

- Using black and red tiles, model the following operations. Be sure to carry out the whole operation; do not short cut by changing signs. In each case, give the complete addition and subtraction sentence.

a)  $+5 + -2$

b)  $+9 + -4$

c)  $-9 + -4$

d)  $+9 - +4$

e)  $+9 - -4$

f)  $-5 - +3$

- Use your black and red tiles and form a minimal Array 1 with Edge Set I = -3 and Edge Set II = -4. Sketch and label your work.

- Using black and red tiles; fill in the table below

Collection	Total # of tiles	# Red tiles	# Black tiles	#R or #B tiles	Integer
1	12	5			+2
2		10	5		
3		3	10		
4	14		9		+4

- You start with a collection of tiles that includes 2 N pieces, and 7 red pieces. This collection is doubled and the net value of the new collection is 10. Use algebra pieces to show your work and figure out the value of N. Explain as you go.
- Create a minimal array with the net value of the edges sets at -3 and +5
- For the following collection, find the unknown number of tiles, sketch the resulting collection, and label it. If the collection can't exist, explain why not. If more than one possibility exists, be sure to show at least 2 examples and explain why there can be more than one collection.


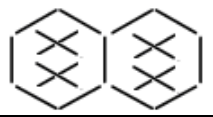

The collection contains an unknown number of even amount of red tiles, an unknown even amount of black tiles, and has a net value of 0.

- Use black and red tiles and edge pieces to solve the equation:  $+18 \div -3 = ?$  Model and explain each step. Which part of your model is the quotient? The dividend? The divisor? Is the array minimal? Why or why not? What is the corresponding multiplication sentence the array and edge sets could show? Explain how the array could also be a multiplication sentence.


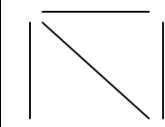

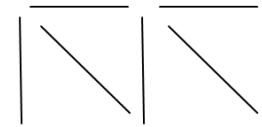
8. For the following multiplication and division questions, use black and red tiles to model each multiplication and division question. Sketch and label your work. Be sure to carry out whole operation; do not short cut by changing signs. In each case, give the completed multiplication and division sentence that the edge sets could show.

- a)  $+3 \times +5 = ?$                       d)  $+15 \div +5 = ?$                       g)  $5 \div 0 = ?$   
 b)  $+3 \times -5 = ?$                       e)  $+15 \div -5 = ?$                       h)  $0 \div 0 = ?$   
 c)  $-3 \times -5 = ?$                       f)  $-15 \div -3 = ?$





9. Loop / Pick count / Words / Symbols

			
Figure #	1	2	3
Total #			

10. Loop the toothpicks and figure out the equation for the sequence. Then describe and figure out the number of toothpicks in the 50th figure.

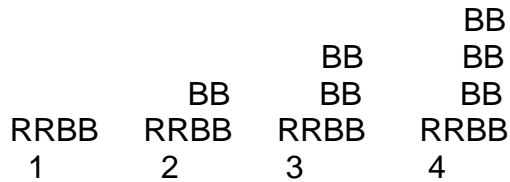
				
Figure #	1	2	3	4
Total #				

11. Use the toothpick figure and looping techniques to find the total number of toothpicks. Then answer the following:

				
Figure #	1	2	3	4
Total #				

- Convert looping ideas into words:
- Convert looping and words into symbols:
- How many toothpicks in the 105th figure?

12. Use the three step Numerical-Words-Symbols framework to analyze the tile figure sequence



- Numerical-Words-Symbols
  - Sketch and describe the  $n$ th figure
  - How many black tiles will be the 100th figure?
  - Create a T-table for the total number of tiles,  $C$ , for figure number inputs  $n = 1, 2, \dots, 6$  and  $n$ . Then plot the ordered pairs and label the axes with appropriate numbers.
  - 50 black tiles were added to a  $n$ th figure and have a total of 100 tiles. Which figure is it? Show algebra piece and symbolic work in a two column table.
13. Figure out the tile count using looping, turn this into words and then into symbols. Using this then describe and figure out the number of tiles in the 60th figure.

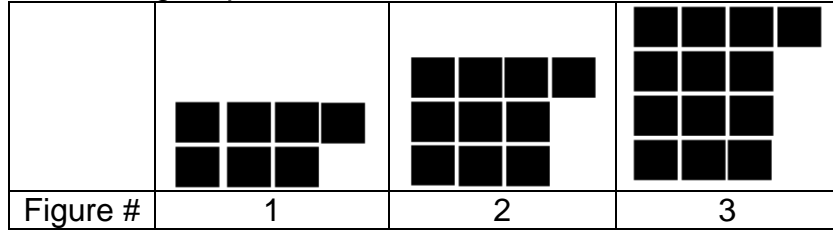
Figure #	1	2	3

14. Explain how you would loop this tile sequence. Convert your looping into words and a symbolic equation. Explain what the  $n$ th figure would look like and use black and red tiles and  $n$ -strips to model the  $n$ th figure. What kind of graph would this tile sequence make?

Figure #	1	2	3	4

15. Consider the equation  $T = 2n + 3$ . Use your black tiles to build figures 1,2,3,and 4 and make a consistent orientation for each figure. Explain how you know what to do as you go and then use algebra pieces to model the  $n$ th figure.
16.  $g(n) = 5n + 6$
- Show the  $n$ th figure.
  - Show the  $(n+1)$ st figure

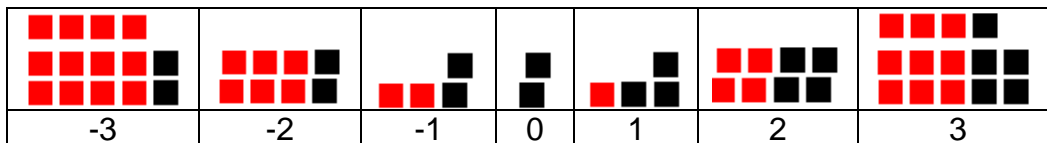
17.  $p(x) = 2x - 2$        $q(x) = 3x - 6$   
 Show the steps, with algebra pieces, to find at which point  $p(x)$  and  $q(x)$  intersect.
18. Use black and red tiles to build figures 1, 2, and 3 for  $T = 2n + 4$ . Explain and model what the  $n$ th figure would look like using the same orientation as your other figures. Describe what the  $(n + 1)$ st figure would look like. If two consecutive figures have a total of 42 black tiles, use algebra pieces to find which two consecutive figures they are.
19. Analyze the following sequence of black tiles



- Loop each figure.
  - Determine the total number of black tiles in each figure.
  - What is the symbolic equation for the figures?
  - Sketch the  $n$ th figure.
  - Describe the 100th figure. What does it look like and how many black tiles are in the 100th figure.
20. Solve the question: When is  $A(n) = B(n)$ ? Use your  $n$ th figures and  $n$ -strip models for  $A(n)$  and  $B(n)$ . Use a two column table to sketch your algebra piece work in the left column and in the right column write the corresponding symbolic steps.

$$A(n) = 4n + 2 \quad B(n) = 3n - 5$$

21. Explain how you would loop this each figure. What would the  $x$ th figure look like and what is the symbolic equation? Model the  $x$ th figure using black and red  $x$ -squares, white and opposite white  $x$ -strips, and black and red tiles. What is the  $y$ -intercept of the graph? Is the graph concave up or concave down? How do you know?



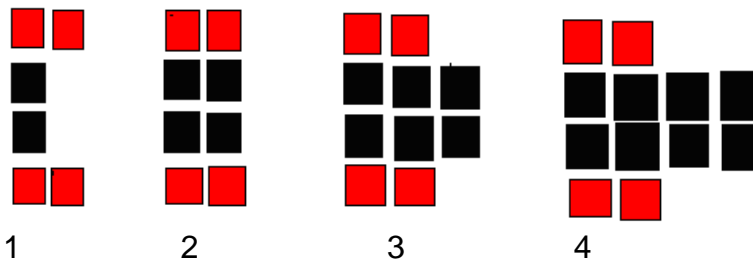
22. Why do we use  $x$ -strips?
23. Factor  $y = x^2 + 12x + 36$
24. Factor  $m^2 = 4m - 12$

25. Model the following sequence, Black-Red Fs tiles.  
 Step one: loop and number each figure and numerically determine the net value of the each figure (but don't simplify). Mark your number counts on the on the figures.

Step two: convert your looping ideas into words.

Step three: convert your looping and word ideas into symbols. Simplify your symbolic equation and check it for  $n=1, 2, 3$  and  $4$ .

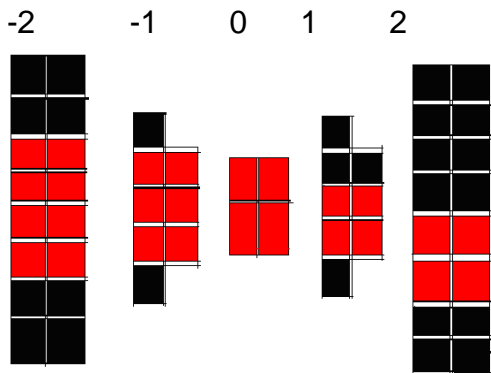
Black- Red tiles Fs



- Describe the 50<sup>th</sup> figure. What does it look like? What is  $F(50)$ ?
  - What does the  $n^{\text{th}}$  Black-Red F look like? Use your black and red  $n$ -strips and tiles, as needed, to model the figure; you do not need to show edge pieces. Sketch and describe the  $n^{\text{th}}$  Black-Red F. Label the pieces clearly.
  - Which Black-Red F will have a total of 2036 black and red tiles? Describe the figure including the number of black tiles and the number of red tiles. For this  $n$ , what is  $F(n)$ ?
  - If the collection of tiles in a certain Black-Red F figure is double and 28 black are added, the new collection of tiles will reduce to a minimal collection of 12 tiles. There are two such Black-Red F figures. One where the resulting 12 tiles are all red and one where the 12 tiles are all black. Which two Black-Red F figures are they? Use Algebra and /or algebra pieces to determine the solution. Clearly show your work and explain your thinking. Hint—there may be more than one solution.
26. Find the  $x$  intercepts, turning point,  $y$  intercept, range, and factored form of  $y = x^2 + 2x - 1$ . Sketch a graph and rectangular model of your findings.
27. Use algebra pieces to find out where  $x^2 - 3x - 12$  and  $-x^2 - 5x$  intersect. Explain your work as you go.
28. Analyze:  $y = x^2 - 6x + 6$ . Give the factored form,  $y$ -intercept,  $x$ -intercepts, range, and the  $y = a(x - h)^2 + k$  form. (If possible)

29.  $m(x) = x^2 + 5x + 6$ .
- Show the -2nd, -1st, 0th, 1st, and 2nd figures.
  - Show the xth figure.
  - What is the y-intercept? How does it relate to the algebra pieces in the xth figure?
  - What are the x-intercepts? How do you find them by using the black and red tiles and the edge pieces?

30. For the following extended sequence of tile figure with domain,  $\mathbb{R}$ :
- Analyze the extended sequence( the x value is below each figure)
  - Sketch the xth figure and give the symbolic for modeling, but don't include them in your sketch of the xth figure).
  - Fill in a variety of useful points on a t-table; try to find the y-intercept.
  - State the range of the function.

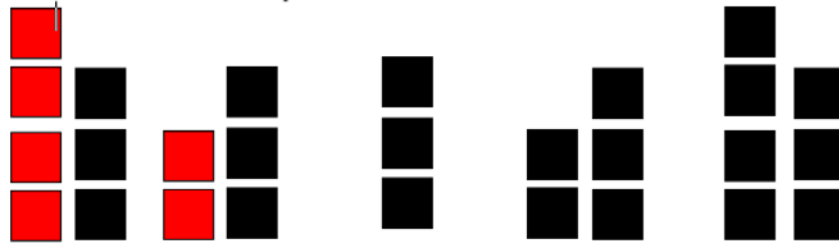


31. Analyze the sequence and figure out the equation. Figure out the y and range of the graph. Then figure out the factored form and x intercepts. Figure out the quadratic formula and turning point.

Figure #	1	2	3

32. Analyze  $y = x^2 + 5x + 4$  and find each of the following (if they exist). Graph  $y = x^2 + 5x + 4$  and label all of the key points: y-intercept; x-intercepts; turning point; range; factored form;  $y = a(x - h)^2 - k$  form

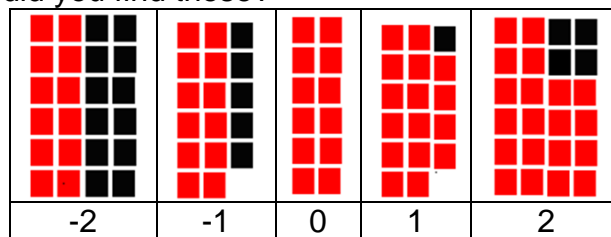
33. Analyze the following sequence of black and red tiles,  $S(n)$ , then determine the following:



Input:	-2	-1	0	1	2
Output:					

- Convert tile sequence into words:
- Convert tile sequence into symbols:
- Using white or opposite white  $n$ -strips and black and red tiles, sketch a representation of the  $n$ th  $S(n)$  tile figure.
- Graph the ordered pairs for  $S(n)$  on a graph, label axes with appropriate numbers, and connect points to draw the linear function.
- With equation  $T(n) = -2x + 1$ , plot on graph and determine where the two lines intersect by the two column method of algebra piece work and corresponding symbolic work. Write the intersection as an ordered pair.

34. Analyze the extended sequence and model the  $x$ th figure using black and red  $x$ -square, white and opposite white  $x$ -strips, and black and red tiles. What would be the symbolic formula for the sequence in  $y = ax^2 + bx + c$  form. Use algebra pieces to model a rectangle by adding zero pairs of white and opposite white strips. What does this rectangle tell you? What is the  $y$ -intercept of this graph? What are the  $x$ -intercepts? How did you find these?

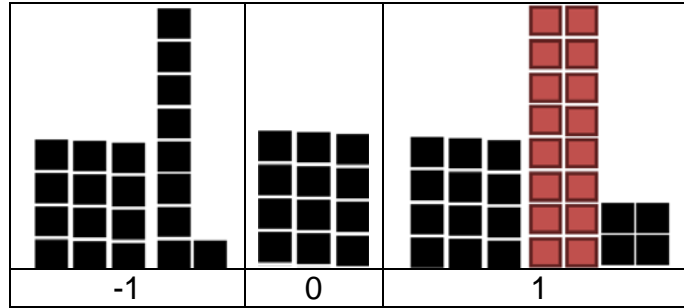


35. Tell me what happens to this equation in terms of shifting and graph it on some graph paper.

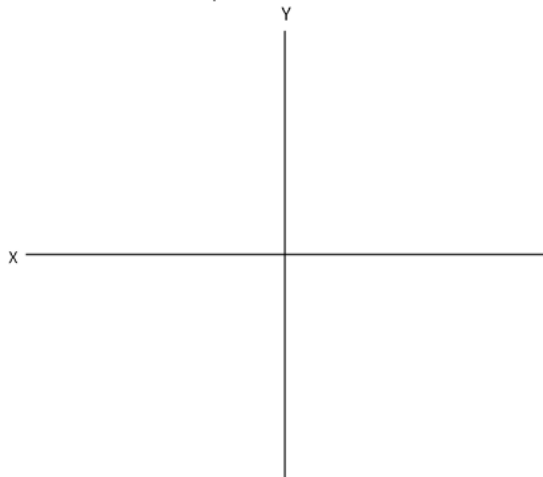
$$y = (x - 3)^2 - 12$$

How do you know what the  $y$  value is? How about the turning point? Point out these points on the graph. How could you find the  $x$ -intercepts? (you do not actually need to do the algebra, just state a why to find them).

36. For the extended sequence of the tile figures  $y=f(x)$ , with domain,  $\mathbb{R}$ .



- Analyze extended sequence, loop figure, give symbolic sequence, simplify into  $y=ax^2+bx+c$ , and sketch the  $x$ th figure.
- Find each of the following if they are possible and show your work.
- Graph the function  $y=f(x)$  and label the important points.

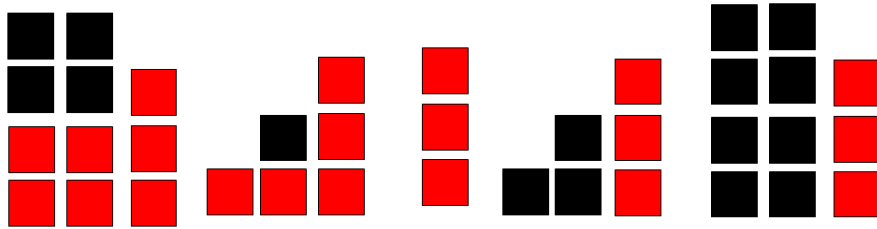


37. Analyze the equation  $y = x^2 - 6x + 8$ . What is the y-intercept? Use the "rectangle model" with algebra pieces to determine the factored form, x-intercepts and turning point. Then graph the function and state the range.

38. Sketch  $b(x) = 2|x + 4|$

- What is the range?
- What is the y intercept?
- What are the x-intercepts?
- Describe in terms of shifting  $y = |x|$
- What are the two linear equations for the two pieces of  $b(x) = 2|x + 4|$ ?

39. Analyze the given tile sequence,  $f(x)$ , and then determine the following:



Input:	-2	-1	0	1	2
Output:					

- Write the function  $f(x)$  in standard form:
- Find the  $y$ -intercept:
- Find the  $x$ -intercepts (if any):
- Determine the factored form (if one):
- Identify the turning point and range:
- Write function  $f(x)$  in vertex form:
- Graph the function  $f(x)$  and label key points and the axes appropriately.
- Is the function  $f(x)$  concave up or concave down?

40. Consider the function:  $y = 2|x + 1| - 3$

- Use your black and red tiles to build figures -2, -1, 0, 1 and 2 for a tile sequence that matches this function.
- Sketch the function  $y = 2|x + 1| - 3$  on graph paper. Label the axes with appropriate numbers.
- What is the range of  $y = 2|x + 1| - 3$ ? Explain.
- What is the  $y$ -intercept of  $y = 2|x + 1| - 3$ ?
- What are the  $x$ -intercepts of  $y = 2|x + 1| - 3$ ?
- How does the graph of  $y = 2|x + 1| - 3$  differ from the graph of  $y = |x|$ ?
- Write  $y = y = 2|x + 1| - 3$  as a split function with two linear components where neither component uses absolute value notation.

41.

- Create an algebra piece model for  $y = x^2 + 4x - 12$ . Use the model to analyze the function and find the  $y$ -intercepts,  $x$ -intercepts, turning points, range and factored form of the quadratic function.
- Create an algebra piece model for  $y = -x^2 + 9x - 14$ . Use the model analyze the function and find the  $y$ -intercepts,  $x$ -intercepts, turning points, range and factored form of the quadratic function.
- Use a two column table to show the algebra piece work and corresponding symbolic work for determining the points of intersection for the parabolas in part a and part b.
- Sketch the parabolas part a and part b together. Label each graph, mark all the key features and label the points of intersection of the two parabolas.