Chapter 12: Rational Functions

12.1 SIMPLIFYING RATIONAL FUNCTIONS
Rational Function

- Function that can be written as a ratio
- Where \( P(x) \) and \( Q(x) \) are polynomials
- As long as \( Q(x) \) is not zero

\[
f(x) = \frac{P(x)}{Q(x)}
\]
Reducing fractions

- Recall that any number divided by itself is 1.
- Recall that fraction bar is a grouping symbol.
- Can reduce fraction if the same FACTOR appears above and below the fraction bar.
- If the same term appears, it is part of a sum, not a factor, and cannot be reduced.
Reducing fractions

- **Like this**
  \[
  \frac{12}{20} = \frac{4 \cdot 3}{4 \cdot 5} = \frac{4}{4} \cdot \frac{3}{5} = 1 \cdot \frac{3}{5} = \frac{3}{5}
  \]

  \[
  \frac{5 + 4}{14 + 4} = \frac{9}{18} = \frac{9}{9} \cdot \frac{1}{2} = 1 \cdot \frac{1}{2}
  \]

- **Not like this**
  \[
  \frac{5 + 4}{14 + 4} \neq \frac{5}{14}
  \]
Reducing Rational Functions

- Factor when possible
- Reduce factors common to numerator and denominator
- Remember that if all factors cancel from either or both, 1 is always another factor present
Reduce this function

- Factor the polynomials
- Cancel factors matching above and below fraction bar: because they equal 1

\[
f(x) = \frac{x^2 - 4}{x - 2} = \frac{(x - 2)(x + 2)}{(x - 2)} = \frac{x - 2}{x - 2} \cdot (x + 2) = x + 2
\]
Evaluate functions

- $f(3)$
- $= 5$
- $f(-4)$
- $= -2$
- $f(2)$
- Does not exist
Domain of functions

- Some are all real numbers
- Some are all real numbers except where denominator is zero
- Often this is not apparent in the simplified function
- Sometimes it appears in graph as vertical asymptote: a gap in the function
- This is an EXCLUDED VALUE of the rational expression
Reduce this function

- Factor the polynomials
- Cancel factors matching above and below fraction bar: because they equal 1

\[
f(x) = \frac{5x + 15}{x^2 + 5x + 6} = \frac{5(x + 3)}{(x + 2)(x + 3)} = \frac{5}{x + 2}
\]
Reduce this function

- Factor the polynomials
- Cancel factors matching above and below fraction bar: because they equal 1

\[
f(x) = \frac{2x^2 - 6x - 20}{2x^2 - 50} = \frac{2(x^2 - 3x - 10)}{2(x^2 - 25)} = \frac{2(x - 5)(x + 2)}{2(x - 5)(x + 5)} = \frac{(x + 2)}{(x + 5)}
\]