You can also help students develop self-monitoring habits after their problem-solving activity is over, when a discussion can focus on what was done to solve the problem. In addition to discussing solution strategies, the *after* phase of the lesson should include opportunities to reflect on the metacognitive questions noted above. This can be accomplished through journals (Roberts & Tayeh, 2007) or classroom discussions, prompted by such questions as:

- What did you do that helped you understand the problem?
- Did you find any numbers or information you didn't need? How did you know?
- How did you decide what to do?
- Did you think about your answer after you got it?
- How did you decide if your answer was right?
- Did you try something that didn't work? How did you figure out it was not going to work out?
- Can something you did in this problem help you solve other problems?

As students become more independent in their study of mathematics, they are less likely to need the support of a teacher to solve problems. Their attitudes and dispositions shift related to what they think mathematics is and how competent they feel in doing mathematics.

Disposition

Disposition refers to the attitudes and beliefs that students possess about doing mathematics. Students' beliefs concerning their abilities to do mathematics and to understand the nature of mathematics have a significant effect on how they approach problems and ultimately on how well they succeed.

Students who enjoy solving problems and feel they will be successful at conquering a perplexing problem are much more likely to persevere, make second and third attempts, and even search out new problems. A lack of productive disposition has just the opposite effect.

Attitudinal Goals

- *Gaining confidence and belief in abilities* is important for a student to want to do mathematics and confront unfamiliar tasks.
- *Being willing to take risks and to persevere* improves a student's willingness to attempt unfamiliar problems and to develop perseverance in solving problems without being discouraged by initial setbacks.
- *Enjoying doing mathematics* helps a student sense personal reward in the process of thinking, searching for patterns, and solving problems.

A classroom environment built on high expectations for all students and respect for each student's thoughts will go a long way toward achieving attitudinal goals. Here are some additional ideas to help with these goals for all students.

- *Build in success.* In the beginning of the year, plan problems that you are confident your students can solve. Avoid creating a false success that depends on your showing the way at every step and barrier.
- *Praise efforts and risk taking.* Students need to hear frequently that they are "good thinkers" capable of good, productive thought. When students volunteer ideas, listen carefully and actively to each idea and give credit for the thinking and the risk that children take by venturing to speak out. Be careful to focus praise on the risk or effort and not the products (i.e., answers) of that effort, as noted earlier.
- *Listen to all students.* Avoid ending a discussion with the first correct answer. As you make nonevaluative responses, you will find many children with different approaches to the same problem or different ways to explain the same strategy. By noting their contribution, use of good mathematical language, or novel approach, you build that student's confidence and increase other students' understanding of what you expect of them.

When students have confidence, show perseverance, and enjoy mathematics, it makes sense that they will achieve at a higher level and want to continue learning about mathematics—opening many doors to them in the future. As noted earlier, though, teaching in this manner is a complete reconceptualization of your role as the teacher and of the student's role as the student. In considering such a transformation, questions are likely to arise. Even if you feel these new methods contain really good ideas, you may be wondering how to accomplish some of the recommendations and how to fit new approaches into a lesson. In the following section, a three-phase lesson plan model is explained. An adaptation of the inquiry-based science lesson model, this process will enable you to engage students in learning through problem solving and learning about problem solving.

A Three-Phase Lesson Format

In a non-problem-based lesson, teachers typically spend a small portion of a lesson explaining or reviewing an idea and then go into "production mode," where students wade through a set of exercises. Lessons organized in this explain-then-practice pattern condition students to focus on procedures, often at the expense of understanding what they are doing. Teachers find themselves going from desk to desk reteaching and explaining to individuals. This approach is in significant contrast to a problem-based lesson that tends to be built around a single problem.

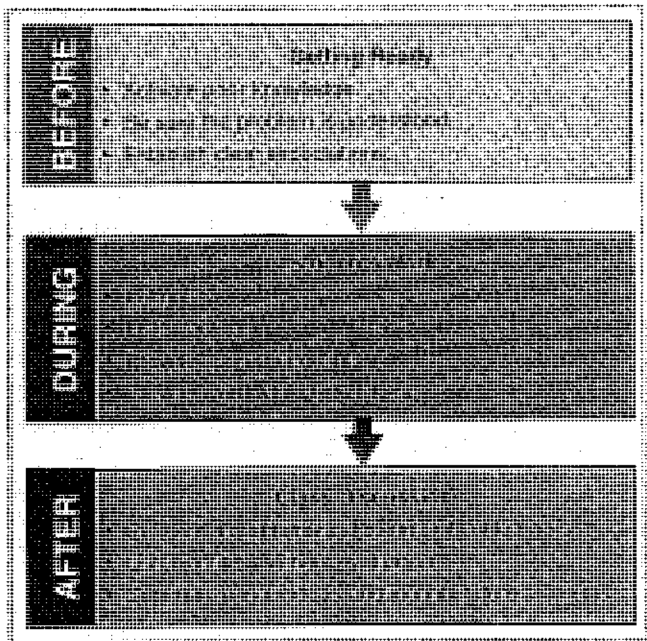


Figure 3.8 Teaching through problem solving lends itself to a three-phase structure for lessons.

It is useful to think of a problem-based lesson as consisting of three parts—before, during, and after (see Figure 3.8). If time is allotted for each segment, one problem may take a full day or even longer. There are times when a task may not merit a full lesson; a mental mathematics activity is a good example. Even here, it is useful to keep the same three components of a lesson in mind. Each part of the lesson has a specific agenda or objective. How you attend to these agendas in each portion of the lesson may vary depending on the class, the problem itself, and the purpose of the lesson.

The *Before* Phase of a Lesson

There are three related agendas for the *before* phase of a lesson:

1. Get students mentally prepared to work on the problem and think about the previous knowledge they have that will be most helpful.
2. Be sure students understand the problem so that they are ready to engage in solving it. You will not need to clarify or explain to individuals later in the lesson.
3. Clarify your expectations to students before they begin working on the problem. This includes both how they will be working (individually or in pairs or small groups) and what product you expect in addition to an answer.

These *before* phase agendas need not be addressed in the order listed. For example, for some lessons you will do a short activity to activate students' prior knowledge for the problem and then present the problem and clarify expectations. Other lessons may begin with a statement of the problem and may or may not have a readiness activity.

Teacher Actions in the *Before* Phase

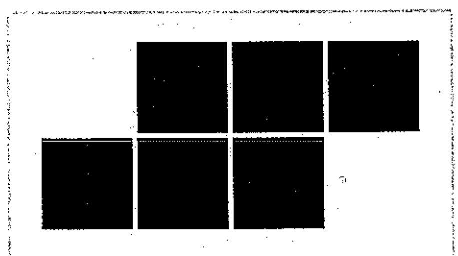
What you do in the *before* portion of a lesson will vary with the task. Some tasks you can begin with immediately. For example, if your students are used to solving story problems and know they are expected to use words, pictures, and numbers to explain their solutions in writing, all that may be required is to read through the problem with them and be sure all understand it. The actual presentation of the task or problem may occur at the beginning or at the end of your *before* actions.

1. Activate Prior Knowledge. Activate specific prior knowledge related to today's concept. What form this preparation activity might take will vary with the topic, as shown in the following options and examples.

Begin with a Simple Version of the Task. Suppose that you are interested in developing some ideas about area and perimeter. Begin by presenting the following task (Lappan & Even, 1989).

Concept: Perimeter
Grades: 4–6

Assume that the edge of a square is 1 unit. Add squares to this shape so that it has a perimeter of 18.



Instead of beginning your lesson with this problem, you might consider activating prior knowledge in one of the following ways:

- Draw a 3-by-5 rectangle of squares on the board and ask students what they know about the shape. (It's a rectangle. It has squares. There are 15 squares. There are three rows of five.) If no one mentions the words *area* and *perimeter*, you could write them on the board and ask if those words can be used in talking about this figure.
- Provide students with some square tiles or grid paper and say, "I want everyone to make a shape that has a perimeter of 12 units. After you make your shape, find out what its area is." After a short time, have several students share their shapes.

Each of these "warm-ups" uses the vocabulary needed for the focus task. The second activity suggests the tiles as a possible model students may elect to use and introduces the idea that there are different figures with the same perimeter.

The following problem is designed to help students use addition to solve a subtraction problem.

Concept: Subtraction
Grades: 2–3

Dad says it is 503 miles to the beach. When we stopped for gas, we had gone 267 miles. How much farther do we have to drive?

Before presenting this problem, you can elicit prior knowledge by asking them to supply the missing part of 100 after you give one part. Try numbers like 80 or 30 at first; then try 47 or 62. When you present the actual task, you might ask students if the answer to the problem is more or less than 300 miles.

Brainstorm Solutions. The following problem is designed to address ratios and data analysis.

Concepts: Ratios and Statistics
Grades: 6–7

Enrollment data for the school provide information about the students and their families from one class as compared to the whole school.

	School	*Class*
Siblings		
None	36	5
One	89	4
Two	134	17
More than two	93	3
Race		
African American	49	11
Asian American	12	0
White	219	15
Travel-to-school method		
Walk	157	10
Bus	182	19
Other	13	0

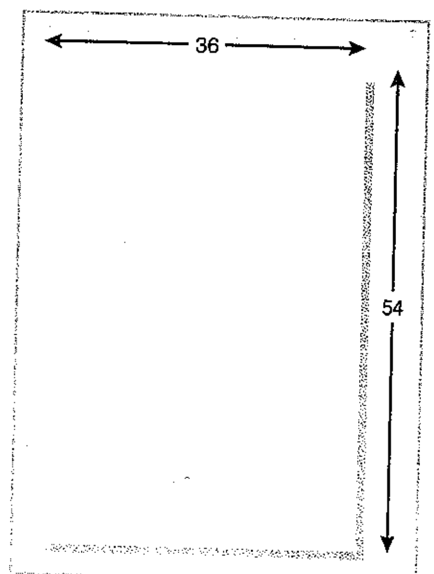
If someone asked you how typical the class was of the rest of the school, how would you answer? Write an explanation of your answer. Include one or more charts or graphs that you think would support your conclusion.

This problem does not lend itself to posing a simpler problem, but instead solicits students' prior knowledge during their thinking about how to approach the problem. For example, students might discuss (e.g., think-pair-share) what "typical" means and how they could determine what a typical class is. The teacher can list the ideas on the board for students to consider when they move into the *during* part of the lesson.

Estimate or Use Mental Computation. When the task is aimed at the development of a computational procedure, a useful *before* action is to have students actually do the computation mentally or suggest an estimated answer. This practice will not spoil the problem for the class; in fact, it may raise curiosity as to what the answer might be. This technique is appropriate for the earlier problem concerning how many more miles to go to the beach. The following task is another example in which preliminary estimates or mental computations would activate prior knowledge.

Concept: Multiplication
Grades: 4–5

How many small unit squares will fit in a rectangle that is 54 units long and 36 units wide? Use base-ten blocks to help you with your solution. Note that base-ten blocks come in ones (one cube), tens (a row of ten cubes), and hundreds (a ten-by-ten grid). Make a plan for figuring out the total number of squares without doing too much counting. Explain how your plan would work on a rectangle that is 27 units by 42 units.



Prior to estimation or mental computation for this problem, beginning with several simpler problems can help—for example, rectangles such as 30 by 8 or 40 by 60.

2. Be Sure the Problem Is Understood. Understanding the problem is not optional! You must always be sure that students understand the problem before setting them to work. It is important for you to analyze the problem in order to anticipate student approaches and possible misinterpretations or misconceptions (Wallace, 2007). Time spent at this stage of the problem-solving process is critical to the rest of the lesson. You can ask questions to clarify student understanding of the problem (i.e., knowing what it means rather than how they will solve it). For example, ask, "What do you know?" and "What do you need to know?" Wallace, a mathematics researcher and teacher, notes, "The more I questioned *prior to* giving the problem, the less help the students needed from me *during* problem solving" (p. 510).

Consider a problem-based approach to mastering the multiplication facts, a term used for the basic multiplication tables. The most difficult facts can each be connected or related to an easier fact already learned.

Concept: Multiplication Facts
Grades: 3–4

Use a "helping fact" (a multiplication fact you already know) to help you solve each of these problems: 4 × 6, 6 × 8, 7 × 6, 3 × 8.

For this task, it is essential that students understand the idea of using a helping fact. They have most likely used helping facts in addition. You can build on this prior knowledge by asking, "When you were learning addition facts, how could knowing 6 + 6 help you figure out 6 + 7?" You may also need to help students understand what is meant by a fact they know—one they have mastered and know without counting.

In the case of a word problem, like the one below, it is important to help them understand the meaning of the sentences, without giving away how to solve the problem.

Concept: Multiplication and Division
Grades: 3–5

The local candy store purchased candy in cartons holding 12 boxes per carton. The price paid for one carton was $42.50. Each box contained 8 candy bars that the store planned to sell individually. What was the candy store's cost for each candy bar?

Questions might include: "What did the candy store do? What is in a carton? What is in a box? What is the price of one carton? What does that mean when it says 'each box'?" The last question here is to identify vocabulary that may be misunderstood. It is also useful to be sure students can explain to you what the problem is asking. Asking students to reread a problem does little good, but asking students to restate the problem in their own words helps them figure out what the problem is asking.

If you have struggling readers or English language learners, additional support may be needed. Explicit attention to vocabulary is critical. Graphic organizers (handouts with places to record needed information) can aid in reading and understanding the text. For more on supporting English language learners, see Table 4.1 in Chapter 4 and Chapter 6.

3. Establish Clear Expectations. There are two components to establishing expectations: how students are to work and what products they are to prepare for the discussion in the third part of the lesson. Each of these is essential; they cannot be skipped.

Whether or not you have students work in groups, it is always a good idea for students to have some opportunity to discuss their ideas with one or more classmates prior to sharing their thoughts in the *after* phase of the lesson. When students work alone, they have no one to look to for an idea or a way to get started if they are stuck. On the other hand, when students work in groups, there is always the possibility of students not contributing or of a dominating student overshadowing the others.

Buschman (2003b), a leader in mathematics education, suggests a *think-write-pair-share* approach, adding that students should first write or illustrate their solutions to the problem before sharing with a partner. With written work to share, the two students have something to talk about. Although appropriate for all students, the think-write-pair-share method is especially helpful for K–1 students who often do not know how to go about discussing a solution or even how to work together.

Teaching through problem solving requires that students focus on not just the solution, but how they reached that solution. Therefore, it is important to model and explain your expectations of what their final product might be. One expectation could be a written explanation of the problem. Writing supports student learning in mathematics (Pugalee, 2005; Steele, 2007) and can be a support to students during discussions, as they can refer to their own written explanation. Students may also or instead choose to prepare an illustration, diagram, or graph with or without a written explanation (see Figure 3.9). In this example, the student was asked to show how many different ways five people could be on the two stories of a house. Discuss with students what they might draw that will show their thinking. Just as it is important to ascertain that students understand the problem itself, it is also important to check that students have a clear understanding of the expectations for the product they will be sharing in the *after* phase of the lesson.

Kindergarten
How many ways can you show what 5 means?

Figure 3.9 A kindergarten student shows her thinking about ways to make 5.

The *During* Phase of a Lesson

In the *during* phase of the lesson students explore the focus task (alone, with partners, or in small groups). There are clear agendas that you will want to attend to:

1. *Let go!* Give students a chance to work without too much guidance. Allow and encourage students to embrace the struggle—it is an important part of doing mathematics.
2. *Listen actively.* Take this time to find out how different students are thinking, what ideas they are using, and how they are approaching the problem. This is a time for observation and assessment—not teaching.
3. *Provide appropriate hints.* Base any hints on students' ideas and ways of thinking. Be careful not to imply that you have the *correct* method of solving the problem.
4. *Provide worthwhile extensions.* Have something prepared for students who finish quickly.

Teacher Actions in the *During* Phase

With the exception of preparing for early finishers, these agendas can challenge teachers who tend to help too much. The teacher is a facilitator, carefully making decisions about when to let go and when to provide a hint. These decisions are based on carefully listening to students and knowing the content goals of the lesson.

1. Let Go! Once students understand what the problem is asking, it is time to LET GO. While it is tempting to "step in front of the struggle" in the *during* phase, you need to hold yourself back. Doing mathematics takes time, and solutions are not always obvious. It is important to communicate to students that spending time on a task, trying different approaches, and consulting each other are important to learning and understanding mathematics. When students are stuck, you can ask questions like, "Is this like another problem we have solved?" "Did you try to make a picture?" "What is it about this problem that is difficult?" This approach is effective in helping students because you are supporting their thinking, yet you are not telling them how to solve the problem.

Students will look to you for approval of their results or ideas. Avoid being the source of right and wrong. When asked if a result or method is correct, respond by saying, "How can you decide?" or "Why do you think that might be right?" or "Can you check that somehow?" Asking "How can we tell if that makes sense?" reminds students that answers without understanding are not acceptable.

Letting go also means allowing students to make mistakes. When you observe an error or incorrect thinking, do not correct it at this point. Students must learn from the very beginning that their mistakes can be opportunities for learning (Boaler & Humphreys, 2005). The best discussions occur when students disagree.

When students make mistakes, ask them to explain their process or approach to you. They may catch their own mistake. In addition, in the *after* portion of the lesson, students will have an opportunity to explain, justify, defend, and challenge solutions and strategies. This process provides an opportunity for mistakes and misconceptions to be treated as opportunities for learning.

2. Listen Actively. "Listening actively" means that you are trying to understand a student's approach to a problem. Consequently your questions must probe your students' thinking; the result may be unexpected. This is different from listening for a particular response or for what you know to be *the* answer and trying to elicit that response. This process is referred to as "funneling" students toward a response that approaches what you have in mind.

The *during* phase is one of two opportunities you have (the other is in the *after* phase) to find out what your students know, how they think, and how they are approaching the task you have given them. You might sit down with a group and simply listen for a while, letting the students explain what they are doing as you take occasional notes. If you want further information, try saying, "Tell me what you are doing" or "I see you have started to multiply these numbers. Can you tell me why you are multiplying?" You want to convey a genuine interest in what students are doing and thinking. This is not the time to evaluate or to tell students how to solve the problem.

"It's easy." "Let me help you." These two simple sentences send two disastrous messages to the student who hears them. For the student who asks for help, it is not easy! Students may think, "If it's easy and I can't get it, I must be stupid." The second sentence can also send a negative message. It implies, "You are not capable of doing this on your own. I have to help you."

Listening actively includes asking questions, such as the following:

- What ideas have you tried so far?
- Can you tell more more about . . . ?
- Why did you . . . ?
- How did you solve it?

By asking questions you find out where students are in their understanding of the concepts.

3. Provide Appropriate Hints. If a group or student is searching for a place to begin, a hint may be appropriate. You might suggest that the students try using a particular manipulative, draw a picture, or make a table if one of these ideas seems appropriate. You might also ask questions like those mentioned in the "Let Go!" section.

Concept: Percent Increase and Decrease
Grades: 6–8

In Fern's Furniture Store, Fern has priced all of her furniture at 20 percent over wholesale. In preparation for a sale, she tells her staff to cut all prices by 10 percent. Will Fern be making 10 percent profit, less than 10 percent profit, or more than 10 percent profit? Explain your answer.

For this problem, consider the following hints:

- Try drawing a picture or a diagram of something that shows what 10 percent off means.
- Try drawing a picture or a diagram that shows what 20 percent more means.
- Maybe you could pick a sample initial price and see what happens when you add 20 percent and then reduce 10 percent.
- Let's try a simpler problem. Suppose that you had 8 blocks and got 25 percent more. Then you lost 25 percent of the new collection.

Notice that these suggestions are not directive but, rather, they serve as starters. Even here, the choice of a hint is best made after listening carefully to what the student has been trying or thinking. After offering a hint, walk away. Don't hover or the student is apt to seek further direction.

4. Provide Worthwhile Extensions. Some students will always finish earlier than their classmates. Early finishers can often be challenged in some manner connected to the problem just solved without it seeming like extra work. (See Chapter 6 for discussion of strategies for students who are talented and gifted.) Ongoing extended projects should be used as another part of your mathematics program. Sometimes students finishing early can use this time to work on their mathematics projects.

Many good problems are simple on the surface. It is the extensions that are challenging. The area and perimeter task in this chapter is a case in point. Many students will quickly come up with one or two solutions. "I see you found one way to do this. Are there any other solutions? Are any of the solutions different or more interesting than others? Which of the shapes with a perimeter of 18 has the largest area and which has the smallest area? Does the perimeter always change when you add another tile?"

Questions that begin "What if . . . ?" or "Would that same idea work for . . . ?" are ways to extend student thinking in a motivating way. For example, "Suppose you tried to find all the shapes possible with a perimeter of 18. What could you find out about the areas?"

The value of students' solving a problem in more than one way cannot be overestimated. It shifts the value system in the classroom from answers to processes and thinking. It is a good way for students to make new and different connections.

As an example, consider the following sixth-grade problem.

Concept: Percent Increase and Decrease
Grades: 6–8

The dress was originally priced at $90. If the sale price is 25 percent off, how much will it cost on sale?

This is an example of a straightforward problem with a single answer. Many students will solve it by multiplying by 0.25 and subtracting the result from $90. The suggestion to find another way may be all that is necessary. Others may require specific directions: "How would you solve it using fractions instead of decimals?" "Draw me a diagram that explains what you did." "How could this be done in just one step?" "Think of a way that you could do this mentally."

Second graders will frequently solve the next problem by counting or using addition.

Concept: Addition and Subtraction
Grades: K–2

Maxine had saved up $9. The next day she received her allowance. Now she has $12. How much allowance did she get?

"How would you do that on a calculator?" and "Can you write two equations that represent this situation?" are ways of encouraging children to connect $9 + ? = 12$ with $12 - 9 = ?$.

The *After* Phase of a Lesson

In the *after* phase of the lesson, your students will work as a community of learners, discussing, justifying, and challenging various solutions to the problem all have just worked on. Here is where much of the learning will occur as students reflect individually and collectively on the ideas they have explored. It is challenging but critical to plan sufficient time for a discussion and make sure the *during* portion does not go on too long. The agendas for the *after* phase are easily stated but difficult to achieve:

1. *Promote a mathematical community of learners.* Includes all learners. Engage the class in productive discussion, helping students work together as a community of learners.

2. *Listen actively without evaluation.* Take this second major opportunity to find out how students are thinking—how they are approaching the problem. Evaluating methods and solutions is the duty of your students.

3. *Summarize main ideas and identify future problems to explore.* You can lay the groundwork for future activities as a natural part of this phase.

Teacher Actions in the *After* Phase

Be certain to plan ample time for this portion of the lesson and then be certain to *save* the time. Twenty minutes or more is not at all unreasonable for a good class discussion and sharing of ideas. It is not necessary for every student to have finished. This is not a time to check answers but for the class to share ideas.

Over time, you will develop your class into a mathematical community of learners where students feel comfortable taking risks and sharing ideas, where students and the teacher respect one another's ideas even when they disagree, where ideas are defended and challenged respectfully, and where logical or mathematical reasoning is valued above all. This atmosphere will not develop easily or quickly. You must teach your students about your expectations for this time and how to interact respectfully with their peers.

1. Promote a Mathematical Community of Learners That Includes All Children. NCTM in its *Standards* documents is very clear in expressing the belief that all children can learn important mathematics. This view is supported by a number of prominent mathematics educators who have worked extensively with at-risk populations (Campbell, 1996; Gutstein, Lipman, Hernandez, & Reyes, 1997; Silver & Stein, 1996; Trafton & Claus, 1994).

Because the needs and abilities of children are different, conducting a large group discussion that is balanced and that includes all children requires skill and practice. Rowan and Bourne (1994) offer excellent suggestions based on their work in an urban, multiethnic, low-socioeconomic school district. They emphasize that the most important factor is to be clear about the purpose of group discussion—that is, to share and explore the variety of strategies, ideas, and solutions generated by the class and to learn to communicate these ideas in a rich mathematical discourse. Every class has a handful of students who are always ready to respond. Other children learn to be passive or do not participate. So, step one is to be sure the discussion involves all students.

Considerable research into how mathematical communities develop and operate provides us with additional insight for developing effective classroom discourse (e.g., Rasmussen, Yackel, & King, 2003; Stephan & Whitenack, 2003; Wood, Williams, & McNeal, 2006; Yackel & Cobb, 1996). Suggestions from this research include the following:

- Encourage student–student dialogue rather than student–teacher conversations that exclude the class. "Juanita, can you answer Lora's question?" "Devon, can you explain that so that LaToya and José can understand what you are saying?" When students have differing solutions, have students work these ideas out as a class. "George, I noticed that you got a different answer than Tomeka. What do you think about her explanation?"
- Request explanations to accompany *all* answers. Soon the request for an explanation will not signal an incorrect response, as children will initially believe. Correct answers may not represent the conceptual thinking you assumed. Incorrect answers may only be the result of an easily corrected error. By requiring explanations, students learn that reasoning in mathematics is important and useful.
- Call on students for their ideas, often calling first on the children who tend to be shy or lack the ability to express themselves well. When asked to participate early and given sufficient time to formulate their thoughts, these reticent children can more easily participate and thus be valued. Asking "Who wants to explain their solution?" will result in the same three or four eager students raising their hands. Other students tend to accept that these students are generally correct and may be reluctant to offer ideas that are different from the well-known leaders. Use the *during* portion of a lesson to walk around the room and identify interesting solutions that will add to your discussion—including those that are incorrect. All students should be prepared to share as part of their everyday expectations.
- Encourage students to ask questions. "Pete, did you understand how they did that? Do you want to ask Antonio a question?"
- Be certain that your students also understand what you understand. Your knowledge of the topic may cause you to accept a less than clear explanation because you hear what the student means to say. Select important points in a student's explanation and express your own "confusion." "Carlos, I don't quite get why you subtracted 9 here in this step. Can you tell us why you did that?" Demonstrate to students that it is okay to be confused and that asking clarifying questions is appropriate. One teaching goal is for students to ask these questions without your input.

- Occasionally ask those who understand to offer explanations for others. "Thandie, perhaps you can explain this idea in your own words so that some of the rest of us can understand better." Don't assume that a student who says he or she understands really does.
- Move students to more conceptually based explanations when appropriate. For example, if a student says that he knows 4.17 is more than 4.1638, you can ask him (or another student) to explain why this is so. Another technique is to use a "fooler." With pretend confusion, ask, "How can this be? It seems like the longer decimal ought to be a larger number." Similarly, move students away from simply listing steps in their solutions. "I see *what* you did but I think some of us are confused about *why* you did it that way and why you think that will give us the correct solution."

2. Listen Actively Without Evaluation. By being a facilitator and not an evaluator, students will be more willing to share their ideas during discussions. This is your window into their thinking and therefore an assessment of their learning. Listen carefully to the discussion without too much interference. You can use this information to plan for tomorrow's lesson and in general to decide on the direction you wish to take in your current unit.

Try to take a neutral position with respect to *all* responses. Resist the temptation to judge the correctness of an answer. You can ask questions to help clarify a response—both right and wrong. When you say, "That's correct, Dewain," there is no longer a reason for students to evaluate the response. Had students disagreed with Dewain's response or had a question about it, they will not challenge or question it since you've said it was correct. Consequently, you will not have the chance to hear and learn from them. You can support student thinking without evaluation. "Does someone have a different idea or want to comment on what Dewain just said?"

Use praise cautiously. Praise offered for correct solutions or excitement over interesting ideas suggests that the students did something unusual or unexpected. This can be negative feedback for those who do not get praise. Comments such as "Good job!" and "Super work!" roll off the tongue easily. However, there is evidence to suggest that we should be careful with expressions of praise, especially with respect to student products and solutions (Kohn, 1993; Schwartz, 1996).

In place of praise that is judgmental, Schwartz (1996) suggests comments of interest and extension: "I wonder what would happen if you tried . . ." or "Please tell me how you figured that out." Notice that these phrases express interest and value the student's thinking.

There will be times when a student will get stuck in the middle of an explanation or when a response is simply not forthcoming. Be sensitive about calling on someone else to "help out." You may be communicating that the child is not capable on his or her own. Always allow ample time. You can sometimes suggest taking additional time to get thoughts together and promise to return to the student later—and then be certain to hear what the student figured out.

3. Summarize Main Ideas and Identify Future Problems. A wide variety of approaches can be used to summarize ideas. A whole class discussion can bring to light main ideas in students' words. There are numerous ways to share verbally, such as a partner exchange, where one partner tells one key idea and the other partner gives an example. Following oral summaries with individual written summaries is important to ensure that you know what each child has learned from the lesson. Exit slips, for example, are handouts with one or two prompts that ask students to explain the main ideas of the lesson (or ask for pictures from younger students). These are handed in as an "exit" from the math lesson. Or ask students to write a newspaper headline to describe the day's activity and a brief column to describe it. There are many different templates and writing starters that could be engaging for your students.

When ideas have been well developed, reinforce appropriate terminology, definitions, or symbols. Vocabulary should come after ideas have been established, not before. If a problem involves creating a procedure such as a method of computing, a strategy for basic facts, or a formula in measurement, record useful methods on the board. These can be labeled with the student's name and an example. These strategies are then available in future lessons for students to try.

Often someone will make a generalization or an observation that he or she strongly believes in but cannot completely justify. Untested ideas can be written up on the board, named after the student with the conjecture—for example, "Andrea's Hypothesis." Explain the meaning of *hypothesis* as an idea that may or may not be true. Testing the hypothesis may become a future problem, or the hypothesis may simply be kept on the board until additional evidence comes up that either supports or disproves it. For example, when comparing fractions, suppose that a group makes this generalization and you write it on the board: *When deciding which fraction is larger, the fraction in which the bottom number is closer to the top number is the larger fraction. Example:* $\frac{4}{7}$ *is not as big as* $\frac{7}{8}$ *because 7 is only 1 from 8 but 4 is 3 away from 7.* This is not an unusual conclusion, but it is not correct in all instances. A problem for a subsequent day would be to decide if the hypothesis is always right or to find fractions for which it is not right (counterexamples).

Even when students have not suggested hypotheses, discussions will often turn up interesting questions that can be used for a follow-up investigation to help further develop an emerging concept.