



Chapter 10: Exponential functions

**10.1 INTEGER EXPONENTS**

**10.2 FRACTIONAL EXPONENTS**

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# Integer

- “The numbers”
- Number that has no fractions. Can be written without decimal or rational component.
- ...— 3, — 2, — 1, 0, 1, 2, 3...
- “...” means “continuing on in the same pattern without end”

# Exponent

- The number of times the base is taken as a factor
- Don't apply exponent to values not part of base
- Base includes numbers or variables attached to the exponent directly
- There may be factors of the exponential that are not part of the base
- Watch for ( ) or not

# Negative exponent

- May or may not be a negative number
- Solve by using quotient rule of exponents

$$\frac{b^3}{b^5} = b^{3-5} = b^{-2}$$

# Negative exponent

- Solve by using quotient rule of exponents

$$\frac{b^3}{b^5} = b^{3-5} = b^{-2}$$

- Implies moving base across fraction bar because the two are equivalent

$$\frac{b^3}{b^5} = \frac{bbb}{bbbbbb} = \frac{1}{bb} = \frac{1}{b^2}$$

Can only combine LIKE bases

$$\frac{b^{-4}}{b^3} = b^{-4-3} = b^{-7} = \frac{1}{b^7}$$

$$\frac{b^{-4}}{c^3} = \frac{1}{b^4 c^3}$$

# Simplify Expressions with Exponents

- No parentheses
- Any values are combined into single value without exponents
- Any variable appears only a single time
- Every exponent is positive

# No parentheses

- Rule:
  - Multiply the exponent inside by the one outside

$$(b^{-3})^4 = b^{-12} = \frac{1}{b^{12}}$$

- Outside exponent means the base, inside, is the factor to be taken that many times

$$(b^{-3})^4 = (b^{-3})(b^{-3})(b^{-3})(b^{-3}) = b^{-12} = \frac{1}{b^{12}}$$

# No parentheses

- Deal with the outside negative first

$$\left(2b^{-5}\right)^{-3} = \frac{1}{\left(2b^{-5}\right)^3}$$

# No parentheses

- Multiply the exponents inside by the one outside

$$\frac{1}{(2b^{-5})^3} = \frac{1}{2^3 b^{(-5)3}}$$

# Values reported without exponents

$$\frac{1}{2^3 b^{-15}} = \frac{1}{8b^{-15}}$$

$$\frac{1}{8b^{-15}} = \frac{b^{15}}{8}$$

- And no negative exponents

# Can be very tedious when complicated

- Steps are not hard
- Keep track of where you are

$$\left( \frac{18b^{-4}c^7}{6b^{-3}c^2} \right)^{-4}$$

- Reduce inside first

$$= \left( 3b^{-4-(-3)}c^{7-2} \right)^{-4} = \left( 3b^{-1}c^5 \right)^{-4}$$

# Now deal with ( )

- Multiply each inside exponent by the outside exponent

$$\left(3b^{-1}c^5\right)^{-4} = 3^{-4}b^{(-1)(-4)}c^{5(-4)}$$

# Then deal with negative exponents

- Any base with a negative exponent needs to be moved across the fraction bar

$$3^{-4} b^4 c^{-20} = \frac{b^4}{3^4 c^{20}} = \frac{b^4}{81c^{20}}$$

- And the numerical exponent needs to be calculated

# Scientific notation

- Move decimal point to have a single digit to left of it
- Multiply by power of 10 to make it equivalent expression
- Common error is to raise number to power instead of 10 to power

$$7 \times 10^3 \quad 7 \times 10^{-3}$$

- $7 \times 1000 = 7,000$
- Move decimal point three places to right
- $7 \times 1/1000 = 0.007$
- Move decimal point three places to left
- Think about a number line!!

845,000,000      0.0000382

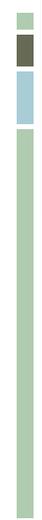
- $8.45 \times 10^8$
- 8 from how many decimal places
- $10^8$  is a very large number...
- $3.82 \times 10^{-5}$
- 5 from how many decimal places
- $10^{-5}$  is a very small number...

778,000,000      0.000012

- $7.78 \times 10^8$
- 8 from how many decimal places
- Did you apply the correct sign of exponent
- $10^8$  is a very large number...LOOK!!
- $1.2 \times 10^{-5}$
- 5 from how many decimal places
- $10^{-5}$  is a very small number...LOOK!!



# Rational exponents: fractions in exponent!!

- Follow all the same rules for exponents
  - Do not assume fractional exponent is a fractional number!!
  - Denominator of exponent means a root
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# Rules for exponents: page 594

- The last rule: doesn't matter if n gets switched with m
- Important note for the rational exponents

$$b^m b^n = b^{m+n}$$

$$\frac{b^m}{b^n} = b^{m-n}$$

$$(bc)^n = b^n c^n$$

$$\left(\frac{b}{c}\right)^n = \frac{b^n}{c^n}$$

$$(b^m)^n = b^{m \cdot n}$$

# Rational exponent $9^{(1/2)}$

- Note you can write 9 as  $3^2$
- Apply rule for nested exponents

$$9^{(1/2)} = (3^2)^{(1/2)} = 3^{2(1/2)} = 3^1 = 3$$

# Watch out for negative base

- If root is odd, retain the negative

$$(-64)^{1/3} = [(-4)^3]^{1/3} = -4$$

- If root is even, it is not a real number

$$(-16)^{1/4} = [(-2)^4]^{1/4} = 2$$

- But  $2^4 = 16$  so above is not true