

Maple Optimization Written HW (file name: LastName1_LastName2_Optimize_W.mw)

- ✓ **Partner work required for the in class portion of this written Maple homework. If you don't finish in class, you may choose to work together out of class, or to work alone out of class.**

REQUIRED REFERENCE

Maple Directions and Reference Page (hard copy or linked to your class homework page)
Previous labs for defining functions and plotting reference

Open and save a new “document mode” Maple worksheet in your home directory.

Questions begin on the next page; there is no lab portion for the Maple Optimization lab

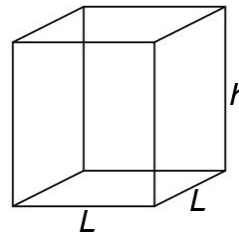
→→Credit for this assignment will only be given if you use the variables defined here. At most ½ credit if you use, for example, (x, f(x)) ←←

A: The Square Bottomed Liter Box

A square bottomed box, with a top, has a volume of 1000 cm^3 .

1. Write the formula for the **Surface Area** of the box, SA , function in terms of only L .

- Show your work by typing in words / math (don't hit enter) in Maple.
- Name the length of a side of the bottom of the box L
- Name the height of the box h
- Write surface area in terms of L and h
- Use the fixed volume to set up a constraint equation; solve for h , in terms of L
- Convert surface area in terms of L and h into SA in terms of L only



2. Give a plausible domain, $a < L < b$ for the **Surface Area** function. Use the constraint (1000 cm^3) to help determine b (let $h = 1$ for an approximation, since $h = 0$ is not helpful). Show and explain your calculations clearly for full credit.

3. Graph the **Surface Area** function in Maple as follows.

- Define SA in terms of L by using the Maple code: $SA := L \rightarrow (\text{formula})$
- In plot, use $SA(L)$
- Define your plot window using $L=a..b$ (instead of x)
- Adjust the range of your plot window ($y=c..d$) until you have a really nice graph.

4. What value of L appears to produce the smallest value of SA ? Use the graph and (right click) *Plot > Probe Info > Nearest datum* trace tool to active the Maple trace tool to determine an **approximate** answer.

5. What should the value of the derivative of the **Surface Area** function at the value of L you approximated in the previous question be? Determine your answer by looking at the graph and then use derivative notation to express your answer.

6. Use Maple to find the derivative of the **Surface Area** function, $SA'(L)$.

7. Use the derivative of the **Surface Area** function to find the exact value of L where the surface area is minimized (find the critical point of SA).

You might find the Maple command: $\text{fsolve}(\{SA'(L), y = 0\})$ to be convenient here.

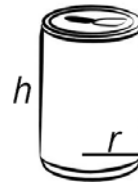
8. Based on your calculations, what are the dimensions of the box that uses the least amount of packaging to contain 1000 cubic centimeters? What is the actual minimal surface area? Give all the dimensions and the surface area for full credit. Show your work (let Maple do the math).

9. How do you know you have minimized the surface area of the box? Explain in terms of calculus. Prove it using the second derivative test. Show your work and explain.

B: The Soda Can

A cylindrical can of volume 355 cm^3 (the volume of a soda can) has closed ends.

1. Write the formula for the **Surface Area**, S , function in terms of only r .
 - Show your work by typing in words / math (don't hit enter) in Maple.
 - Name the radius of the can r
 - Name the height of the can h
 - Write surface area in terms of r and h
 - Convert surface area in terms of r and h into S in terms of r only



2. Give a plausible domain, $a < r < b$ for the **Surface Area** function Use the constraint (355 cm^3) to determine b (again, let $h = 1$ for an approximation, since $h = 0$ is not helpful). Show and explain your calculations clearly for full credit.
3. Graph the **Surface Area** function, $S(r)$, in Maple, using the domain $a < r < b$.
4. What value of r appears to produce the smallest value of S ? Use the graph and *Plot > Probe Info > Nearest datum* trace tool to determine an **approximate** answer.
5. What is the value of the derivative of the **Surface Area** function at the value of r you approximated in the previous question? Determine your answer by looking at the graph and use derivative notation to express your answer.
6. Use Maple to find the derivative of the **Surface Area** function, $S'(r)$.
7. Use the derivative of the **Surface Area** function to find the exact value of r where the surface area is minimized (find the critical point of S).
8. Based on your calculations, what are the dimensions of the soda can that use the least amount of material to construct that contains 355 cubic centimeters of soda? What is the actual minimal surface area? Give all the dimensions and the surface area for full credit. Show your work (let Maple do the math).

Note, *simplify(%)* entered on the line after a calculation that gives the answer in terms of π will give the decimal for that number
9. How do you know you have minimized the surface area of the soda can? Explain in terms of calculus. Prove it using the second derivative test. Show your work and explain.

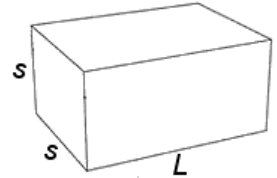
C: The Postal Package

Consider a box-shaped package with square ends. The US Postal Service will accept such a package if the sum of its length plus its girth is less than or equal to 108 inches.

→Girth is the perimeter of the square end←

1. Write the formula for the **Volume**, V , function in terms of s . You will need to assume the sum of the length of the box plus its girth of the box is equal to 108 to simplify to one variable.

- Show your work by typing in words / math (don't hit enter) in Maple.
- Name the length of a side of the square end s
- Name the length of the box L
- Write the volume in terms of s and L
- Convert volume in terms of s and L into V in terms of s only



2. Give a plausible domain, $a < s < b$ for the **Volume** function. Use the constraint (108 inches) to determine b . Show and explain your calculations clearly for full credit.
3. Graph the **Volume** function, $V(s)$, in Maple, using the domain $a < s < b$.
4. What value of s appears to produce the largest value of V ? Use the graph and *Plot > Probe Info > Nearest datum* to determine an **approximate** answer.
5. What is the value of the derivative of the **Volume** function at the value of s you approximated in the previous question? Determine your answer by looking at the graph and use derivative notation to express your answer.
6. Use Maple to find the derivative of the **Volume** function, $V'(s)$.
7. Use the derivative of the **Volume** function to find the exact value of s where the volume is maximized. (find the critical point of V). Remember you can adjust *fsolve*.
Example `fsolve({V'(s), y = 0}, {s = -1..1, y = -1..1})`
8. Based on your calculations, what are the dimensions of the box with the greatest volume that meets U.S. Postal regulations? What is the actual greatest volume? Give all the dimensions and the surface area for full credit. Show your work (let Maple do the math).
9. How do you know you have maximized the volume of the package? Explain in terms of calculus. Prove it using the second derivative test. Show your work and explain.

Email subject line: Maple Optimize