



BREAK THE

By Amy Roth McDuffie
and Norma Eve

AAR BOUND

Understanding the concept of area is a challenge for children. Beyond following procedures with formulas, studying area has been a source of confusion for students at all mathematical levels (Nitabach and Lehrer 1996; Thompson and Preston 2004). In fact, National Assessment of Educational Progress (NAEP) data from fourth and eighth graders suggest that many students have incomplete or superficial understanding of area (Martin and Strutchens 2000). Although

students continue to struggle, the 2003 NAEP data shows that performance on area items is significantly improving, which may be due to an increased curricular focus on area concepts and the use of manipulatives and visualization (Blume, Galindo, and Walcott 2007).

In the past, instruction about area often focused more on learning procedures for measuring rather than on learning underlying concepts (Stephan and Clements 2003). To develop conceptual understanding, primary students

Follow this professional learning team's lesson design aimed at helping second graders understand area by covering physical surfaces with many identical units.

AREA ARIES



need experiences with (1) partitioning a region with a two-dimensional unit of measure; (2) iterating a unit to cover the region without space or overlap; (3) exploring conservation of area (i.e., the area stays the same even if a region is cut and rearranged into different shapes); and (4) structuring an array. Structuring an array is considered the most sophisticated concept to learn in preparation for using formal procedures of calculating area by multiplying two lengths (Stephan and Clements 2003).

A team effort

With these challenges and concepts in mind, primary-grade teachers who participated on a professional learning team (PLT) designed and studied a lesson for second graders. They decided to focus on the first three area concepts identified at left. Structuring an array was not included because the concept is a fourth-grade focus in the team's state standards and is not introduced before third grade in *Curriculum Focal Points* (NCTM 2006). Consistent with



research on children's learning of area, the PLT members recognized from past experiences that building understanding of area is difficult for students, and they sought ways to improve their teaching and their students' learning of this concept.

The team formed as part of a schoolwide, practice-based professional learning program. School-based collaborative teams provide a promising approach to improving teaching and learning (Ebby, Otinger, and Silver 2007; Little et al. 2003). This team adapted approaches from lesson study to design, implement, analyze, and reflect on lessons (Lewis 2002; Post and Varoz 2008; Wang-Iverson and Yoshida 2005). The team comprised all four second-grade teachers at the school and article co-authors Roth McDuffie (a university mathematics educator) and Eve. As the school mathematics specialist, Eve works with teachers and students at grade levels K–5 to improve mathematics teaching and learning. She facilitated the PLT meetings, compiled the notes, and recorded lesson plans. Roth McDuffie provided external support by recommending readings, reviewing and commenting on lesson plans (often via e-mail), observing lessons, and participating in postlesson discussions.

Prior teaching and learning

Before the lesson on area was presented, students studied the unit “Shapes, Halves, and Symmetry” from district-adopted curriculum materials in the Investigations in Number, Data, and Space series (TERC 1998). Students first explored pattern block relationships (e.g., using a blue rhombus and a green triangle to build a red trapezoid). Next, students completed activity sheets that contained various shapes with prompts to predict how many of a given pattern block they would need to cover

the shapes (e.g., how many blue rhombuses would cover a large parallelogram). After predicting, students covered a shape with pattern blocks, counted the number of blocks they used, and then recorded the count. Finally, in a computer-based activity that presented shapes, students identified pattern blocks to cover the shapes with the fewest number of blocks and then recorded the number in a table.

PLT members appreciated the Investigation approach to teaching and learning area; however, after implementing the unit, the teachers perceived that students needed additional experiences to solidify their emerging understanding of area. Teachers realized not only their need to provide more hands-on experiences with partitioning and covering but also their failure to fully capitalize on opportunities for students to communicate about their approaches, including justifying and reflecting on their solutions. Although the teachers could have implemented the Investigations lesson with an emphasis on communication, they believed that most students were not yet ready to share and reflect on their approaches—they needed time to explore the shapes.

However, without the focus on teaching students to communicate and justify their solutions, teachers remained convinced that not all students understood how to partition shapes without overlap or space. Nor did they understand that a shape could be covered in more than one way (a fundamental understanding for conservation of area). Moreover, PLT members wanted to extend the Investigations goals for learning about area by explicitly discussing the blocks as units of measure for area. In sum, the team teachers sought to develop a lesson that provided additional experiences covering shapes yet was consistent with the students' previous experiences with the Investigations unit. The teachers also wanted to encourage students to communicate and justify their solutions and to build more formal understanding of units of measure.

The new lesson plan

After examining Washington State grade-level expectations, *Principles and Standards* (NCTM 2000), and students' assessment data, the school's PLT members (across all grades) had decided to focus on improving teaching and

Student strategies to find area

To support Marcia's second graders' ability to clearly explain their reasoning, she encouraged them to take specific actions as they recorded their solutions:

- **Cover** the shapes with blocks to find the area.
- **Compute** the area with numbers. For example, if you cover a shape with trapezoids, you might add groups of three to find the area in triangle units.
- **Communicate** your thinking with numbers (by writing equations) and with words (by labeling).

learning of measurement and communication. This team narrowed the measurement focus—to area—and reviewed relevant research and theory, using Van de Walle (2004) as a resource. Team members also examined their use of the Investigations unit “Shapes, Halves, and Symmetry” (TERC 1998), determining that Investigations materials are consistent with research on teaching and learning area. Nevertheless, they found that implementing the materials did not result in all students developing a strong understanding of area concepts. It was at this point that they had identified the need for an additional lesson as described earlier.

As a result of these realizations, the team decided to supplement the Investigations materials with a lesson from the Bridges in Mathematics curriculum (Snider and Burk 1999). The Bridges materials align with Investigations to build on students’ prior experiences, but they also provide opportunities to meet identified needs. The lesson involved using a triangle pattern block as a unit to determine the area of other shapes (e.g., rhombus, hexagon, and trapezoid) and then to “prove” the answer. Students were encouraged to show their work—either by tracing on paper the blocks that they had used to cover the shape or by gluing die-cut pieces onto a cutout of the shape. This added step of tracing or gluing (beyond covering a shape with blocks) provided an opportunity for students to recognize the importance of not overlapping units or leaving space—especially as students shared their work with one another.

Teachers next presented more complex shapes, such as the letter **A**. The team recognized that the lesson also provided opportunities for students to compute with addition and formalize their thinking by writing equations. Students were to find total area by partitioning a complex area into known shapes (e.g., rhombuses, worth two triangle units each, are used in combination with triangles to cover the **A** shape, and the area can be found by adding the areas of the smaller shapes).

The PLT developed a detailed lesson plan to meet these goals. The lesson design includes a launch-explore-summary structure also used in Connected Mathematics materials (Lappan et al. 2002). With such a structure, the teacher launches a lesson by posing a problem situated in a meaningful context, which generates interest



and allows the teacher to assess student knowledge about the topic. The explore phase involves students (often in small groups) in investigating problems and various solution methods. In the summary phase, which highlights the important ideas, the teacher leads a discussion as students share their understanding and approaches. Although Investigations does not use the same *launch-explore-summary* terms, the approach is consistent with the Connected Mathematics materials’ design, so the structure resonated with the teachers’ past practices.

Discussing this structure helped the teachers clearly understand the goals and purposes for parts of the lesson. They decided as a team to explicitly plan each part of the lesson and work on the summary portion in particular. The teachers found that they often spent too much classroom time in the explore phase, which did not allow enough time for the summary. Moreover, they often left to chance which solution methods were shared, depending on student volunteers. Thus, the team decided to deliberately plan when to end the explore phase and how to select students to share their solutions.

After planning the lesson, Marcia (a pseudonym) volunteered to teach the lesson. All the PLT members agreed to focus their observations on one of three questions about students’

Team members knew that all the students would not be able to complete the entire activity sheet in the time provided, so the teacher prompted students to use the last few minutes of exploration time to finish at least two shapes—an **A** and a rocket—that the class would discuss together.

learning and work during the lesson (see **table 1**).

The principal arranged for substitute teachers so that PLT members had a free half-day to observe Marcia teaching the lesson and to

discuss it afterward. A district mathematics coach also observed the lesson and participated in the postlesson discussion, providing an additional perspective.

Before Marcia presented the lesson, the PLT met to review plans and confirm teachers' assignments for observation foci. Immediately following the lesson, the PLT met again for an hour to discuss their observations, analyze and reflect on the lesson, and develop ideas for revising the lesson.

Implementation

Marcia followed the lesson plan designed by the PLT, intentionally focusing on communicating and justifying methods and using triangles as a unit of measure. To launch the lesson, Marcia showed students a green equilateral triangle pattern block and asked, "If the area of this triangle is one unit, what is the area of each of the shapes below? Find a way to prove your answers. So instead of just telling me a number, I want you to tell me how you figured out the answer."

Next, Marcia placed a blue rhombus on the document camera and asked, "How many triangles would it take to cover the blue rhombus?" The PLT's learning targets of using triangle units and justifying were evident in how Marcia directed the ensuing discussion.

Consistent with the PLT plans, Marcia directed the students' discussion toward proving their answers, and she provided opportunities for students to use the blocks to explore how to cover shapes. This focus aligns with *Principles and Standards*' (2000) call for teachers to develop students' reasoning and proof in the early grades by having them make conjectures and justify their thinking "empirically or with reasonable arguments" (p. 126).

Marcia then shifted the discussion to using other ways to find area, including using other shapes that represent more than one unit (e.g., covering a shape with trapezoids that are worth three unit triangles each). This shift helped to meet the PLT goals of focusing on a common unit and conservation of area. Students also discussed using just one triangle and tracing and moving it until the first shape is covered.

Next, Marcia began the lesson's exploration portion by instructing students to look in their packets and locate the shapes for which they needed to find the area. She encouraged stu-

TABLE 1

PLT members agreed to focus their classroom observations, analysis, and reflection on one of three questions.

Focus Question	Why This Question?
1. Are students engaged, resourceful mathematics learners?	To assess a schoolwide goal for engaging students and encouraging their ownership of mathematics learning
2. How do students record and prove their mathematical work?	To assess communication goals and explain reasoning
3. How do students measure and calculate area? Do they use knowledge of pattern block relationships, or do they count by ones?	To assess goals for students' understanding of— (a) partitioning the shape and iterating the unit, (b) geometric relationships between pattern block shapes, (c) strategies for measuring, and (d) computation methods

Teacher-directed discussion

The PLT's learning targets were evident in how Marcia guided this discussion:

Marcia: How many triangles would it take to cover the blue rhombus?

Marcus: Two.

Marcia: How do you know that?

Robin: Because if you split it apart, then that would be one triangle and another triangle.

Marcia: Can you come show me what you mean by "split it apart?" [*Using the document camera, Marcus covers the rhombus with two triangles and then separates them slightly.*] So you've shown me you can split that apart into two...?

Marcus: Two triangles.

Marcia: OK. What if I wanted to know the same thing about the hexagon? If I know that the rhombus is worth two, how much will that hexagon be worth?

Sarah: Four?

Marcia: Can you come show me why you think it's going to be worth four?

Sarah: [*arranging four triangles on the hexagon*] Hmmm.

Marcia: Are you revising your thinking? So, how many do you think it is worth?

Sarah: Eight? Or seven? [*She continues to put triangles on the hexagon.*]

Marcia: Well, try it and see how many actually cover it up. So, how many cover it?

Sarah: Six.

Marcia: So, when you finally had it covered, it had six. So, what is that hexagon worth?

Sarah: Six.

Marcia: So, Marcus told me this [rhombus] is worth two. And this [hexagon] is worth six. I want you to think of another way to prove that the hexagon is worth six. Think about if you didn't have six green triangles to use. What's another way you could do it? I want you to turn to your partner; whisper to your partner.

dents to clearly explain their reasoning as they recorded their solutions and gave them strategies to do so. Once students began to work, Marcia circulated, asking them to self-assess whether they had provided enough information for others to understand their thinking.

On the basis of having observed students participating in the Investigations unit, the PLT anticipated that the youngsters might count the pattern blocks they had used instead of attending to the triangle units (i.e., recording an area of 2 when two trapezoids covered the shape instead of recording 6 triangle units). Indeed, Marcia and other PLT members observed several students making this error. Marcia prompted students as the team had planned:

When we did other problems, we only wanted to know how many blocks we used. [But with these problems] I really want to know how many triangles it would take—even if you use other blocks. If you count it in triangles, how many triangles would it be?

As students explored, Marcia observed and noted students who solved the problems in the various ways anticipated in PTL planning, including covering with triangles only and then drawing and labeling each triangle with counting numbers to find the total; covering with another shape and then counting by the shape's value (i.e., using a rhombus and then counting by 2s for each rhombus); or using a variety of shapes to chunk the triangles (i.e., as trapezoids and rhombuses) and then labeling each shape with its value in terms of unit triangles.

Knowing that all students would not complete the entire activity sheet in the time provided, the team had planned to begin the summary after everyone finished at least two of the more complex shapes, an **A** shape and another shape that resembled a rocket. So, after most students had completed these shapes, Marcia prompted the class to use the last few minutes of the exploration time to make sure they had completed a minimum of these two shapes because the class would soon be discussing them.

Marcia began the summary by gathering the class around a chart she had prepared with four **A** shapes. She explained that she had selected four students to share so that the class could see a variety of ways to find area. Throughout

the summary, Marcia deliberately focused on the learning targets, asking questions and offering prompts such as the following:

- **Which** blocks did you use to cover the **A**?
- **Show** me how you covered the **A** and then counted to find the area.
- **Write** your equation to show how you found the area.

Figure 1 shows the four different approaches that the students used to find area:

- **Student 1** drew and labeled the value of each trapezoid as 3 and the triangle as 1 and then wrote an equation to find the sum of 13.
- **Student 2** used the same approach but covered the **A** with blocks instead of drawing the blocks on the **A**.
- **Student 3** covered the **A** with a combination of rhombuses and triangles, counted by 2s for each rhombus for a total of 8, counted the 5 triangles, and combined for a sum of 13. (If you look closely, you can see writing on the poster that indicates this third student struggled to add $8 + 5$. The rest of the class eventually helped her see that the sum is 13.)
- **Student 4** covered the **A** entirely with triangles and counted 13 triangles.

Throughout the discussion, Marcia commented that all four students arrived at an area of 13 triangle units, but each used a different approach.

A look back

After teaching and observing the lesson, the team teachers and guest observer met to discuss their reflections and analyze their obser-

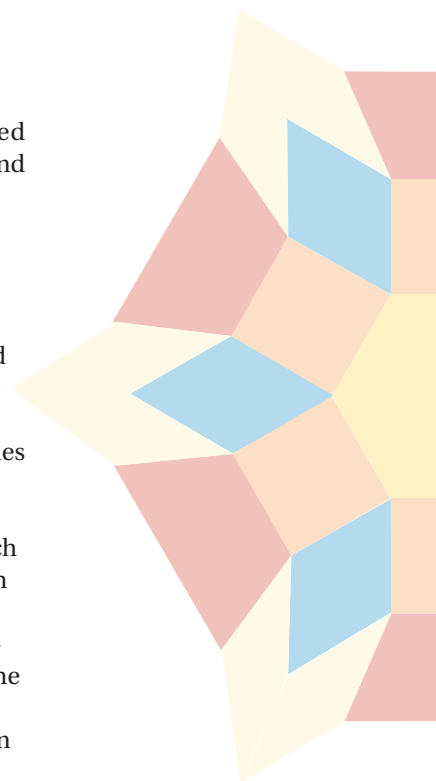


FIGURE 1

Four students arrived at the same answer of 13 triangle units for the area of **A**, but each used a different approach.





Students found total area by partitioning a complex area into known shapes.

variations, supporting the latter with examples of students' work and comments. The goals for this reflection session were to discuss students' methods and difficulties, the objectives of this and previous lessons, ideas to challenge students, and ways to follow up the lesson. The group noticed much more about students' methods than any one person could have seen during the teaching process.

Many students had struggled—more than the team had expected—with using the unit triangle to find the areas of shapes. Several students initially counted blocks without regard to the unit. One student said that the **A** had an area of 5 because she covered it with 4 trapezoids and 1 triangle. In another case, a teacher observed a student counting only triangles and disregarding other blocks; the youngster said that the **A** had an area of 1 because she used only 1 triangle.

By the end of the lesson, students had changed their approaches and had begun to attend to the triangle unit. However, many appeared hesitant to use triangle units, checking and re-doing their work several times before they were convinced they could move ahead.

After sharing these initial observations, team members analyzed why students had more difficulties than expected and whether the lesson was appropriate for students' needs. The team revisited one of the identified purposes for this lesson: to provide students with more experiences covering shapes, focusing on a triangle unit, and translating to that unit. Prior experience seemed to lead students to believe that they simply needed to count blocks. Previous lessons about area involved covering shapes with square tiles or a given block. Students had not needed to consider the unit; they had simply counted the number of blocks they used. Moreover, earlier units had focused on spatial sense; students had covered shapes with more than one kind of pattern block, and again, had no need to focus on a common unit.

Acceptable struggles

Subsequent to questioning the purposes of these earlier lessons and whether they were an impediment to developing understanding of area, the PLT decided that the earlier lessons served an important role in developing spatial sense and understanding of covering and that such understanding was foundational to the current lesson. However, they also decided that supplementing the unit with this lesson was important for focusing directly on a single unit of measure. One teacher asked, "Do they need a better understanding of area and units *before* this lesson [to alleviate the struggle], or should that come *after* this lesson?"

The team perceived that students need the experiences of exploring and confronting their limited understanding that finding area means to "just cover with blocks and count the blocks." Then students must recognize the importance of a common unit in finding area.

Acceptable challenges

Another issue that arose in PLT discussions was how to challenge students who are ready to take on more complex problems. As part of the initial planning, the team intended to ask students who completed the packet to create their own shape and find its area in triangle units. Students were limited to a shape with an area no greater than 80 triangle units (to maintain focus on area, rather than on making designs). The teachers observed that the few students

who worked on this activity were engaged in creating the shape and finding more efficient strategies to measure area (e.g., chunking blocks into a larger shape and using repeated addition or multiplication).

Other challenges could include pursuing the inverse relationship between the size of the block and the number of blocks required to cover a shape (i.e., a smaller unit requires a larger quantity of units to cover a shape). For example, teachers could ask students, “If you use a trapezoid as a unit of measure instead of a triangle, would the area of the shape take a larger or smaller quantity? Is the actual area still the same?” Students could predict and then test their conjecture using different shapes as units.

The variety of methods that students used to find area impressed the PLT members. Their role of providing opportunities for students to use their own approaches to develop understanding was underscored when the teachers

New questions

Collaborative lesson design deliberately concentrates on students striking specific learning targets—more so than lessons that any single member could design. This process generated additional questions for the team:

1. What does proof look like for primary students?
2. How much do we need to justify thinking?
3. We selected a range of methods for students to share; should we have had them share in a different order?
4. The student who chunked the units as indicated on the sheet shared first; instead should the last student (who covered the **A** entirely with triangles) have shared first?
5. We planned to end the explore portion after the students had attempted two key shapes; was this appropriate?

saw the range of both correct and incorrect practices in their summary and additional observations. However, most students were not yet confident in approaching these problems, and students needed to continue this activity the next day to solidify their understanding. Indeed, several students had asked permission to continue their work the next day. A teacher suggested—and the PLT agreed to—a discussion on the second day: The teacher would use the

second student's approach and ask the class, "Why doesn't the area of this **A** equal 5?" to directly confront the misconception that finding area always means counting the blocks.

More questions

Collaborative work during planning and postlesson discussions provided the PLT with opportunities to study students' thinking about area. Observing the lesson allowed teachers to notice complexities and nuances in students' thinking and learning. The teamwork better prepared teachers to anticipate students' responses, to make deliberate instructional decisions throughout the lesson, and to adjust the lesson for a second day. However, questions for further study remained.

Positive, lasting effects

Although the process created more questions for the team, they recognized that their collaborative lesson-designing effort was more

intentional and deliberate in ensuring that students hit specific learning targets than lessons any team member could have designed individually. Teachers later heard students refer to this lesson when they worked on problems involving area and equivalent relationships, indicating that the lesson had positive, lasting effects on students' understanding. Moreover, the PLT continued to collaborate on other lessons, finding that their work generated new perspectives on teaching and learning that informed their practices outside of math as well.

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Reflect and discuss: "Break the area boundaries"

Reflective teaching is a process of self-observation and self-evaluation. It means looking at your classroom practice, thinking about what you do and why you do it, and then evaluating whether it works. By collecting information about what goes on in our classrooms—and then analyzing and evaluating this information—we identify and explore our own practices and underlying beliefs.

The following questions related to "Break the area boundaries" by Amy Roth McDuffie and Norma Eve are suggested prompts to aid you in reflecting independently on the article, discussing it with your colleagues, and using the authors' ideas to benefit your own classroom practice.

1. What are common gaps or misconceptions in students' understanding of the concept of area? How might a student show there is a misconception about using a consistent unit to measure area? What beginning concepts need to be understood before students are exposed to deeper understanding of area?
2. How does planning explicitly for the launch, explore, and summarize portions of a lesson lead to deeper student understanding of the lesson's learning targets? What is an important aspect of the lesson's explore portion? How does anticipating the various ways students will respond to the explore section improve the impact of the lesson? What parts of the lesson's summarize portion should be considered to improve the impact of the lesson?
3. How can working together as a PLT deepen teachers' knowledge of mathematics? What are the important considerations for teachers working together in a PLT for a single lesson?

Tell us how you used "Reflect and discuss" as part of your professional development. Submit letters to *Teaching Children Mathematics* at tcm@nctm.org. Include "readers exchange" in the subject line. Find more information at tcm.msubmit.net.



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