sides. Apparently, they realized how representations of mathematical ideas could help them communicate their thinking. One

Shape posters

Sample discussion prompts for exploring student-created shape posters might include the following:

1. Describe each group of *n*-sided shapes; what do the shapes in each group have in common?
2. What is different about the shapes within each of these groups?
3. Do any groups share something in common with another group? If so, what?
4. Describe the location of the longest side of each shape within that shape.
5. Describe the location of the shortest side of each shape within that shape.
6. When two sides are the same within a shape, describe the angles of that shape.
7. Do any of the groups have symmetry? If so, which ones, and how do you know this? If not, why not?
8. What can you say about the shapes that have "a dip"? How are they alike and different from the other shapes?
9. Compare the angles of the shapes on each shape poster.
10. Explain how you could create an *n*-sided shape using shapes on other posters.
11. How could you further sort the shapes in the convex group?
12. How could you further sort the shapes in the concave group?

student explained, “Sometimes it is easier to just trace around some of these to show what I’m talking about.”

When explaining and defending how to be certain that they sorted shapes correctly for each condition or sorting rule, youngsters often traced their own shapes.

Communicating in this way, students meet one aspect of the Representation Standard, which states that students of all grade levels should “create and use representations to organize, record, and communicate mathematics ideas” (NCTM 2000, p. 206).

PAULETTE R. RODRIGUE

The “Shape sort” activity continues until all groups have completed the recording sheet for at least four sorting conditions. Then each group presents one condition to the class. Although two or more groups sometimes present the same characteristic, how students verbalize the feature is usually different and somewhat original. For instance, the spokesperson for a group that sorted by the number of sides described how group members used the number of vertices:

> These have three points; these have four points; these have five points; these have six points; these have seven points; and these have eight points. But it’s the same as counting the sides. We just counted the points. And it doesn’t matter if the points are on the inside or the outside [referring to concavity].

Following group presentations of the sorting, each group is assigned one particular sorting condition and is instructed to create a poster to illustrate it (see **fig. 1**). The posters contain labels for the categories within the sort as well as drawings or tracings of the shapes that belong to each category. Some of the posters will be used in subsequent lessons when the class investigates specific polygon properties. The posters also become the focal point for discussions that lead to the riddle activity.

Polygon properties

After students have completed the shape-sorting tasks and are familiar with a particular set of shapes, they receive geoboards and are asked to construct some shapes from the various posters. They also draw these shapes on dot paper, which encourages them to think about the properties of the shapes, although they may not necessarily realize that they are doing so.

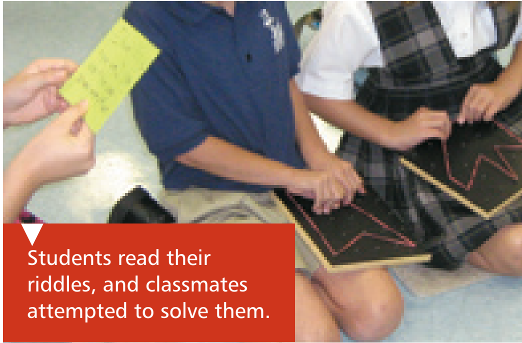
Once students are comfortable with constructing various shapes on geoboards and on dot paper, the class discusses specific properties and definitions. One part of the discussion might center on triangle properties. Students would be asked questions or given discussion prompts about properties of the shapes on their “three-sided shapes” poster or dot paper. The discussion might then move toward the properties of quadrilaterals.

Such discussion makes students more aware of polygon properties and better able to articu-

Wrap-up discussions

After solving several “Who am I?” riddles, students become more and more engaged and motivated to create challenging riddles, both possible and impossible. The excitement that using riddles creates is evident: Students keep their riddles hidden until they are ready to reveal them, and they do not want the activity to end. In a teacher-led wrap-up discussion, students are encouraged to share what they have learned about polygons and their properties through both the shape-sorting and the riddle activities. During such discussions, students are asked questions, such as those that follow, and are prompted to explain their answers:

1. Can a triangle have more than one obtuse angle?
2. Can a triangle have three acute angles?
3. Can a triangle or a quadrilateral have exactly two right angles?
4. Can a trapezoid have exactly one right angle?
5. Can a quadrilateral have two obtuse angles?
6. Can a quadrilateral have exactly three right angles?
7. Can all four sides of a quadrilateral have different lengths?
8. Can a five-sided shape, a pentagon, have five acute angles?
9. Can a six-sided shape, a hexagon, have four obtuse angles?
10. Can an eight-sided shape, an octagon, have eight right angles?
11. Describe what you have learned about each kind of polygon.


PAULETTE R. RODRIGUE

Students read their riddles, and classmates attempted to solve them.

late their thinking about those properties. One student remarked, "The longest side of a triangle is always in the 'mouth' of the largest angle."

Riddle activities

After discussing the assorted posters, students are introduced to the idea of polygon "Who am I?" riddles. (A list of **sample riddles** accompanies the online version of this article.) To solve a riddle, students must construct the answer on their geoboards as "proof" of their answer. Early riddles are basic and somewhat obvious; as the riddles gradually become more challenging, the teacher introduces the notion of an "impossible" riddle. Through trial and error, students learn that not every riddle can be solved.

Riddles are presented to students in a grab bag, each written on a card, color-coded by difficulty. After solving a riddle, students either explain and defend their answer (including their geoboard shape) or explain why the riddle is impossible. While a student explains a solution, classmates determine whether they agree with the proposed riddle solution. If anyone disagrees with the proposed solution, then that student must explain and defend why the proposed solution is not the solution.

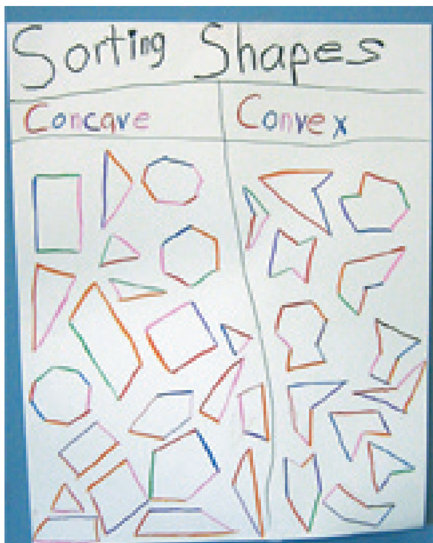
After this counterexample, students continue to discuss the solution, as needed. The presentation does not end until all agree as a class on the correct solution. Students thus have an opportunity to "develop and evaluate mathematical arguments and proofs," one strand of the Reasoning and Proof Standard (NCTM 2000, p. 188).

After students have solved several riddles and can defend why some are impossible, the final task within the activity is for them to create their own riddles, both possible and impossible, and then to solve one another's riddles. Each student creates at least two possible and two impossible riddles that can be swapped with a partner or read to the entire class and then solved. Students write a riddle on one side of an index card. On the other side, they write the solution, including a brief explanation of why it is either possible or impossible.

Student pairs trade riddles, solve them, compare their solution with the solution on the back

FIGURE 1

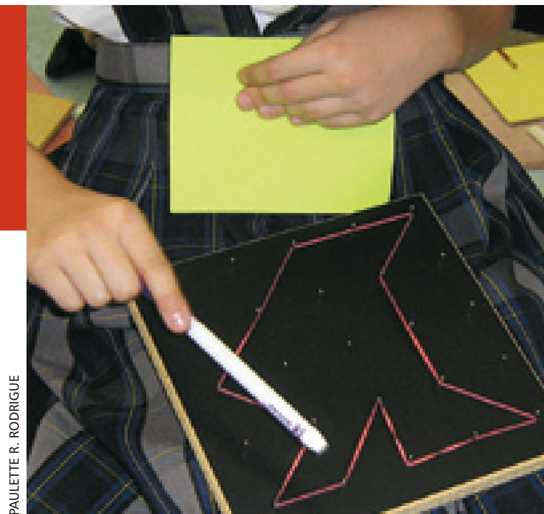
After the sorting presentations, each group creates a poster illustrating one particular sorting condition. This poster shows a convex and concave sort.



PAULETTE R. RODRIGUE

Students had to either explain and defend their answer (including their geoboard shape) or explain why a riddle is impossible.

Discussing specific polygon properties and definitions will make students more aware of and better able to articulate their thinking about those properties.


PAULETTE R. RODRIGUE

of the card, and then discuss with their partners why they agree or disagree with the solution.

After listening to a student read a riddle, her classmate proclaimed, "That one could use more information; we're not given any information about the angles. How many are acute or obtuse?"

Students may modify their solutions in response to the discussion. The final version of students' riddles on index cards can be collected and used as an assessment of students' understanding of the properties of the shapes.

Explaining how he created his riddle, one student remarked, "I made an eight-pointed star, then moved one of the sides. It's like a star at first; then just move one side. When you move one side, you can add another side, so it went from sixteen to seventeen sides."

Typically, students with a deeper understanding of the properties will have more challenging riddles, and students with a surface understanding will get basic riddles. Regardless, this task allows all students to demonstrate what they do understand, rather than what they do not understand, which follows the spirit of the NCTM Assessment Principle (NCTM 2000).

If various geometric terms (e.g., *isosceles, scalene, regular, irregular, parallel*) have not yet been introduced, they should be presented during a wrap-up discussion, assuming that students now understand the concepts associated with such terms. Once students understand the concepts, introducing words to describe the concepts helps them explain their thinking. Furthermore, students have a more meaningful understanding of abstract geometric terms after they have previous experiences with the underlying concepts associated with those terms.

Final thoughts

Discourse deepens students' understanding of polygon properties and promotes an awareness of which combinations of riddle criteria are possible and impossible. Through these activities, elementary school students learn to view polygons as more than flat, *n*-sided shapes. They begin to think of polygons in terms of the number and types of angles as well as which combinations of sides and angles are impossible and why this is so. They also begin to recognize polygons in the world around them—such as crackers and stop signs—and connect these everyday sights with their polygon properties. Such learning can be extended further by having your students do several more activities:

- **Measure** sides and angles of various shapes to integrate the Measurement Standard (NCTM 2000) into the shape-sorting tasks.
- **Draw** new shapes on colored card stock, cut them out, and add them to the collection rather than drawing a new shape on the recording sheet only. Other students can sort the new shapes at a subsequent time.
- **Record** geoboard shapes onto dot paper along with the riddle that was solved by that geoboard shape. (This will develop students' spatial visualization during the "Who am I?" riddle activity.)

Elementary school students have immense potential and capacity for geometric thinking beyond the visual level (van Hiele 1999). By engaging in activities such as those described in this article, students can and do progress from nonverbal thinking to highly verbal explanations, providing evidence of thought at the informal deduction level. On completion of the sorting tasks, one student commented, "I like finding the shapes in the classroom; it's easier to remember things about the sides and angles when you find [a shape] in the classroom."

After further exploration, focused thought, and solving possible and impossible "Who am I?" riddles, students' verbal explanations become much more driven by the relationships among the various shapes' properties, evidenced

by comments during the riddle activities and the wrap-up discussions. Discussing why a riddle is impossible, a student made these remarks:

> You just can't have four sides with exactly three right angles. If you can only have four sides, then when you put in the third right angle, this side becomes even [parallel] with this one, and then you have four right angles. You can't just have three. If I could have five sides, then I could do it with three right angles.

In terms of the entire sequence of geometric activities, another student commented:

> My favorite part of geometry is making shapes with the geoboard; you don't have to erase to get a new shape, you just move the rubber bands; it's easy to make acute and obtuse angles with the geoboards. You just have to be careful that your rubber bands don't break.

By listening to students verbalize their geometric understanding throughout the activities, teachers gain new insights into both what their students are capable of understanding and how their students are thinking in terms of polygon properties. Hands-on, engaging, exploratory activities such as those described in this article provide students with meaningful mathematical learning experiences while promoting not only the NCTM Geometry Standard but also the Process Standards of Communication, Reasoning and Proof, Connections, and Problem Solving.

PAULETTE R. RODRIGUE

REFERENCES

Clements, Douglas H., and Julie Sarama. "Young Children's Ideas about Geometric Shapes." *Teaching Children Mathematics* 6, no. 8 (April 2000): 482–88.

Kay, Cynthia S. "Is a Square a Rectangle? The Development of First-Grade Students' Understanding of Quadrilaterals with Implications for the van Hiele Theory of the Development of Geometric Thought." PhD Diss., University of Georgia, 1987. *Dissertation Abstracts International* 47, no. 8 (1987): 2934A.

National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics*. Reston, VA: NCTM, 2000.

Van de Walle, John A. *Elementary and Middle School Mathematics: Teaching Developmentally.* 6th ed. Boston, MA: Pearson Education, 2007.

van Hiele, Pierre. "Developing Geometric Thinking through Activities That Begin with Play." *Teaching Children Mathematics* 5, no. 6 (February 1999): 310–16.

The authors extend their deepest gratitude to the third graders of St. Joseph's Elementary School in Thibodaux, Louisiana, for their help.

Rebecca R. Robichaux, rrr102@msstate.edu, an assistant professor of mathematics education at Mississippi State University, is interested in standards-based mathematics instruction, the development of spatial visualization, and the content knowledge and beliefs of preservice and in-service mathematics teachers. **Paulette R. Rodrigue, paulette.rodrigue@nicholls.edu**, teaches undergraduate and graduate mathematics methods courses at Nicholls State University in Thibodaux, Louisiana. She is interested in standards-based mathematics instruction with an emphasis on cooperative learning and authentic assessment.

A recording sheet **template** and a sample list of "Who am I?" **riddles** accompany the online version of this article at **www.nctm.org/tcm**.

Name______________________________

Polygon properties: What is possible?

Shape-sorting Recording Sheet

Labels for Your Sort	
Some Shapes from Each Group	
Shapes in the Classroom	
New Shapes	

Written explanation (use the back if needed)

Polygon properties: What is possible?

"Who am I?" Sample Riddles

Who am I? I have 3 sides, and 2 of my sides are equal.

Who am I? I have 4 sides and no right angles.

Who am I? I have 5 sides. I am concave.

Who am I? I have 3 sides and 2 obtuse angles.

Who am I? I have 8 sides. None of my sides are equal.

Who am I? I have 3 sides. All of my angles are acute.

Who am I? I have 4 sides. Exactly 2 of my sides are equal.

Who am I? I have 3 sides and exactly 1 obtuse angle.

Who am I? I have 4 sides and exactly 1 right angle. I am convex.

Who am I? I have 3 sides and 3 obtuse angles.

Who am I? I have 8 sides and exactly 4 right angles.

Who am I? I have 4 sides and 2 pairs of parallel sides.

Who am I? I have 5 sides and exactly 1 right angle.

Who am I? I have 4 sides and exactly 3 right angles.

Who am I? I have 6 sides and exactly 3 right angles.

Who am I? I have 3 sides and exactly 1 right angle.

Who am I? I have 5 sides, and 2 of my angles are acute angles.

Who am I? I have 3 sides and exactly 2 right angles.

Who am I? I have 6 sides, and 4 of my angles are acute.

Who am I? I have 4 sides and exactly 2 right angles.

Who am I? I have 3 sides and exactly 1 acute angle.

Who am I? I have 8 sides and 2 pairs of parallel sides.