

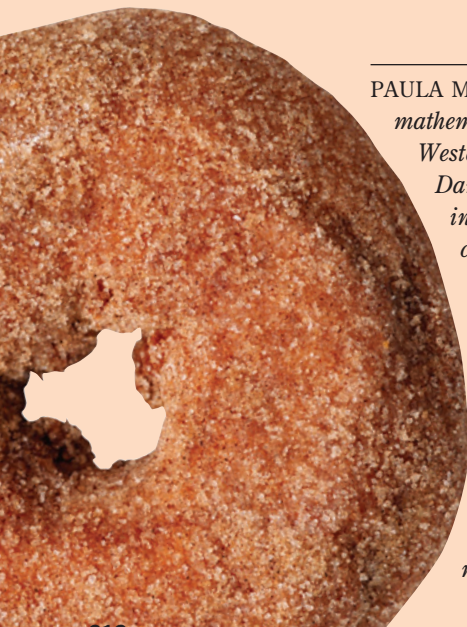


# HOW DOES YOUR DOUGHNUT MEASURE UP?

PAULA MAIDA AND MICHAEL MAIDA

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IT SHOULD COME AS NO SURPRISE THAT IMAGES of doughnuts and coffee have escalated recently, with doughnut stores erupting on many street corners and online stores available for any coffee item imaginable. In keeping with this latest trend, we created an activity for middle-grades students to apply mathematical formulas while exploring the geometry of a doughnut. The rationale for this activity was for the students to develop more sophisticated thinking of surface area and volume. We conducted this activity in two sixth-grade classes at an all-girls school. An entire forty-five-minute class period was used to complete the activity and answer the majority of the discovery questions. The remaining questions were completed as a homework assignment.



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## How Does Your Doughnut Measure Up?

### A. INITIAL THOUGHTS

Estimate the volume of your doughnut in cubic centimeters. \_\_\_\_\_

Estimate the surface area of your doughnut in square centimeters. \_\_\_\_\_

Estimate the volume of the doughnut box in cubic centimeters. \_\_\_\_\_

Estimate the surface area of the doughnut box in square centimeters. \_\_\_\_\_

### B. MEASUREMENTS

Measure the following dimensions of your doughnut and box. Record them in the chart.






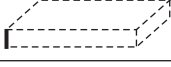

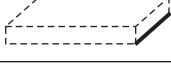
Circumference of outer circle (in cm)		
Circumference of inner circle (in cm)		
Diameter of outer circle (in cm)		
Diameter of inner circle (in cm)		
Height of doughnut (in cm)		
Height of box (in cm)		
Length of box (in cm)		
Width of box (in cm)		

Fig. 1 Parts A and B started students rolling on the activity.

## Supplies and Preparation

THESE ITEMS ARE NEEDED FOR THE ACTIVITY:

- Double-sided, two-page, stapled handout (figs. 1–3)
- One doughnut per student (use a typical doughnut that has a hole in the center)
- Empty doughnut boxes that hold a dozen doughnuts (ask a doughnut store manager for these and explain that they are for a school experiment)
- Centimeter cubes
- Flexible tape measures

## How Does Your Doughnut Measure Up?—Continued

### C. CALCULATIONS

Calculate the following areas, showing your work in the workspace that is provided. Note that your calculations will be approximations because of the irregularities of your doughnut. Record your answer in the answer column. Do not forget to label the units.







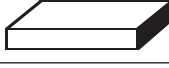

	Workspace	Answer
Area of outer circle		
Area of inner circle		
Area of top of doughnut		
Area of outer lateral surface of doughnut		
Area of inner lateral surface of doughnut		
Area of front face of box		
Area of side face of box		
Area of top face of box		

Fig. 2 In part C, students calculate measurements.

- Calculators
- Napkins

Each student received a napkin, doughnut, a handful of centimeter cubes, and the stapled handout. Students brought calculators to class. The students shared the tape measures and the doughnut boxes. Because of taste preferences, the doughnuts used in this activity were chocolate and vanilla frosted, however, any variety would be appropriate (e.g., plain, glazed) as long as they have holes in the center.



In Battista's research regarding students' thinking about area and volume measurement, he claims, "Instructional tasks must also encourage and support students' construction of personally meaningful enumeration strategies (i.e., those that are based on properly structured mental models). Students' construction of such strategies is facilitated, not by 'giving' them formulas, but by encouraging students to invent, reflect on, test, and discuss enumeration strategies in a spirit of inquiry and problem solving" (Battista 2003, p. 135). After the students explored and discussed possible attributes to be measured, we prompted them to decide on sensible units for their measurements. They unanimously agreed on centimeters (which they confessed was because they saw the centimeter cubes on the table and thought this manipulative could be useful in their estimations). As a class, we explored other units of measurement, such as millimeters (metric) and inches (customary), and agreed that although many of the choices given would be suitable, the centimeter was the unit with the greatest vote.

Students began estimating the volume and surface area of their doughnut and the volume and surface area of the doughnut box in part A of this activity (see **fig. 1**). All students chose to use the centimeter cubes for their estimation. Some students had difficulty doing this part quickly. As opposed to an educated guess, they wanted to measure and use their calculators to "get it right." Others attempted to completely build a replica of the doughnut using their centimeter cubes. "An important point from a mathematical perspective is that such (layer) structuring is more general and powerful than using standard area and volume formulas. For example, layer structuring is extremely useful for thinking about the volumes of cylinders and many problems in calculus" (Battista 2003, p. 129).

The teachers briefly reminded the students about sensible ranges versus exact answers and the ability to choose an estimate rapidly. This part of the activity illustrated that although these students have had frequent opportunities to estimate, multiple estimation experiences and frequent exposure are necessary for students to build comfort and skill in their estimation abilities.

Before leaving part A of this activity, we spent more time focusing on the volume and surface area of the doughnut only. The students had just finished making their estimates and now we wanted them to think of ways to refine those measurements with better accuracy. It took awhile for them to recognize an approach. A student commented that she knew the volume of a right circular cylinder and that the doughnuts looked like short right circular cylinders. Another student debated that this was not true be-



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### Initial Thoughts

WE BEGAN BY ASKING THE STUDENTS TO concentrate on their doughnut and list its various attributes. Replies included "has a hole in the middle" and "has frosting on the top." We continued by asking them to reflect on its mathematical characteristics. Responses included direct comparison measures ("Mine is wider than hers"), nonstandard measures ("About one large marshmallow can fit in the space inside my doughnut"), and standard measures ("My doughnut is about three and a half centimeters high"). We questioned students further: "What types of attributes might a doughnut manager be concerned about and why?" Students commented that he or she would want a dozen doughnuts to fit inside a box "without getting smushed" and that the box should have enough room to leave space for the frosting. We also asked them to distinguish among one-, two-, and three-dimensional measures. Sample responses included the "height of the doughnut" as one-dimensional; the "volume of the doughnut box" as three-dimensional; the "area of the top of the doughnut box or of the top of the doughnut" as two-dimensional; and the "diameter of the doughnut" as one-dimensional. The students had knowledge of geometric terminology and formulas from previous class lessons, so this activity reinforced these concepts. Consequently, this activity was more self-guided for students than teacher-guided. After hearing many more answers, we considered this set of questions to be a smooth segue into the planned activity.

cause of the doughnut's hole. After another moment, another student reasoned that the hole itself looked like it was a right circular cylinder that had been removed. A brief discussion continued as students reasoned that they could calculate the difference between the volumes of two right circular cylinders, the "larger" one minus the "smaller" one, to calculate the volume of the doughnut. Although the teachers supported this discussion, we could not let the activity branch off without citing some mathematical inaccuracies. First, we showed students a manipulative of a *torus*, which is a doughnut-shaped solid. Second, we used a miniature Slinky toy and a circle to illustrate how a circle can be revolved to generate (or trace out) that torus shape. We briefly explained that determining the volume of this solid of revolution requires calculus, which is discussed in later grades, and that since integrals are not readily accessible to them, we would be content with close approximations of a short right circular cylinder with an inner cylinder removed. Actually, one student commented that these particular frosted doughnuts did actually look more "cylinder-like" than "torus-like," possibly because of the specific cutter used and that doughnut's consistency. We admit that her observation was accurate but nonetheless agreed that the students' real-life doughnuts with miscellaneous bumps could not be identified as any perfect shape. Therefore, we agreed to be comfortable with their suggestions of using cylinder formulas, acknowledging their final calculations as very clever approximations.

Similar discussions occurred relating to the surface area of the doughnut, as discussed in detail in the discovery section addressed in **figure 3**. Incidentally, this initial discussion already raised the students to a level of thinking that was higher than their previous work with geometric formulas. This brainstorming session produced greater results than the teachers had anticipated.

## Measurements

IN PART B OF THE ACTIVITY IN **FIGURE 1**, TITLED "Measurements," students measured and recorded various dimensions of their doughnut. Although the worksheet had been predesigned by the teachers, the students had already suggested measuring the majority of these dimensions during the initial classroom discussion. We encouraged students to measure as accurately as possible, to the nearest tenth of a centimeter. They shared flexible tape measures for this part of the experiment. The students did an excellent job with labeling units. Comments made by the students such as "This is cool" and "This is awesome" suggested that they were enjoying the

## How Does Your Doughnut Measure Up?—Continued

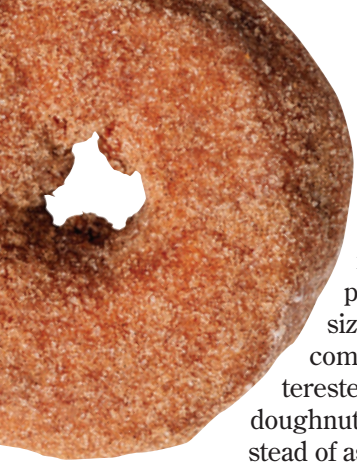
### D. DISCOVERIES

1. (a) Calculate the volume of your doughnut. Show the step(s) you used. Label the units. Note that your calculations will be approximations because of the irregularities of your doughnut.
  - (b) Do you think the volume you calculated for your doughnut is an overestimate or an underestimate of the actual volume of the doughnut? Why?
2. (a) Calculate the surface area of your doughnut. Show the step(s) you used. Label the units. Note that your calculations will be approximations because of the irregularities of your doughnut.
  - (b) Your younger sister asks you, "What do you mean by surface area of that doughnut?" What types of doughnuts might be helpful in your explanation? How would you word your answer to your sister?
3. (a) Calculate the volume of the air inside the doughnut box. Show the step(s) you used. Label the units.
  - (b) If you put a dozen of your doughnuts into the box, explain in words how you would determine the volume of the air around the doughnuts inside the box.
  - (c) Now use the method you described to find the volume of this air.
4. (a) Calculate the surface area of the doughnut box. Show the step(s) you used. Label the units.
  - (b) Compare this calculation with your original estimate of the box's surface area in part A. What did you notice?
5. (a) According to a Dunkin' Donuts Web site, "Dunkin' Donuts sells more than 6 million donuts a day, a whopping 2.3 billion a year." If the 2.3 billion doughnuts sold each year were only powdered doughnuts, estimate the amount of powdered sugar needed to cover them.
 

*Source:* Dunkin' Donuts. "Dunkin' Donuts 5 Points." [www.dd5points.com/donuts.htm](http://www.dd5points.com/donuts.htm)

  - (b) Which did you rely on for your answer: the surface area of the doughnut or the volume of the doughnut? Why?

**Fig. 3 Part D, titled "Discoveries," asked students to explain their doughnut exploration.**



activity. They were truly immersed at this point and were also interested in comparing their own measurement with their peers' as they progressed. We also emphasized our expectations of clear mathematical communication. For example, if they were interested in the diameter measure of a classmate's doughnut, they needed to use the term *diameter* instead of asking, "How far across is yours?" The average of the measurements found by the twenty-four students is as follows:

- Circumference of outer circle (in cm): 30.3 cm
- Circumference of inner circle (in cm): 7.5 cm
- Diameter of outer circle (in cm): 9.4 cm
- Diameter of inner circle (in cm): 2.7 cm
- Height of doughnut (in cm): 3.3 cm
- Height of box (in cm): 5.0 cm
- Length of box (in cm): 38.0 cm
- Width of box (in cm): 29.3 cm

This activity helped students realize that when real-world objects do not perfectly "fit the mold," then adjusting measurements might be considered. For instance, when some doughnuts were more oval than circular, students made reasonable adjustments for their doughnut's diameter. They learned that tweaking measurements can be an acceptable and sensible way of working with real-world information.

## Calculations

THE STUDENTS HAD BEEN DROOLING FROM THE smell of their doughnuts while taking measurements, so we said that they could go ahead and eat them after they performed their calculations, since all measurements were complete. Students used their own

calculators to complete the table in **figure 2**.

The pictorial representations displayed on the worksheet provided additional clarification of the terminology. Those diagrams were created in The Geometer's Sketchpad computer program using corresponding measurements. This means, for instance, that the rectangle illustrating the "area of the outer lateral surface of the doughnut" was purposely created with its length being equal to the circumference of the circle depicted for "area of the outer circle."

The only recurring question in part C of **figure 2** was the meaning of the outer and inner lateral surface of the doughnut. Foreseeing the need for this explanation, we prepared a manipulative in advance. It was created from a Styrofoam right circular cylinder purchased from a craft store. The cylinder was approximately the size of a doughnut, so we simply cut and discarded a small cylinder from the center. A rectangle was cut from bright colored paper to perfectly fit the outer lateral surface of the manipulative and another rectangle was cut to fit the inner lateral surface. When a student asked for clarification of the outer lateral surface, we asked her to wrap the colored rectangle around the corresponding surface of the doughnut manipulative. Each student who completed this task recognized that the length of the rectangle is the circumference of the outer circle and that the height of the rectangle is the height of the doughnut. Since they knew how to calculate the area of a rectangle, they consequently could obtain the area of the outer lateral surface. They recognized that the inner lateral surface used similar reasoning.

## Discoveries

"IN THE *SPATIAL STRUCTURING* PROCESS, INDIVIDUALS abstract an object's composition and form by identifying, interrelating, and organizing its components" (Battista 2003, p. 123). The teachers witnessed this structuring process and observed this level of thinking during conversations that students had with one another as they worked on part D of this activity, shown in **figure 3**. In part D, students were asked to reflect on their work by answering discovery questions designed by the teachers. The students chose their own approaches to answer the questions by recognizing the fundamental component pieces from their earlier work and relating them to the perceived object composition. Again, the students were engaged and active, applying formulas and clarifying terminology with one another while the teachers provided very little assistance.

Two approaches were used to determine the volume of their doughnut. In one approach, students calculated the product of "area of top of doughnut"

1. (a) Calculate the volume of your doughnut. Show the step(s) you used. Label the units. Note that your calculations will be approximations because of the irregularities of your doughnut.

Student 1

$$73.1 \cdot 4.1 = 299.71 \text{ cm}^3$$

Area of the outer circle - Area of the inner circle =  $73.1 \cdot$  the height (4.1) = 299.71, which is the volume.

Student 2

<p>Volume of outside</p> $V = \pi r^2 h$ $\approx 3.14 \cdot 5^2 \cdot 3.5$ $\approx 3.14 \cdot 25 \cdot 3.5$ $\approx 274.8 \text{ cm}^3$	<p>Volume of inside</p> $V = \pi r^2 h$ $\approx 3.14 \cdot 1.5^2 \cdot 3.5$ $\approx 3.14 \cdot 2.25 \cdot 3.5$ $\approx 24.7 \text{ cm}^3$
$\underline{274.8}$	$\underline{24.7}$
$250.1 \text{ cm}^3$	

**Fig. 4** These students showed how they calculated the volume of their doughnuts.

1. (b) Do you think the volume you calculated for your doughnut is an overestimate or an underestimate of the actual volume of the doughnut? Why?

Overestimate because if it is squared off there is extra space.

I think I overestimated the volume because the donut is not a perfect cylinder. Since I measured the widest part of the donut, my calculation overestimated the volume of the donut.

Fig. 5 Students discussed their estimates.

and “height of doughnut.” In another approach, students calculated the volume of two right circular cylinders and found their difference (see fig. 4).

The students were asked to consider if their calculated volume was an overestimate or an underestimate of the actual volume of the doughnut. Some students reasoned that their calculated volume was an overestimate because their measurements were based on a right circular cylinder with a right circular cylinder removed. That is, in their calculations, the lateral surfaces meet the bases at “sharp” rims, whereas the curvature of real doughnuts creates a “shaved off” appearance and consequently a smaller true volume (see fig. 5). A student who answered “underestimate” reasoned that she did not account for the bumps that were on her doughnut, giving it extra actual volume. Although there was no correct answer to this question, it was interesting to note that the majority of students (sixteen versus eight) answered “overestimate.” Many students who answered “overestimate” used their perception of size to answer this question, claiming that their doughnut simply did not look as big as the number they calculated.

To determine the surface area of their doughnut, most students calculated it this way: twice the area of the doughnut’s base + outer lateral surface + inner lateral surface (see fig. 6), clarifying again that their surface area calculation was an approximation to the doughnut’s true surface area. Students who answered this question incorrectly neglected the doughnut’s hole. They calculated it this way: twice the area of the outer circle + outer lateral surface. This latter response prompted the question, “Are there doughnut choices matching that surface area?” Students replied, “Yes, the filled ones such as Boston cream or jelly-filled powdered doughnuts.” While answering question 2a, “Calculate the surface area of your doughnut,” some students made extra work for themselves by referenc-

2. (a) Calculate the surface area of your doughnut. Show the step(s) you used. Label the units. Note that your calculations will be approximations because of the irregularities of your doughnut.

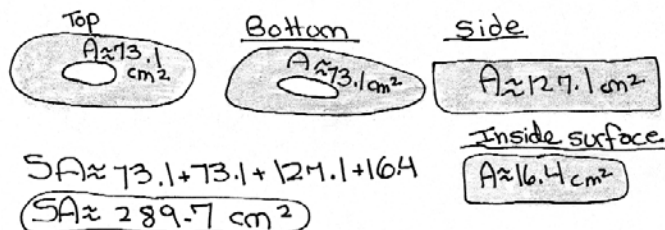


Fig. 6 A doughnut’s surface area calculations are illustrated.

2. (b) Your younger sister asks you, “What do you mean by surface area of that doughnut?” What types of doughnuts might be helpful in your explanation? How would you word your answer to your sister?

*The flavor that would be most helpful in my explanation would probably be a chocolate glazed. This is because unlike the frosted donuts the glazed donuts cover not only the top of the donut in glazed, but the entire donut. I would word my answer by saying something along the lines of “The surface area of a donut is the part of the donut that is covered in glaze. Not the inside, but outside surface of the donut.”*

*It’s like the area that covers the surface. In other words, what is wrapped around it. For example a glazed doughnut. The glaze is the surface area because it covers all of the outside. Same thing with powdered doughnuts, they are covered in powder.*

Fig. 7 These students explained surface area.

ing their initial measurements and recalculating it in the space provided, instead of using the calculations they had already recorded in part C’s table.

Next, students were asked to describe the surface area, relevant to the doughnut, to a younger child. They were encouraged to consider doughnut flavors to aid in their explanation. The responses to this question were right on target. Their clear, correct explanations suggested that they had a good conceptual grasp of surface area (see fig. 7). One student reasoned that the surface area relates to the amount of doughnut you can see without taking a bite; another described it as the area of everything on the outside of the doughnut that you can touch.

3. (b) If you put a dozen of your doughnuts into the box, explain in words how you would determine the volume of the air around the doughnuts inside the box.

First, I would find the volume of one donut, then multiply it by 12 (dozen). This would let me know how much room the donuts take up. Then I would find the volume of the box. This shows me how much air there is. Then I would subtract the 12 donuts' volume from the air in the box to get the air around the donuts in the box.

**Fig. 8** The focus having moved from doughnut to doughnut box, this student investigated the volume of the air around the doughnuts inside the box.

The focus then turned from their doughnuts to the doughnut box. The students calculated the volume of the air inside the empty box and then imagined placing a dozen doughnuts inside the box. Using descriptive sentences first (asked in question 3b), then mathematical calculations (requested in question 3c), students determined the volume of the air around the dozen doughnuts inside the box (see **fig. 8**). (It should be noted that the students' calculations were slightly flawed since they measured the exterior of the box, and the volume of the air inside the empty box does not include the cardboard material. We discussed this important detail with the students, and they adjusted their measurements accordingly.)

The students did a good job calculating the surface area of the doughnut box. However, as was the situation for question 2a, "Calculate the surface area of your doughnut," we recognized that many students duplicated their work. They showed thorough calculations using their initial measurements and recalculated the areas of the faces of the box. After questioning the students, we learned that their extra work was a result of the directions asking them to "Show the step(s) you used." The intention was for them to quickly reference the last three rows of their table in part C, which already contained the necessary work shown in the workspace, and only show the summation step calculation. In future renditions of this activity, the teachers will place greater emphasis on using results from part C. According to a Dunkin'

5. (b) Which did you rely on for your answer: the surface area of the doughnut or the volume of the doughnut? Why?

*I relied on the surface area of a donut. This is because the surface area is the outside of the donut, and the part of the donut which you put the sugar on. If I used the volume, then I would be filling the inside with sugar.*

**Fig. 9** This student explained how the earlier question was answered.

Donuts Web site, "Dunkin' Donuts sells more than 6 million donuts a day, a whopping 2.3 billion a year" (Dunkin' Donuts 2004). The students were asked to estimate the amount of powdered sugar needed to cover those 2.3 billion doughnuts. They used various units for their answers (including  $\text{cm}^2$ ,  $\text{in.}^2$ , cups, grams). Most students multiplied the surface area of their doughnut (found in question 2a) by 2.3 billion, using  $\text{cm}^2$  as units. We intended for the students to think in terms of units different than  $\text{cm}^2$ , such as cups, tablespoons, or teaspoons. However, the question was not clear on this point. When the activity is implemented again, we will consider revising the last sentence in question 5a so that it reads "estimate the number of cups of powdered sugar needed to cover them." All students answered "surface area" for question 5b (see **fig. 9**).

## Extensions

IN ONE OF THE TWO CLASSES WE WORKED WITH, we extended the applications by stopping the students after they calculated their doughnut's volume (see question 1a in **fig. 3**). The students were then asked to estimate the number of doughnuts that Dunkin' Donuts sells each year. The teachers shared the Web site estimate described above, and asked, "What is the volume of 2,300,000,000 of *your* doughnuts per year?" Students multiplied the volume of their own doughnut by 2.3 billion. The teachers continued, "If the volume of Earth is approximately  $1.09 \times 10^{27} \text{ cm}^3$ , then how many years would it take to fill up Earth with your doughnuts?" We briefly discussed the idea of breaking the doughnuts up so that gaps and holes were filled in. This was a loaded problem with connections to science, calculations with large numbers, the use of scientific notation, and real-world applications. With answers near  $2 \times 10^{15}$  years, students were surprised at how large planet Earth really is. The teachers asked questions: "Will your great, great, great, great grandchildren still be busy filling up Earth



with your doughnuts?” It was difficult for students to fathom this number of years. There was a lot of chatter in the room and comments of amazement.

## Conclusions

IN BONOTTO’S RESEARCH REGARDING STUDENTS’ learning and understanding of surface area, she writes, “Traditional classroom teaching often seems to favor the separation between classroom and real-life experience. This rift is particularly significant with respect to the concept of *surface*” (Bonotto 2003, p. 157). In addition to surface area, this same argument can undoubtedly be made for volume as well. Applications of measurement and geometric vocabulary (including circumference, diameter, face, lateral surface, volume, surface area) enhance comprehension and recollection of the terminology. If we are to expect our students to recognize the geometry and measurements surrounding them, we should offer frequent, engaging activities for reflection on their physical environment.

In this activity, students used verbal discussions and clever reasoning to further develop their level of thinking related to measurement. Their written expla-

nations and documentation, feedback, and subsequent assessments revealed a greater understanding of the concepts of volume and surface area.

Furthermore, this activity encouraged students to analyze a three-dimensional shape that did not exactly correspond with the definition of a right circular cylinder. The hole and curvature of this concrete model required more discerning thought during measurements and calculations. This classroom activity using geometry and measurement allowed students to witness how powerful and attainable mathematics truly is.

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